

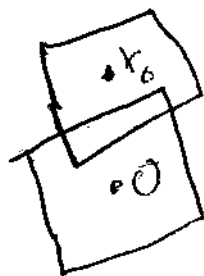
## Geometry of Linear Systems

Let  $A$  be  $m \times n$  real matrix  
let  $a_1, \dots, a_m$  be the rows  
Then  $Ax=0 \iff a_1 \cdot x=0$  and  $a_2 \cdot x=0$   
and  $\dots$  and  $a_m \cdot x=0$   
 $\implies x$  is orthogonal (normal)  
to  $a_1, \dots, a_m$

Solving  $Ax=0$  (e.g. by Gaussian elimination)  
gives a parametric form for this set  
of solutions

$$t_1 u_1 + \dots + t_s u_s$$

The set of solutions to  $Ax=b$   
is the translate of the set of solutions  
to  $Ax=0$  by any solution  $x_0$  to  $Ax=b$   
i.e.  $A(x-x_0)=0$  iff  $Ax=b$   
if  $Ax_0=b$



Cross Product  $\mathbb{R}^3$   
 $u, w$  vectors in  $\mathbb{R}^3$  (AKA 3 space)

$$u = (u_1, u_2, u_3) \quad w = (w_1, w_2, w_3)$$

$$u \times w = \left( \begin{vmatrix} u_2 & u_3 \\ w_2 & w_3 \end{vmatrix}, -\begin{vmatrix} u_1 & u_3 \\ w_1 & w_3 \end{vmatrix}, \begin{vmatrix} u_1 & u_2 \\ w_1 & w_2 \end{vmatrix} \right)$$