2A03 Assignment 1

§2.1 22. $F(x,y) = (a,a)$

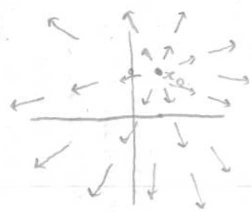
where $\|(a,a)\| = \|(x,y)\|$

i.e. $\sqrt{2a^2} = \|(x,y)\|$

i.e. $a = \frac{\|(x,y)\|}{\sqrt{2}}$

so $F(x,y) = \left(\frac{\|(x,y)\|}{\sqrt{2}}, \frac{\|(x,y)\|}{\sqrt{2}} \right)$

26.



pointing away from x_0, with magnitude distance from x_0

§2.2 14. Let L_1 and L_2 be level curves

say $L_i = \{ \underline{x} \mid f(\underline{x}) = c_i \}$

and suppose they intersect, say $\underline{a}$ is in both.

Then $f(\underline{a}) = c_1$ and $f(\underline{a}) = c_2$

so $c_1 = c_2$

so $L_1 = L_2$

So if two level curves intersect, they are the same curve.

§2.3 6. $f(x,y) := \frac{(y-2)}{(x+1)}$

If $\lim_{(x,y)\to(-1,2)} f(x,y)$ exists, and equals c say,

then for some $\delta > 0$, setting $\underline{x}_0 := (-1,2)$,

$|f(\underline{x}_0 + \underline{h}) - c| < 1$ whenever $\|\underline{h}\| < \delta$.

But

$f(\underline{x}_0 + (h, \frac{\delta}{2})) = f(-1+h, 2+\frac{\delta}{2}) = \frac{\delta/2}{h} \xrightarrow[h \to 0^+]{} \infty$

so for small enough h, $\|(h, \frac{\delta}{2})\| < \delta$

but $f(\underline{x}_0 + (h, \frac{\delta}{2})) > c+1$

X

§2.3 34. $\underline{F}(\underline{x}) = \underline{x} \times \underline{a}$ say $\underline{a} = (a,b,c)$

$F(x,y,z) = (yc - zb, xc - za, xb - ya)$

components are clearly continuous everywhere

so $\underline{F}$ is continuous on $\mathbb{R}^3$

$\underline{G}(\underline{x}) := \frac{\underline{x} \times \underline{a}}{\|\underline{x} \times \underline{a}\|} = h(F(\underline{x}))$ where $h(\underline{y}) = \frac{\underline{y}}{\|\underline{y}\|}$

h is cont^s except at 0

composition of cont^s is cont^s

so $\underline{G}$ is cont^s when $\|\underline{x} \times \underline{a}\| \neq 0$

i.e. when $\underline{x} \times \underline{a} \neq \underline{0}$

i.e. when $\underline{x}$ is not collinear with $\underline{a}$.

When $\underline{x}$ is collinear with $\underline{a}$, G isn't even defined.

So $\underline{G}$ is cont^s precisely off the line through $\underline{a}$

(assuming $\underline{a} \neq \underline{0}$; if $\underline{a} = \underline{0}$, then $\underline{G}$ is nowhere defined)

§2.4 10. $f(x,y,z) = xe^{yz^2}$

$f_x = \frac{d}{dx}(xe^{yz^2}) = e^{yz^2}$

$f_y = xz^2 e^{yz^2}$

$f_z = 2zxy\, e^{yz^2}$

§2.4 64. $f(x,y) = \frac{3xy}{x-2y}$

$\nabla f = (f_x, f_y) = \left(\frac{3y}{x-2y} - \frac{3xy}{(x-2y)^2}, \frac{3x}{x-2y} + 2\frac{3xy}{(x-2y)^2} \right)$

$= \left(\frac{-6y^2}{(x-2y)^2}, \frac{3x^2}{(x-2y)^2} \right)$

$\nabla f(3,1) = \left(\frac{-6}{9}, \frac{27}{9} \right) = \left(\frac{-2}{3}, 3 \right)$

$f(3,1) = 9$

so tangent plane at $(3,1)$

passes through $(3,1,9)$

has x-slope $\frac{-2}{3}$

and y-slope 3

so writing its equation as $z = ax + by + c$

we have: $a = \frac{-2}{3}$

$b = 3$

$9 = a3 + b1 + c = \frac{-2}{3}3 + 3 \cdot 1 + c$

$= -6 + 3 + c = c - 3$

so $c = 12$

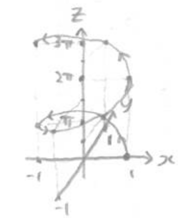
$\boxed{z = \frac{-2x}{\ldots}}$ [illegible]

§2.5 ~~20~~ 18

$\underline{c}(t) = (\cos t, \sin t, t)$ $t \in [0, 3\pi]$

$(\cos t, \sin t)$ is point on unit circle t radians around

So $\underline{c}(t)$ traces out corkscrew ascending anticlockwise from $(1,0,0)$:



§2.5 20. Similar, but rotated:

§2.6 6. $f(x,y) = ye^{xy}$

$\nabla f = (y^2 e^{xy}, (1+xy)e^{xy})$

$\underline{c}(t) = (t, \ln t)$

$\underline{c}'(t) = (1, \frac{1}{t})$

chain rule: $D(f \circ \underline{c}) = (Df \circ \underline{c}) \cdot D\underline{c}$

i.e. $(f \circ \underline{c})' = (\nabla f \circ \underline{c}) \cdot \underline{c}'$

i.e. $\frac{d}{dt} f(\underline{c}(t)) = \nabla f(\underline{c}(t)) \cdot \underline{c}'(t)$

$= \nabla f(t, \ln t) \cdot (1, \frac{1}{t})$

$= ((\ln t)^2 e^{t \ln t}, (1 + t\ln t)e^{t \ln t}) \cdot (1, \frac{1}{t})$

$= ((\ln t)^2 + \frac{1}{t} + \ln t) e^{t \ln t}$

directly: $f(\underline{c}(t)) = f(t, \ln t) = t e^{t \ln t}$

so $\frac{d}{dt} f(\underline{c}(t)) = \frac{d}{dt} (\ln t) e^{t \ln t}$

$= \frac{e^{t \ln t}}{t} + (\ln t)(\ln t + \frac{t}{t}) e^{t \ln t}$

$= ((\ln t)^2 + \frac{1}{t} + \ln t) e^{t \ln t}$

§2.6 26.

$\underline{F}(\underline{x}) = A \cdot \underline{x}$ $\quad \underline{G}(\underline{x}) = B \cdot \underline{x}$

$D\underline{F}(\underline{x}) = A$ $\quad D\underline{G}(\underline{x}) = B$

so $D(\underline{G} \circ \underline{F})(\underline{x}) = (D\underline{G} \circ \underline{F}) \cdot D\underline{F}$

$= B \cdot A$

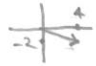
matrix mult^n

2A03 Ass1 contd

§2.7 30. height $(x,y) = h(x,y) = 3000 - x^2 + 2y^2 - xy$

$\nabla h(x,y) = (2x - y, 4y - x)$

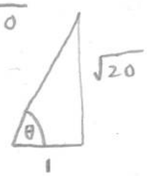
(a) $\nabla h(2,0) = (4, -2)$



so climber should climb that way.
("east-by-southeast"!)

(b) $\|\nabla h(2,0)\| = \sqrt{20}$

$\theta = \operatorname{asin}(\sqrt{20})$



(c) level curve

$h(x,y) = 2996$

$\Leftrightarrow -x^2 + 2y^2 - xy = -4$

$\Leftrightarrow x^2 + xy - 2y^2 = 4$

which is a hyperbola.