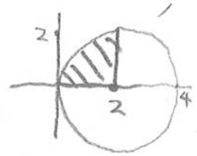
2A03 Ass 5

§16.4 6. $4x - x^2 = -(x-2)^2 + 4$, so $y^2 = 4x - x^2 \iff y^2 + (x-2)^2 = 2^2$

equation of circle of radius 2 centre $(2,0)$

so $D = \{(x,y) \mid 0 \le x \le 2,\ 0 \le y \le \sqrt{4x - x^2}\}$

=



$= \{(r,\theta) \mid 0 \le r \le 2,\ \pi/2 \le \theta \le \pi\}$

in polar co-ords centred at $(2,0)$

so $(x,y) = (r\cos\theta + 2,\ r\sin\theta)$

Jacobian det^{nt} $= r$

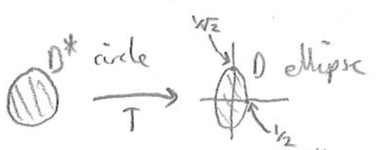
so $\displaystyle\int_0^2 \int_0^{\sqrt{4x-x^2}} (x^2+y^2)^{3/4}\, dy\, dx$

$\displaystyle = \int_{\pi/2}^{\pi}\int_0^2 \left((r\cos\theta + 2)^2 + (r\sin\theta)^2\right)^{3/4} r\, dr\, d\theta$

Jacobian term

which would be painful to evaluate, but luckily we weren't asked to!

28.



$T(u,v) = (\frac{u}{\sqrt{2}}, v)$ will work:

$D^* =$ circle of radius $\frac{1}{\sqrt{2}}$

if $x = \frac{u}{\sqrt{2}}$, $y = v$

then $u^2 + v^2 \le \frac{1}{2}$

$\iff 2x^2 + y^2 \le \frac{1}{2}$

$\iff 4x^2 + 2y^2 \le 1$

$\|DT\| = \frac{1}{\sqrt{2}}$

so $\displaystyle\iint_D 5\, dA = \iint_{D^*} \frac{5}{\sqrt{2}}\, dA^*$

$= \frac{5}{\sqrt{2}}(\text{area of } D^*)$

$= \frac{5\pi}{2\sqrt{2}}$

§7.1 8. cone

for given z, circle of radius $|z|$

so could parametrise each circle polarly:

$\underline{r} : \mathbb{R} \times [0, 2\pi] \to S$

$\underline{r}(z,\theta) = (|z|\cos\theta,\ |z|\sin\theta,\ z)$

§7.2

12. $\underline{r}(t,u) = (\cos t\,(1 + u\cos(t/2)),\ \sin t\,(1 + u\cos(t/2)),\ u\sin(t/2))$

$\underline{N} = \underline{r}_t \times \underline{r}_u = (-\sin t\,(1+u\cos\frac{t}{2}) - \frac{u}{2}\cos t\sin\frac{t}{2},\ \cos t\,(1+u\cos\frac{t}{2}) - \frac{u}{2}\sin t\sin\frac{t}{2},\ \frac{u}{2}\cos\frac{t}{2})$

$\times (\cos t\cos\frac{t}{2},\ \sin t\cos\frac{t}{2},\ \sin\frac{t}{2})$

$= (\cos t\sin\frac{t}{2}(1+u\cos\frac{t}{2}) - \frac{u}{2}\sin t,\ \sin t\sin\frac{t}{2}(1+u\cos\frac{t}{2}) + \frac{u}{2}\cos t,\ -\cos\frac{t}{2}(1+u\cos\frac{t}{2}$

(using $\sin^2 + \cos^2 = 1$ repeatedly)

so $\underline{N}(0,u) = (0,\ 0,\ -(1+u))$

$\underline{N}(2\pi,u) = (0,\ 0,\ 1+u)$

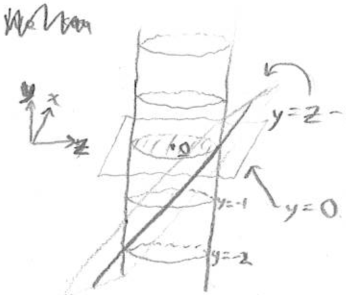
so normal flips direction as we go round

so this does not parametrise an oriented surface

corresponding to the Möbius strip not being orientable.

§7.3 14.

S =



could e.g. parametrise with (y,θ)

$D = \{(y,\theta) \mid -2 \le y \le 0,\ -\arccos(y+1) \le \theta \le \arccos(y+1)\}$

but let's be clever:

we're getting exactly half of the cylinder between $y=-2$ and $y=0$

so area $= \frac{1}{2}((0-(-2))\,2\pi r) = \underline{2\pi}$

2A03 Ass 5

§7.4 4.

parametrise : $\underline{r}: [0,1]^2 \to \mathbb{R}^3$

$\underline{r}(u,v) = (2,2,1) + u\underline{U} + v\underline{V}$ where

$\underline{V} = (0,-2,3)$

$\underline{U}(-2,0,0)$

$\underline{N} = \underline{r}_u \times \underline{r}_v = \underline{u} \times \underline{v} = (0,6,4)$

so $\iint_S \underline{F}\cdot d\underline{S} = \int_0^1\int_0^1 \underline{F}(\underline{r}(u,v))\cdot\underline{N}\,du\,dv$

$= \int_0^1\int_0^1 (1,-2,4)\cdot(0,6,4)\,du\,dv$

$= 4$

14. Can consider S as graph of function

$z = 1 - y/4 - x/8$

so, parametrising by (x,y),

$\underline{N} = (-f_x, -f_y, 1) = (1/8, 1/4, 1)$

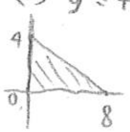
"first octant" means $x,y,z \geq 0$

so $x \geq 0$, $y \geq 0$, $z = 1 - y/4 - x/8 \geq 0$

$\Leftrightarrow 2y + x \leq 8$

$\Leftrightarrow y \leq 4 - x/2$

so D = triangle



so flux $= \iint_D \underline{F}\cdot\underline{N}\,dA = \int_0^8\int_0^{4-x/2} \left(\frac{x^2 y}{8} - \frac{(x+y)^2}{2} - (1 - \frac{y}{4} - \frac{x}{8})^2 x\right) dy\,dx$

20. 0, by symmetry!

or by noting $\text{div}\,\underline{F} = 0$ and using Gauss!

If we wanted to actually compute the surface integrals, we should take care to have the outwards orientation for the outer sphere, inwards for the inner.

§8.2 2. Approximation; assume that $\text{div}\,\underline{F} = 4$ throughout the little sphere

so then flux $= \iiint_{\text{sphere}} \text{div}\,\underline{F}\,dV \approx \iiint_{\text{sphere}} 4\,dV$ = volume of sphere $\times 4$

$= \frac{4^2}{3}\pi(0.1)^3$

4. 0, by symmetry!

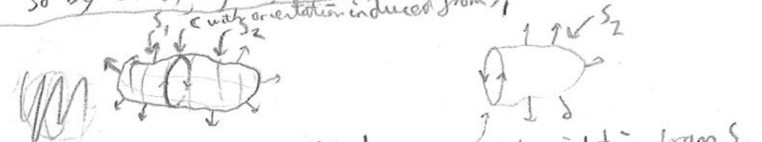
or: $\text{div}\,\underline{F} = \text{div}(\underline{c}\times\underline{r}) = \text{div}(c_2 z - c_3 y, \ldots)$

$= 0+0+0$

$= 0$

so by Gauss, flux is 0

§8.3 2.



By Stokes, $\iint_{S_1} \text{curl}\,\underline{F}\cdot d\underline{S} = \int_C \underline{F}\cdot d\underline{s}$

but orientation on S_2 induces opposite orientation on C

so By Stokes, $\iint_{S_2} \text{curl}(\underline{F}\cdot d\underline{S} = -\int_C \underline{F}\cdot d\underline{s}$

so $\iint_{S_1} \text{curl}(\underline{F}\cdot d\underline{S} = -\iint_{S_2} \text{curl}(\underline{F}\cdot d\underline{S}$

so $\iint_S \text{curl}\,\underline{F}\cdot d\underline{S} = \iint_{S_1} \text{curl}(\underline{F}\cdot d\underline{S} + \iint_{S_2} \text{curl}(\underline{F}\cdot d\underline{S}$

$= 0$

12. Stokes; $\int_C \underline{F}\cdot d\underline{s} = \iint_{\text{cone oriented up}} \text{curl}(\underline{F}\cdot d\underline{S}$

$= \int_0^1\int_0^{2\pi} \text{curl}(\underline{F}\cdot(-\underline{N})\,d\theta\,dz$

$= \int_0^1\int_0^{2\pi} (z^2\sin^2\theta + z)\,d\theta\,dz$

$= \int (\pi z^2 + 2\pi z)\,dz = \frac{\pi}{3} + \pi = \frac{4\pi}{3}$

parametrise as in §7.1:8.

$\underline{r}(\theta, z) = (z\cos\theta, z\sin\theta, z)$

$\underline{N}(\theta, z) = (-z\sin\theta, z\cos\theta, 0)\times(\cos\theta, \sin\theta, 1)$

$= (z\cos\theta, z\sin\theta, -z)$

oops, wrong orientation; so we use $-\underline{N}$

$\text{curl}\,\underline{F} = (2y, 0, 1)$

$(\int_0^{2\pi} \sin^2\theta\,d\theta = \pi)$