MATH 2A03 Fall 2014 Midterm 2 SAMPLE TEST

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Midterm 2 SAMPLE TEST
Duration of test: 50 minutes
McMaster University
2014

Name:_____

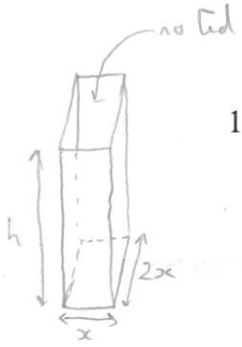
Student No.:_____

There are 4 questions each worth 5 marks. The test is marked out of 15. You may answer all 4 questions, but **only the 3 highest marks will count towards your total mark**.
You are not required to show your working or reasoning except where it is explicitly asked for, but partial credit may be given for correct working despite an incorrect answer.
Please be sure to head each sheet of paper you write on with your name and student number.

You may consult your textbook and course notes during the test.
Only the McMaster standard student calculator may be used for this test.
The maximal number of marks on this test is 15.

Score

Question	1	2	3	4	Total
Points	5	5	5	5	15
Score					



1. [5] Suppose that, in order to save the world, you must design a cuboid box with no lid, whose height is h cm, whose length is x cm, whose width is $2x$ cm, whose volume is 10 cm^3, and whose surface area is as small as possible.

   (a) [1] Write down formulae in terms of h and x for the volume and surface area of the box. Bear in mind that the box has no top, so has only 5 sides.

   (b) [4] What should be the height of the box?

2. [5] The position in metres $\mathbf{c}(t)$ of an object at time t seconds is given by the path

$$\mathbf{c} : [0,2] \to \mathbb{R}^3$$
$$\mathbf{c}(t) = (t^{\frac{3}{2}}, t, t).$$

   (a) [4] Find a parametrisation by arc-length for the path. You do *not* need to simplify your answer.

   (b) [1] At what time has the object travelled a distance of 1 metre since time $t = 0$? You do *not* need to simplify your answer.

3. [5] Consider the curve C in the plane consisting of the points (x,y) which satisfy the equation

$$x^3 = (y-1)^2$$

   and are such that $y \geq 2$ and $x \leq 4$.

   Consider the orientation of the curve which starts at $(1,2)$ and ends at $(4,9)$.

   Let $\mathbf{F} : \mathbb{R}^2 \to \mathbb{R}^2$ be the vector field

$$\mathbf{F}(x,y) = (y-1, x).$$

   By considering the map $\mathbf{c}(t) = (t^2, t^3+1)$ with a suitable domain, or otherwise, find the path integral of $\mathbf{F}$ along C with the given orientation.

4. [5] Consider the vector field

$$\mathbf{F}(x,y,z) = (0, \frac{y}{y^2+z^2}, \frac{z}{y^2+z^2}).$$

   (a) [1] Find $\mathrm{dom}\,\mathbf{F}$, the domain of definition of $\mathbf{F}$.

   (b) [1] Is $\mathrm{dom}\,\mathbf{F}$ simply connected?

   (c) [3] Is $\mathrm{dom}\,\mathbf{F}$ a gradient vector field? Briefly explain your answer.