MATH 2A03 Fall 2014 Midterm 2

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Midterm 2
Duration of test: 50 minutes
McMaster University
2014

Name:____

Student No.:______________________________________

There are 4 questions each worth 5 marks. The test is marked out of 15. You may answer all 4 questions, but **only the 3 highest marks will count towards your total mark**.
Please write your answers on the paper provided, not directly on the question paper.
You should show your working. Partial credit may be given for correct working despite an incorrect answer.
Please be sure to head each sheet of paper you write on with your name and student number.

You may consult your textbook and course notes during the test.
Only the McMaster standard student calculator may be used for this test.
The maximal number of marks on this test is 15.

Score

Question	1	2	3	4	Total
Points	5	5	5	5	15
Score					

1. [5] What is the greatest value attained by the function $f(x,y) = (y+2)x$ on the circle of radius 2 centred at the origin, $x^2 + y^2 = 4$?

2. [5] Consider the curve parametrised by $\mathbf{c} : [0,2] \to \mathbb{R}^3$

$$\mathbf{c}(t) = (t, \frac{2\sqrt{2}}{3}t^{\frac{3}{2}}, \frac{1}{2}t^2).$$

   What is the curvature of the curve at $(1, \frac{2\sqrt{2}}{3}, \frac{1}{2})$? You may find it helpful to recall the formula $(t+1)^2 = t^2 + 2t + 1$.

   Hint: if you calculate correctly, you should be able to write your final answer in the form $2^{(\frac{n}{m})}$.

3. [5] Consider the ellipse E with equation

$$4x^2 + y^2 = 4$$

   (a) [2] Find positive numbers a and b such that the path $\mathbf{c} : [0,2\pi] \to \mathbb{R}^2$

$$\mathbf{c}(t) = (a\cos(t), b\sin(t))$$

   is a parametrisation of the ellipse E.

   (b) [3] What is the average value of the distance from $(0,0)$ to a point on E? Express your answer in terms of definite integrals of functions of one variable - you do not need (and will not be able!) to actually evaluate these integrals.

4. [5] Let $\mathbf{c}$ be the path $\mathbf{c} : [0,1] \to \mathbb{R}^3$

$$\mathbf{c}(t) = (t, t^2, t^3).$$

   Suppose $\mathbf{F} : \mathbb{R}^3 \to \mathbb{R}^3$ is a C^1 vector field, defined on the whole of $\mathbb{R}^3$, and suppose $\operatorname{curl}\mathbf{F} = \mathbf{0}$ everywhere on $\mathbb{R}^3$.

   However, you only know the value of $\mathbf{F}(x,y,z)$ when $x = y = z$: for all $t \in \mathbb{R}$, the value at (t,t,t) is

$$\mathbf{F}(t,t,t) = (t^2, t^2, t^2).$$

   Find

$$\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{s}.$$