2A03 Ass3 3.1:20, 3.3:26, 3.4:24, 5.1:20, 5.2:4,12,28

§3.1 20 Local solutions for x to $(x^2+y^2)^2-x^2+y^2=0$

· Rearrange; $0=(x^2)^2+2x^2y^2+y^4-x^2+y^2$

$$=(x^2)^2+(2y^2-1)x^2+(y^4+y^2)$$

· Solve this quadratic in x^2:

$$x^2=\tfrac12\left((1-2y^2)\pm\sqrt{(2y^2-1)^2-4(y^4+y^2)}\right)$$

$$=\tfrac12\left((1-2y^2)\pm\sqrt{1-8y^2}\right)$$

so $x=\pm\sqrt{\tfrac12((1-2y^2)\pm\sqrt{1-8y^2})}$

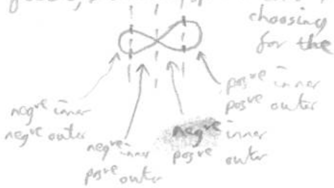
This holds at every point on the curve, but is not the graph of a f^n

Near $(1,0)$: x is >0, so want pos^ve outer square root, and to get 1 when $y=0$ need pos^ve inner square root too.

So $x=\sqrt{\tfrac12((1-2y^2))+\sqrt{1-8y^2}}$ is the graph of a f^n, and defines the curve near $(1,0)$

Near $(-1,0)$, it's $x=-\sqrt{\tfrac12((1-2y^2))+\sqrt{1-8y^2}}$

In fact, the curve splits into 4 such graphs, by choosing different signs for the square roots.



§3.3 26 BAD QUESTION!

$(-4)^{3/2}$ is not defined!

§3.4 24. $c(x):=(x,\,x+\ln x)$

$c'(x)=(1,\,1+1/x)$

$\|c'(x)\|=\sqrt{1^2+(1+1/x)^2}$

$=\sqrt{2+2/x+1/x^2}$

$$T_c(x)=\frac{c'(x)}{\|c'(x)\|}=\frac{(1,\,1+1/x)}{\sqrt{2+2/x+1/x^2}}$$

$$T_c'(x)=\Big(\left(\tfrac{-2}{x^2}-\tfrac{2}{x^3}\right)\tfrac{-1}{2}\left(2+2/x+1/x^2\right)^{-3/2},\ (1+1/x)(1/x^2+1/x^3)(2+2/x+1/x^2)^{-3/2}+\tfrac{-1}{x^2}(2+2/x+1/x^2)^{-1/2}\Big)$$

$$=\frac{\left(1/x^2+1/x^3,\ (1+1/x)(1/x^2+1/x^3)+\frac{-1}{x^2}(2+2/x+1/x^2)\right)}{(2+2/x+1/x^2)^{3/2}}$$

$$=\frac{\left(1/x^2+1/x^3,\ -1/x^2\right)}{(2+2/x+1/x^2)^{3/2}}$$

$$\kappa_c(x)=\frac{\|T_c'(x)\|}{\|c'(x)\|}=(2+2/x+1/x^2)^{-2}\sqrt{(1/x^2+1/x^3)^2+(-1/x^2)^2}$$

$$=(2+2/x+1/x^2)^{-2}\sqrt{2/x^4+2/x^5+1/x^6}$$

$$=(2+2/x+1/x^2)^{-2}\sqrt{x^{-4}(2+2/x+1/x^2)}$$

$$=(2+2/x+1/x^2)^{-3/2}x^{-2}$$

so curvature at $(1,1)$ = $\kappa_c(1)=5^{-3/2}$

and lim as $x\to\infty$ of curvature is

$$\lim_{x\to\infty}\kappa_c(x)=0$$

§5.1 20 (a) No, ϕ not 1-1 ($\phi(-1)=\phi(1)$)

(b) Yes; $c(\phi(t))=(2t,\,4t^2)$

The curve is the same: the bit of the graph of $y=x^2$ between $x=-1$ and $x=2$

(c) No, ϕ is not diff^ble at 0

(d) Yes; $c(\phi(t))=(-3t,\,9t^2)$

the curve is still the same (but the orientation is reversed)

§5.2 4. $$\int_c f\,ds=\int_0^{\pi/2}xy\sqrt{(-2\sin t)^2+(3\cos t)^2+5^2}\,dt$$

$$=\int_0^{\pi/2}6\sin t\cos t\sqrt{25+(4\sin^2t+4\cos^2t)+5\cos^2t}\,dt$$

$$=6\int_0^{\pi/2}\sin t\cos t\sqrt{29+5\cos^2t}\,dt$$

$$=-6\int_1^0u\sqrt{29+5u^2}\,du\qquad u=\cos t$$

$$=-3\int_1^0\sqrt{29+5v}\,dv\qquad v=u^2$$

$$=-3\int_1^0\frac{d}{dv}\left(\tfrac15\tfrac23(29+5v)^{3/2}\right)dt$$

$$=\tfrac25\left((29+5)^{3/2}-29^{3/2}\right)$$

12 $$\int_c f\,ds=\int_{c_1}f\,ds+\int_{c_2}f\,ds$$

$$=\int_0^1(t+2t^2)\sqrt{1^2+4t^2}\,dt+\int_0^1(1+2-4t-t^2)\sqrt{1+4+1}\,dt$$

$c_1(t)=(t,t^2,0)$
$c_2(t)=(1,1-2t,t)$

$$=\int_0^1t(1+4t^2)^{1/2}dt+2\int_0^1t^2(1+4t^2)^{1/2}dt+\sqrt6\int_0^1(3-4t-t^2)\,dt$$

$$\int_0^1t(1+4t^2)^{1/2}dt=\tfrac12\int_0^1(1+4u)^{1/2}du\qquad u=t^2$$

$$=\tfrac1{12}(5^{3/2}-1)$$

$\int_0^1t^2(1+4t^2)^{1/2}dt=$ ... sinh inverse sub... sorry...

$\int\cosh^2\theta\sinh^2\theta\,d\theta$ $t=\tfrac12\sinh\theta$

$=\int\sinh^2\theta(1+\sinh^2\theta)^{1/2}\cosh\theta\,d\theta$ $\frac{dt}{d\theta}=\tfrac12\cosh\theta$

$=8\int t^2(1+4t^2)^{1/2}dt$

so $\int_0^1t^2(1+4t^2)^{1/2}dt=8\int_0^{\operatorname{asinh}1}\cosh^2\theta\sinh^2\theta\,d\theta$

$$=8\left(\left[\cosh\theta\,\frac{\sinh^3\theta}{3}\right]_0^{\operatorname{asinh}1}-\frac13\int_0^{\operatorname{asinh}1}\sinh^4\theta\,d\theta\right)$$

(by parts, $\int\cosh\theta\sinh^2\theta\,d\theta=\int u^2du$ $u=\sinh\theta$
$=\frac{u^3}{3}=\frac{\sinh^3\theta}{3}$)

Ah, but for $\sinh^4$ you need to use the "double-angle" formulae, which I don't think are covered in 1st year calculus. Poor student

§5.2 28

Parametrise : $\underline{c}(\theta) = (\cos\theta, \sin\theta)$ $\theta \in [0, \pi/2]$

$$\text{Mass} = \int_0^{\pi/2} (3 + 2\cos\theta\sin\theta)\, d\theta$$

$$= \frac{3\pi}{2} + 2 \underbrace{\int_0^{\pi/2} \cos\theta \sin\theta \, d\theta}$$

$$= \int_0^1 u \, d\theta \qquad (u = \sin\theta)$$

$$= 1/2$$

$$= \frac{3\pi}{2} + 1$$