Mathematics 3Q03: Mid-Term Test 1
Instructor: Dr. D. Pelinovsky
Date: October 4, 2012, 9:30-10:20

NAME: ______________________________

STUDENT NUMBER: __________________________

Instruction: Textbooks, lecture notes, and McMaster calculators are allowed on the test. The duration of this test is 50 minutes. The test paper has 4 questions, where the marks are specified next to each question. Total marks = 20. For full mark, show all your work.

Problem	Points	Score
1	6	
2	6	
3	6	
4	2	
Total	20	

1. Consider the function

$$f(x,y) = \frac{xy(x^2+y^4)}{x^2+y^2}, \quad (x,y) \in \mathbb{R}^2 \backslash \{(0,0)\}.$$

[2] (a) Show that f is continuous at $(0,0)$ and find the limit

$$f(0,0) := \lim_{(x,y)\to(0,0)} f(x,y).$$

[2] (b) Compute the x-partial derivative at $(0,0)$ by using the definition

$$\frac{\partial f}{\partial x}(0,0) := \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}.$$

[2] (c) Compute $\frac{\partial f}{\partial x}(x,y)$ by using chain rule and prove that it is continuous at $(0,0)$.

2. Consider the function

$$f(x, y) = x^4 - 2x^2 + y^2 + 3,$$

in the disk $D = \{(x, y) \in \mathbb{R}^2 : \quad x^2 + y^2 \leq 1\}$.

[2] (a) Find critical points by using the first derivative test.

[2] (b) Identify local minima and maxima in D by using the second derivative test.

[2] (c) Find the global minimum and global maximum in D.

3. Consider the function

$$f(x, y) = e^x \sin(2x - y), \quad (x, y) \in \mathbb{R}^2.$$

[2] (a) Find the gradient vector of f at any point (x, y).

[2] (b) Find the directional derivative of f at the point $(1, 2)$ in the direction of the line $y = 3x - 1$, for increasing values of x.

[2] (c) Find the unit direction vector $\mathbf{u}$ at the point $(1, 2)$, along which the function has the maximal increase.

4. TRUE or FALSE:

[1] (a) The function $Q(x,y) = 3xy + x$ is the quadratic approximation of the function $f(x,y) = x(1+y)^3$ at the point $(0,0)$.

[1] (b) If a function $f(x,y)$ is continuous at the point (x_0, y_0) in its domain, then it is differentiable at the point (x_0, y_0).