MATH 3TP3 Assignment #2

Due: Friday, 5th of October, in class

1. For each of the following wff's, determine whether it is a tautology, a contradiction, or neither, by drawing up a truth table.

   (a) $\langle P \supset \langle Q \supset P\rangle\rangle$

   (b) $\langle\langle P \supset Q\rangle \supset \langle Q \supset P\rangle\rangle$

   (c) $\langle\langle P \supset Q\rangle \supset \langle \sim Q \supset \sim P\rangle\rangle$

   (d) $\langle\langle P \supset \sim P\rangle \wedge \langle \sim P \supset P\rangle\rangle$

   (e) $\langle\langle\langle\langle P \supset Q\rangle \supset R\rangle \supset \langle\langle R \supset P\rangle \supset Q\rangle\rangle \supset \langle\langle Q \supset R\rangle \supset P\rangle\rangle$

2. Write down a tautology in which the propositional variables P, Q and R all occur, and which does not use any logical operation other than $\supset$ (i.e. none of '$\sim$','$\wedge$' or '$\vee$' occur).

3. This question is set on Smullyan Island. Each inhabitant of Smullyan Island is either a Knight or a Knave. Every statement made by a Knight is true, while every statement made by a Knave is false. They are all well-trained in formal logic.

   (a) Lost in a maze, walking down a corridor, you encounter a pair of inhabitants of the island.

   Having seen this kind of puzzle before, you know that one of them is a knight and the other is a knave; but you don't know which is which.

   As you approach the first, he points in the direction you were going and exclaims "The exit lies that way, or I'm a knight!".

   You approach the second, and she points in the same direction and also exclaims "The exit lies that way, or I'm a knight!"

   Let P be the proposition that the first inhabitant is a knight, and let Q be the proposition that the exit lies in the direction they pointed. Using truth tables, or otherwise, determine whether you should proceed or turn back.

   (b) You reach a fork in the maze; one path leads East, the other West. Three inhabitants of the island are standing guard. You approach and ask for directions.

One of them announces "My two colleagues here are both knights.". Then one of the other two, scowling and pointing at the one who spoke, says "Don't trust him, he never tells the truth. My friend here, on the other hand, " he continues, gesturing at the third inhabitant, "certainly is a knight.". "Yes," the third says, "I *never* lie!".

Bemused, you start to turn back the way you came - but then you hear one of the three (you're not sure which!) say "The exit lies along the Eastern path.", and another (though it could have been the same one) say "The exit lies along the Western path". They refuse to say anything further.

What should you do? Explain your reasoning.

Hint: first try to use the three statements of the three inhabitants in the second paragraph to work out which of the three are knights and which knaves. You might like to use truth tables for this.

Bonus Question Having finally escaped the maze, you come to a bush of delicious-looking red berries. While you ponder their safety, an inhabitant passes by, dressed in the formal regalia of the Balda sect. You studied your guidebook well, and know that this sect, in addition to being split into Knights and Knaves, and having excellent botanical knowledge, has a unique peculiarity. Although they understand English perfectly, they speak only in their own language. You know only two words of the language: "bal" and "da". One of them means "yes" and the other "no", but you forget which is which! The inhabitant (who remember is either a knight or a knave) looks hurried, but she would probably answer a single question if you ask one quickly.

(a) Find a single yes-no question you can ask such that the answer will allow you to determine whether the berries are poisonous.

(b) For bonus points, find a way to phrase the question in natural-sounding English, without any awkward phrasings involving "if and only if" or suchlike.