MATH 3TP3 Assignment #4

Due: Friday, 12th of October, in class

1. Show that the following wffs corresponding to rules of PROP are tautologies:

   (i) $\langle\langle P \wedge \langle P \supset Q\rangle\rangle \supset Q\rangle$

   (ii) $\langle\langle\sim P \wedge \sim Q\rangle \supset \sim \langle P \vee Q\rangle\rangle$

   (iii) $\langle\langle\sim \langle P \vee Q\rangle \supset \langle\sim P \wedge \sim Q\rangle\rangle\rangle$

2. (a) Suppose σ and τ are wffs, and suppose that σ is a tautology. Let α be the result of replacing every occurence of the propositional variable P in σ with τ. Show that α is a tautology.

   (b) Explain how you can deduce from part (a) and question 1 that

   $$\langle\langle\langle P \supset \langle R \wedge P\rangle\rangle \wedge \langle\langle P \supset \langle R \wedge P\rangle\rangle \supset \langle Q \supset P\rangle\rangle\rangle \supset \langle Q \supset P\rangle\rangle$$

   is a tautology.

3. For each of the following wffs, either write out a complete PROP-derivation, or explain why no PROP-derivation exists:

   (i) $\langle\sim \langle P \wedge Q\rangle \supset \langle\sim P \vee \sim Q\rangle\rangle$

   (ii) $\langle\langle P \supset Q\rangle \supset \langle Q \supset P\rangle\rangle$

   (iii) $\langle\langle P \supset \langle Q \wedge \sim Q\rangle\rangle \supset \sim P\rangle$

4. Show that each of the following tautologies is a PROP-theorem (we need this for our completeness proof):

   (i) $\langle P \supset \sim\sim P\rangle$

   (ii) $\langle\sim P \supset \sim P\rangle$

   (iii) $\langle\langle P \wedge Q\rangle \supset \langle P \wedge Q\rangle\rangle$

   (iv) $\langle\langle\sim P \wedge \sim Q\rangle \supset \sim \langle P \wedge Q\rangle\rangle$

   (v) $\langle\langle P \wedge Q\rangle \supset \langle P \supset Q\rangle\rangle$

   (vi) $\langle\langle P \wedge \sim Q\rangle \supset \sim \langle P \supset Q\rangle\rangle$
   Hint: this one is slightly tricky. I suggest you first see how to deduce $\sim \langle\sim P \vee Q\rangle$ from $\langle P \wedge \sim Q\rangle$, and $\langle\sim P \vee Q\rangle$ from $\langle P \supset Q\rangle$. Both of these involve finding ways to introduce double-negations.

(vii) $\langle\langle\sim P \wedge \sim Q\rangle \supset \langle P \supset Q\rangle\rangle$

(viii) $\langle\langle P \wedge Q\rangle \supset \langle P \vee Q\rangle\rangle$

(ix) $\langle\langle\sim P \wedge Q\rangle \supset \langle P \vee Q\rangle\rangle$

(x) $\langle\langle\sim P \wedge \sim Q\rangle \supset \sim \langle P \vee Q\rangle\rangle$