MATH 3TP3 Assignment #5
Due: Wednesday, October 24, in class

1. For each of the following strings, determine if it is a term of TNT, a wff of TNT, or neither. Indicate the free variables of those strings that are wff's. You do not need to produce parse trees for the wff's and terms.
   (a) $(SS0 + S0 + a)$
   (b) $S((a - b) \cdot Sa')$
   (c) $\sim (S(a + a') \cdot SS0)$
   (d) $(S0 \cdot S0) = (S0 + S0)$
   (e) $\sim \forall a : \langle\langle a = b \wedge a' = SSb\rangle \vee \exists b' : \exists c' : c' = (c' \cdot b')\rangle$
   (f) $\langle(S0 + SS0) < SSSS0 \vee \exists a : (a + a = SSS0)\rangle$
2. Produce wff's that express each of the following:
   (a) The number x is a perfect cube.
   (b) There are no solutions to $x^3 + y^3 = z^3$ in the natural numbers.
   (c) The sum of any two odd numbers is even.
   (d) There are infinitely many pythagorean triples (natural numbers x, y, z such that $x^2 + y^2 = z^2$).
3. Translate each of the following wff's into English. For those wff's that are sentences, decide if they are true.
   (a) $\exists a : \exists b : (a \cdot a) = (Sa + (b + b))$
   (b) $\forall a : \exists b : \exists c : \exists d : a = ((b \cdot b) + ((c \cdot c) + (d \cdot d)))$
   (c) $\exists a : (a \cdot a) = S((S0 + SS0) \cdot S(SS0 \cdot SS0))$
   (d) $\exists a' : \exists b' : \langle(c \cdot a') = a \wedge (c \cdot b') = b\rangle$
   (e) $\langle\alpha \wedge \forall d : \langle\beta \supset \exists e : (d + Se) = c\rangle\rangle$, where α is the wff from part (d) and β is obtained from α by replacing all occurrences of the variable c by the variable d.