MATH 3TP3 Assignment #7

Due: Friday, November 9, in class

1. On Pages 225-227 of GEB, there is a long TNT-derivation. Read it carefully, and get a feel for how it works. As the answer to this question, please write honestly "I have read the derivation carefully, and have a feel for how it works".

2. Show that the following sentences are theorems of TNT.

   (i) $\forall x : Sx = (x + S0)$.

   (ii) $\forall x : (S0 \cdot x) = x$.

   (iii) $\forall x : \forall y : \forall z : ((x + y) + z) = (x + (y + z))$. Hint: try induction on z.

   (iv) $\forall x : \langle (x \cdot x) = 0 \supset x = 0 \rangle$.

3. Show that the sentence $\forall x : \langle x = 0 \vee \exists y : Sy = x \rangle$ is not a TNT'-theorem. Hint: Consider adding a new element "∞" to the natural numbers.

Bonus Question Consider the one-element structure in the language of arithmetic: this is the structure $\mathcal{D} = \langle \{0\}; 0, S', +', \cdot' \rangle$ where successor, addition and multiplication are defined in the only possible ways:

$$S'0 = 0; \;\; 0 +' 0 = 0; \;\; 0 \cdot' 0 = 0.$$

Consider the system obtained by adding the single axiom

$$\forall x : x = 0$$

to PRED. Show that this system is complete for $\mathcal{D}$, i.e. that every sentence which is true in $\mathcal{D}$ is a theorem of PRED+$\{\forall x : x = 0\}$.