MATH 3TP3 Assignment #8
Due: Friday, November 16, in class

Consider the following formal system S (which is a fragment of the formal system version of PROP given in the lecture notes):

- Alphabet: $\mathtt{P}$ $\mathtt{'}$ $\supset$ $\langle$ $\rangle$ $\vdash$ $\mathtt{?}$ $\mathtt{W}$ $\mathtt{F}$ $\mathtt{F}$ $\mathtt{:}$
- Axioms: $\vdash$, $\mathtt{WFF{:}P}$
- Rules:

(I)	$\mathtt{WFF{:}P}x$	$\mapsto$	$\mathtt{WFF{:}P'}x$	(well-formedness)
(II)	$(\mathtt{WFF{:}}x,\ \mathtt{WFF{:}}y)$	$\mapsto$	$\mathtt{WFF{:}}\langle x \supset y\rangle$	(well-formedness)
(III)	$(x \vdash y,\ \mathtt{WFF{:}}z)$	$\mapsto$	$x?z \vdash z$	(push)
(IV)	$(x \vdash y,\ \mathtt{WFF{:}}z)$	$\mapsto$	$x?z \vdash y$	(carry-over)
(V)	$(x?y \vdash z,\ \mathtt{WFF{:}}y)$	$\mapsto$	$x \vdash \langle y \supset z\rangle$	(pop)
(VI)	$(x \vdash \langle y \supset z\rangle,\ \ x \vdash y)$	$\mapsto$	$x \vdash z$	(detachment)

(a) Give a derivation in this system of

$$\vdash \langle \mathtt{P} \supset \langle\langle \mathtt{P} \supset \mathtt{P'}\rangle \supset \mathtt{P'}\rangle\rangle$$

Hint: First think how you'd derive it in PROP, then try to translate the PROP-derivation into an S-derivation. Remember that the empty string is a valid value for the string variables x, y, and z in the production rules of S.

(b) Define a Gödel numbering of strings in this alphabet, and explain in some detail why there exists a TNT-formula $Theorem_S(x)$ with one free variable x which is true in the natural numbers precisely when x takes value the Gödel number of a theorem of S.

Your explanation should be sufficiently detailed that it is clear that such a TNT-formula exists. One way to give a sufficiently detailed explanation would be to actually write out such a formula without any abbreviations, but that is likely to be rather painful and prone to error. I suggest you make use of abbreviations like those in lectures.

You may assume the β Lemma, and furthermore may assume as understood the existence and properties of the formulae

$$ListElement(x, y, z);\ \ Exp(x, y, z);\ \ HasLength(x, y);\ \ Concat(x, y, z)$$

defined in lectures, and any notation defined in lectures.

When finding formulae corresponding to the production rules, I would consider it acceptable to consider only (III) and (VI) in detail and say that the others can be handled similarly.