MATH 3TP3 Assignment #2 Solutions

1. Truth tables.

2. e.g. $\langle P \supset \langle Q \supset \langle R \supset P \rangle\rangle$

3. (a) One of them is a knave, so is making a false claim. That claim is a disjunction, so both disjuncts are false: the exit isn't ahead, and the speaker isn't a knight (as you already knew). So turn back!

   (b) There's a bug in this question. As the question is stated, it describes an impossible situation. The second speaker can't be a knight, because he claims that the third is also a knight, so the first speaker is telling the truth, contradicting the second speaker's first statement. But nor can the second speaker be a knave, since then **every** statement he makes is false; in particular, his first statement is false, meaning that the first speaker is a knight; but the first speaker claims the second is a knight, giving a contradiction.

   In terms of propositional logic, if we let P, Q, and respectively R refer to the propositions that the first, second, and respectively third speaker is a knight, then the situation described implies the truth of

$$\begin{array}{lr} \langle\langle & \langle P \supset \langle Q \wedge R \rangle \wedge \langle\langle Q \wedge R \rangle \supset P \rangle\rangle \wedge \\ \langle & \langle\langle Q \supset \ P \rangle \wedge \langle\ P \supset Q \rangle\rangle \wedge \\ & \langle\langle Q \supset R \rangle \wedge \langle R \supset Q \rangle\rangle\rangle\rangle \end{array}$$

   but this is a contradiction (false for all truth values of P,Q,R).

   The bug was having the second speaker make two separate statements. He should instead have said "The first speaker is a knave and the third is a knight". Now we know the truth of

$$\begin{aligned} \sigma = \langle\langle\langle P \supset \langle Q \wedge R \rangle \wedge \langle\langle Q \wedge R \rangle \supset P \rangle\rangle \wedge \\ \langle\langle\langle Q \supset \langle\ P \wedge R \rangle\rangle \wedge \langle\langle\ P \wedge R \rangle \supset Q \rangle\rangle\rangle \end{aligned}$$

   which is quite a different proposition! If we draw a truth table, we get:

P	Q	R	σ
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	T

so we can conclude that they're all knaves. Alternatively, we could argue it out in words like this: if the first is a knight, then the second's statement is false, contradicting the first's claim. So the first is a knave. If the second is a knight then so is the third, confirming the first's statement. So the second is also a knave. Hence so is the third.

Trust no-one! Turn back!

Bonus question "If I were to ask you whether the berries are safe, would you answer 'bal'?". Think about it!