MATH 3TP3 Assignment #5 Solutions

1. (a) Neither: missing brackets.
   (b) Neither: - is not in our alphabet.
   (c) Neither: what goes after $\sim$ has to be a wff, not a term.
   (d) A sentence.
   (e) A wff with free variables a' and b.
   (f) Neither: < is not in our alphabet.

2. (a) $\exists y : (y \cdot (y \cdot y)) = x$
   (b) $\sim \exists x : \exists y : \exists z : \langle \sim z = 0 \wedge ((x \cdot (x \cdot x)) + (y \cdot (y \cdot y))) = (z \cdot (z \cdot z)) \rangle$ (without the assumption that $z \neq 0$, there is the solution $0^3 + 0^3 = 0\ldots$)
   (c) $\forall x : Ay \langle \langle Ex' : x = S(x'+x') \wedge Ey' : y = S(y'+y') \rangle \supset \exists z : (x+y) = (z+z) \rangle$
   (d) Let make the more interesting (and true) statement that there are infinitely many **coprime** Pythagorean triples (noting that if two of the three are coprime, then they're all coprime): $\forall z : Ez' : \exists x : \exists y : \langle ((x \cdot x) + (y \cdot y)) = ((z+z') \cdot (z+z')) \wedge Az'' \langle \langle \langle Ex' : x = (x' \cdot z'') \wedge Ey' : y = (y' \cdot z'') \rangle \supset x' = S0 \rangle \rangle$

3. (a) There exists a solution to $x^2 = x + 1 \bmod 2$. False.
   (b) Every natural number is the sum of three squares. False - e.g. 7 is a counterexample. (Something to think about: what if we allowed four squares?)
   (c) 16 is a square. True.
   (d) c is a common divisor of a and b.
   (e) c is the greatest common divisor of a and b.