TOPICS IN KOSZUL DUALITY – COURSE SYLLABUS

Michaelmas 2019, Oxford University

Course description. The goal of these lectures is to give an introduction to Koszul duality, a simple, yet powerful, principle applicable in algebraic geometry and topology.

Outline. Given a field k and an augmented k-algebra A, we can construct a new k-algebra

$$E(A) := \mathbb{R}\operatorname{Hom}_A(k, k),$$

where the multiplication is given by the Yoneda product. In good cases, performing this construction twice brings us back to where we started, i.e. $A \cong E(E(A))$. If this happens, we say that A and $E(A)$ *lie in Koszul duality.* For example, this links $A = k[t]$ to the exterior algebra $E(A) = \Lambda(x)$. Whenever A and $E(A)$ lie in Koszul duality, their derived categories are closely related.

This idea has applications in the study of derived categories of coherent sheaves, the homology of loop spaces, and the stable homotopy groups of spheres.

Categories of modules (like $\mathrm{Mod}_{\mathbb{R}}$ or $\mathrm{Mod}_{M_n(\mathbb{C})}$) are governed by associative rings (e.g. $\mathbb{R}$ or $M_n(\mathbb{C})$). More general categories of algebraic structures (such as rings, Poisson algebras, or Lie algebras) are governed by monads; these are simply associative algebras in suitable categories of endofunctors. The Koszul duality construction $A \mapsto E(A)$ from above generalises to this context.

This abstraction has several concrete consequences such as the close relation between Lie algebras and deformation theory, as well as the Bogomolov-Tian-Todorov theorem on the unobstructedness of Calabi-Yau varieties.

Instructor. Lukas Brantner

E-mail. brantner@maths.ox.ac.uk

Meeting time and place. Fridays 11-12 am, lecture room L6.

Office hours. Mondays 9-10 am (or by appointment), room N2.32.

Course website. https://people.maths.ox.ac.uk/brantner/Koszul_lectures.html

Text. Course notes (including relevant references) will be provided on the course webpage.

Prerequisites. Material from courses on homological and commutative algebra, basic algebraic geometry, and algebraic topology.