Workshop on
"New Perspectives in Nonlocal and Nonlinear PDEs"

Anacapri, 07-11 July, 2025

https://people.maths.ox.ac.uk/carrillo/Capri/anacapri5.html

Schedule

Day	Monday 07	Tuesday 08	Wednesday 09	Thursday 10	Friday 11
09:00–09:40	Registration & opening	Kovařík	Di Francesco	Ferone	Cañizo (9:30)
09:45–10:25	Vázquez	del Teso	Cavagnari	Muratori	Tassi (10:10)
10:30–11:00	**Coffee Break**				
11:00–11:40	Warnett/Kamath	Koo/Ghaderi Z.	Mainini	Guo/Z. Huang	Dolbeault
11:45–12:25	Segatti	Y. Huang	Fernández-Jimenez	Punzo	Greetings
12:30–15:00	**Lunch**				
15:00–15:40	Schmidtchen	Schlichting	Free	Indrei	
15:45–16:25	Skrzeczkowski	Holzinger	Free	Schulz	
16:25–17:00	**Coffee Break**				
17:00–17:40	Free Discussion	Gómez-Castro	Free	Grillo	

Practical information

- *The workshop will take place at Villa Rosa (Via Axel Munthe, 4) of the Comune di Anacapri.*
- *The social dinner will be held at the restaurant of the Hotel San Michele, on Tuesday, July* 8^{th}.

Contents

List of Abstracts

Entropy methods in slightly new contexts

José Alfredo Cañizo

University of Granada, Spain

In some recent works with Alejandro Gárriz, Diego Marín and Fernando Quirós we have been using standard entropy methods in contexts in which a PDE does not exactly have an entropy structure, but is asymptotically close to it for large times. We will present examples of this for the standard heat equation on an exterior domain ($\mathbb{R}^d$ minus an open, bounded set), the porous media equation on an exterior domain, and the heat equation in hyperbolic space. In all of these cases one can find rates of convergence and improve existing results by using relative entropies to well-chosen "transient equilibria".

Wasserstein vs Hilbert: approaches to multi-particle control problems

Giulia Cavagnari

Politecnico di Milano, Italy

We present two approaches to optimal control problems for multi-particle systems: one in the Wasserstein space of probability measures, using tools from optimal transport, and one in a Hilbert space of parametrizations, following a Lions-inspired approach. By linking Eulerian and Lagrangian dynamics, we prove the equivalence of the value functions of the two settings, emphasizing their respective strengths and the role of an intermediate formulation. These results also shed light on recent developments in the theory of viscosity solutions for Hamilton-Jacobi equations in this context. Joint work with Stefano Lisini, Carlo Orrieri (University of Pavia - Italy) and Giuseppe Savaré (Bocconi University - Italy).

Mean Value Properties and Finite Difference Schemes for Local and Nonlocal p-Laplace Problems

Félix del Teso

Universidad Autónoma de Madrid, Spain

The aim of this talk is to present recent advances on asymptotic expansions, mean value properties, and finite difference schemes for parabolic and elliptic equations involving the p-Laplacian and fractional p-Laplacian operators.

Deterministic particles and smoothing effects

Marco Di Francesco

University of L'Aquila, Italy

Deterministic particle approximations allow to construct solutions to scalar conservation laws, to nonlinear diffusion equations, and to some Wasserstein gradient flows. Despite their finite-dimensional nature, they are able to catch certain "smoothig-effect" properties of the relevant many-particle continuum limit. We will review some recent results on this aspect, focusing in particular on discrete counterparts of two celebrated properties: the Oleinik one-sided Lipschitz property for scalar conservation laws and the Aronson-Bénilan estimate for the porous medium equation. We will also consider the case of the one-dimensional nonlocal interaction equation with repulsive Morse potential. The results involve M. Schmidtchen (TU Dresden), V. Iorio (L'Aquila), M. D. Rosini (Chieti-Pescara), D. Matthes (TU Munich) as co-authors.

Symmetry, symmetry breaking, and phase transitions in interpolation inequalities

Jean Dolbeault

Université Paris Dauphine, France

This lecture will be devoted to a short review of recent results on nonlinear interpolation inequalities, branches of optimizers, and the classification of some phase transitions between symmetry and symmetry breaking regimes. When symmetry occurs, spectral and entropy methods can be used to obtain improved inequalities, improved rates of convergence for the corresponding nonlinear diffusion flows, and stability results.

A Li–Yau and Aronson Bénilam approach to study Keller–Segel

Alejandro Fernández-Jiménez

University of Oxford, UK

In this talk we will focus on the Keller–Segel system

$$\begin{cases} \partial_t \rho = \Delta \rho^m - \operatorname{div}(\rho \nabla u) & \text{in } (0,\infty)\times\mathbb{R}^d, \\ -\Delta u = \rho & \text{in } (0,\infty)\times\mathbb{R}^d, \end{cases}$$

for $d \geq 2$ and $m = 2 - \frac{2}{d}$, i.e. the critical exponent. The main goal of this talk will be to present a new approach to study this family of equations. In order to do that, we prove a Li–Yau and Aronson–Bénilam type estimate that entail L^∞ bounds depending on its initial mass, up to the critical mass case. This argument is based on a careful analysis of the subsolutions of the Liouville and the Lane–Emden equations. As a consequence, we are able to show existence of smooth solutions with fewer regularity assumptions than previous literature and we also provide new estimates for the solutions of the Keller–Segel system.

The talk presents ongoing work together with C. Elbar and F. Santambrogio.

Symmetrization for nonlocal problems

Vincenzo Ferone

University of Naples "Federico II", Italy

We discuss Talenti-type symmetrization results in the form of mass concentration (i.e., integral comparison) for various nonlocal elliptic and parabolic problems. The general approach, via Riesz inequality, will be discussed both in relation to the model case of the fractional Laplacian, and in relation to more general operators, also considering the case of eigenvalue problem. The results are contained in various papers in collaboration with B. Brandolini, I. de Bonis, G. Piscitelli and B. Volzone.

Aggregation-Diffusion Equations with saturation in bounded domains

David Gómez-Castro

Universidad Autónoma de Madrid, Spain

In this talk we will discuss the well-posedness, asymptotics, and numerical analysis of aggregation-diffusion equations with non-linear mobility

$$\partial_t \rho = \operatorname{div}(\mathrm{m}(\rho)\nabla(U'(\rho) + V + W * \rho)).$$

We are interested in non-linear mobilities of saturation type where $\mathsf{m}(0) = \mathsf{m}(\alpha) = 0$ and $0 \leq \rho_0 \leq \alpha$. We consider bounded domains and no-flux conditions. These are formally gradient flows of a modified Wasserstein-type distances. For the case $W = 0$ we will discuss an existence theory of semigroup solutions with comparison principle, and time-asymptotics. We will also discuss properties and convergence of numerical schemes of finite-volume type in the general setting based on discrete Sobolev and Wasserstein-type estimates.

The talk presents joint work with J.A. Carrillo and A. Fernández-Jiménez (U. Oxford).

Kermack–McKendrick type models for epidemics with nonlocal aggregation terms

Fatemeh Ghaderi Zefreh

University of L'Aquila, Italy

We propose an approach to model spatial heterogeneity in SIR-type models for the spread of epidemics via nonlocal aggregation terms. We first consider an SIR model with spatial movements driven by nonlocal aggregation terms, in which the inter-compartment and intra-compartment interaction terms are distinct and modelled through smooth interaction kernels. For the Cauchy problem we provide a full well-posedness theory for H^1 solutions. In the second part we discuss on the existence of steady states for these type of models and display a specific example of non-trivial steady states for an SIS model with aggregations, the existence of which is determined by a threshold condition for a suitable "space-dependent" basic reproduction rate. This is a project in collaboration with M. Di Francesco.

Widder theory for the porous medium equation with rough kernels

Gabriele Grillo

Politecnico di Milano, Italy

We consider weighted porous medium equation with rough and inhomogeneous density that may be singular at a point and tends to zero at spatial infinity. We identify a class X of initial measure data that give rise to very weak solutions, we show that non-negative very weak solutions necessarily admit an initial trace in X at time $t = 0$, and we prove that any two non-negative solutions having the same initial trace are equal. The corresponding theory for the classical (unweighted) equation was established by exploiting various properties that are not available in our weighted setting, such as the continuity of solutions, the explicit scale invariance of the equation, Aleksandrov's reflection principle, and the Aronson–Bénilan inequality. We stress that the results are new even for weights which are bounded and bounded away from zero, but are just measurable.

What can we know from Liouville equations for many-particle systems?

Shuchen Guo

University of Oxford, UK

This talk focuses on the analysis of high-dimensional Liouville equations, particularly those arising from space-homogenous Landau equations. By showing the monotonicity of entropy or Fisher information, we gain enough compactness to derive BBGKY-type Landau hierarchies. Furthermore, the propagation of the exponential moments implies the uniqueness of certain Landau hierarchies. These two steps together lead to the propagation of chaos.

Nonlinear Stability for chemotactic clustering with discontinuous advection

Franca Hoffmann

Caltech, USA

Bacterial chemotaxis describes the ability of single-cell organisms to respond to chemical signals. In the case where the bacterial response to these chemical signals is sharp, the corresponding chemotaxis model for bacterial self-organization exhibits a discontinuous advection speed. This is a key challenge for analysis. We present a perturbative approach, both from a macroscopic and a kinetic viewpoint, where the shape of the cellular profile is clearly separated from its global motion. As a result, we obtain exponential relaxation to equilibrium with an explicit rate. This is joint work with Vincent Calvez and Gianluca Favre. CANCELLED.

Mean-field convergence in relative entropy for multi-species models and fluctuations in the moderate regime

Alexandra Holzinger

University of Oxford, UK

In this talk, I will show how to prove a mean-field limit in relative entropy for a multi-species model of moderately interacting particles. In our setting, the inter- and intra-species interaction kernels approximate attractive/repulsive singular potentials up to Newtonian/Coulomb singularities without additional cut-off on the particle level. A crucial argument in the proof is to show a convergence in probability result between the particle level and the corresponding smooth approximation. Moreover, I will explain how this crucial convergence in probability can be used as a new technique to extend Oelschläger's well-known result for fluctuations around the porous media equation to the case of aggregating particles. This talk contains joint work with José A. Carrillo and Shuchen Guo as well as Li Chen and Ansgar Jüngel.

Ground states for the Choquard equation in the Thomas–Fermi limit

Yanghong Huang

University of Manchester, UK

The ground states for the Choquard equation in the Thomas–Fermi limit are studied. They are related to optimizers of a Gagliardo–Nirenberg type inequality

$$\iint_{\mathbb{R}^N\times\mathbb{R}^N} \frac{|u(x)|^p\,|u(y)|^p}{|x-y|^{N-\alpha}}dx\,dy \le C\Big(\int_{\mathbb{R}^N}|u|^2dx\Big)^{p\theta}\Big(\int_{\mathbb{R}^N}|u|^q dx\Big)^{2p(1-\theta)/q},$$

that involves the nonlocal Riesz energy with $0<\alpha<N$, $p>\frac{N+\alpha}{N}$, $q>\frac{2Np}{N+\alpha}$ and $\theta=\frac{(N+\alpha)q-2Np}{Np(q-2)}$. For $p=2$, the equivalent problem has been studied in connection with the Keller–Segel diffusion–aggregation models in the past few decades. The general case $p\neq 2$ considered here appears in the study of Thomas–Fermi limit regime for the Choquard equations with local repulsion. We establish, optimal ranges of parameters for the validity of the above interpolation inequality, discuss the existence and qualitative properties of the nonnegative maximizers, and in some special cases estimate the optimal constant. For $p=2$ it is known that the maximizers are Hölder continuous and compactly supported on a ball. We show that for $p<2$ the maximizers are smooth functions supported on $\mathbb{R}^N$, while for $p>2$ the maximizers consist of a characteristic function of a ball and a nonconstant nonincreasing Hölder continuous function supported on the same ball. These results are complemented with numerical examples and asymptotic behaviours in limiting parameter regimes.

Mean field error estimate of the random batch method for large interacting particle system

Zhenyu Huang

Shanghai Jiao Tong University, China

The random batch method (RBM) proposed in [Jin et al., J. Comput. Phys., 400(2020), 108877] for large interacting particle systems is an efficient with linear complexity in particle numbers and highly scalable algorithm for N-particle interacting systems and their mean-field limits when N is large. We consider in this work the quantitative error estimate of RBM toward its mean-field limit, the Fokker-Planck equation. Under mild assumptions, we obtain a uniform-in-time $O(\tau^2 + 1/N)$ bound on the scaled relative entropy between the joint law of the random batch particles and the tensorized law at the mean-field limit, where τ is the time step size and N is the number of particles. Our result can be seen as an improvement over the previous works about the random batch, and we fill the gap to understand the approximation error of the RBM as a numerical method for its mean-field limit.

The geometry of the free boundary

Emanuel Indrei

Kennesaw State University, USA

The seminar will address the non-transversal intersection of the free boundary with the fixed boundary for nonlinear uniformly elliptic operators when $\Omega = \{\nabla u \neq 0\} \cap \{x_n > 0\}$ which solves a problem in elliptic theory that in the case of the Laplacian is completely understood but has remained arcane in the nonlinear setting in higher dimension. Also, I will discuss some new ideas which illuminate that the free boundary is C^1 in a neighborhood of the fixed if the solution is physical and if $n = 2$ in the absolute general context. The regularity is even new for the Laplacian. The innovation is via geometric configurations on how free boundary points converge to the fixed boundary and investigating the spacing between free boundary points.

Bulk-surface Cahn Hilliard model for chemically active wetting

Megha Kamath

Unversity of L'Aquila, Italy

In this talk we present a bulk-surface degenerate Cahn-Hilliard system with an active source term that models the wetting of a solute-solvent mixture(in the bulk) on a membrane(surface). The active binding process at the surface maintains the system away from equilibrium leading to steady, non-equilibrium states as seen in numerical simulations. We show some a priori estimates and existence of solutions to the system via space/time discretization and techniques to deal with degernerate mobilities. (Ongoing work with Jan-Frederik Pietschmann and Simone Fagioli)

Large time behavior for flocking hydrodynamic models

Dowan Koo

Yonsei University, South Korea

In this talk, we study the large-time behavior of flocking hydrodynamic models. More precisely, we consider pressureless Euler-Alignment models with nonlocal interaction forces. In the one-dimensional case, we consider λ-convex potential and a repulsive-attractive potential. In both cases, we establish the uniform-in-time diameter estimate for the support of the density profiles, where the communication kernel is assumed to be merely positive. This significantly improves the tail conditions for unconditional flocking in the context of interaction potentials. We study the time decay rate of the solution profiles depending on the short-range assumptions on the communication kernels. For the λ-convex interaction potential, we show a sharp polynomial decay depending on the weak singularity of the communication protocol near the origin. Finally, we analyse the multi-dimensional case with λ-convex potentials, obtaining decay rates or convergence under several short-range assumptions on the communication kernel. This talk is based on a joint work with J.A. Carrillo, Y.-P. Choi, and O. Tse.

Weak perturbations on the $p-$Laplacian

Hynek Kovařík

Universiy of Brescia, Italy

In this talk I consider the p-Laplacian in $\mathbb{R}^N$ with a critical background potential and a weak perturbation. I will discuss the asymptotic behaviour of the lowest eigenvalue of such an operator in the weak coupling limit separately for $p > N$, $p = N$ and $p < N$. The talk is based on joint works with T.Ekholm, R. Frank, U.Das and Y.Pinchover.

Smoothing effect and uniqueness for aggregation-diffusion models

Edoardo Mainini

University of Genova, Italy

We consider aggregation-diffusion models in the fair competition and diffusion dominated regimes. We discuss the L^∞ regularizing effect, showing that the small-time estimates coincide with the classical ones of the porous medium equation. In combination with the gradient flow structure of these models, we deduce uniqueness of the solution.

The concentration comparison for nonlinear diffusion on model manifolds

Matteo Muratori

Politecnico di Milano, Italy

We investigate the validity of the mass concentration comparison for a class of nonlinear diffusion equations, commonly known as filtration equations, posed on Riemannian manifolds that are spherically symmetric, that is, model manifolds. The concentration comparison states that the solution of a certain parabolic PDE that takes the radially decreasing rearrangement $u_0^\star$ as its initial datum is more concentrated than the solution starting from the original datum u_0. This is known to hold in $\mathbb{R}^n$ as a consequence of the celebrated Pólya-Szegő inequality, which asserts that the L^2 norm of the gradient of a function f is always larger than the L^2 norm of the gradient of its rearrangement $f^\star$. However, on a general model manifold, it is not for granted that the Pólya-Szegő inequality holds; in fact, we provide a simple condition, involving the scalar curvature, under which such an inequality actually fails. Our main result states that, given any continuous bijection $\phi : [0, +\infty) \to [0, +\infty)$, the filtration equation $\partial_t u = \Delta\phi(u)$ satisfies the concentration comparison if and only if the underlying model manifold supports the Pólya-Szegő inequality. As a simple corollary, the validity of such a comparison for the heat equation is sufficient to guarantee that the same holds for all filtration equations. Moreover, we prove that if the manifold supports a centered isoperimetric inequality then the Pólya-Szegő inequality holds, allowing us to include important examples such as the hyperbolic space and the sphere. This is a joint work with Bruno Volzone.

Phragmèn-Lindelöf type results for a class of parabolic equations on infinite graphs

Fabio Punzo

Politecnico di Milano, Italy

We discuss Phragmèn-Lindelöf type theorems for parabolic equations with a variable density, posed on infinite weighted combinatorial graphs. In particular, we address the relation between the behaviour of the density and the uniqueness class. We also show that our conditions on the density are sharp. The results have been recently obtained jointly with S. Biagi (Politecnico di Milano) and G. Meglioli (Bielefeld University).

Breakdown of the mean-field description of interacting systems: Phase transitions, metastability and coarsening

André Schlichting

University of Ulm, Germany

We present results concerning the qualitative and quantitative description of interacting systems, with particular emphasis on those possessing a phase transition under the change of relevant system parameters.

For this, we first discuss and identify continuous and discontinuous phase for mean-field limits of interacting particle systems on the torus and spheres.

Since phase transitions are intimately related to the metastability of the stochastic particle system, we show how a suitable mountain pass theorem in the space of probability measures can describe the metastable behaviour of the underlying finite particle system.

We also show that the mean-field description of the particle system in the regime of strong local interaction breaks down. In this regime, coarsening is observed, where smaller clusters grow through coagulation events. We provide numerical experiments with a positivity preserving numerical scheme for a SPDE of Dean-Kawasaki type, consisting of the McKean-Vlasov equation and conservative noise.

Joint works with José Carrillo (U Oxford), Nicolai Gerber (U Ulm), Rishabh Gvalani (ETH Zürich), Greg Pavliotis (Imperial London) and Anna Shalova (TU Eindhoven).

Nonlocal approximation of an anisotropic cross-diffusion system

Markus Schmidtchen

TU Dresden, Germany

Localisation limits and nonlocal approximations of degenerate parabolic systems have experienced a renaissance in recent years. However, only few results cover anisotropic systems. This work addresses this gap by establishing the nonlocal-to-limit for a specific anisotropic cross-diffusion system encountered in population dynamics featuring phase-separation phenomena, i.e., internal layers between different species.

Regularity and Uniqueness for a Model of Active Particles with Angle-Averaged Diffusions

Simon Schulz

SNS Pisa, Italy

We study the regularity and uniqueness of weak solutions of a degenerate parabolic equation, arising as the limit of a stochastic lattice model of self-propelled particles. The angle-average of the solution appears as a coefficient in the diffusive and drift terms, making the equation nonlocal. We prove that, under unrestrictive non-degeneracy assumptions on the initial data, weak solutions are smooth for positive times. Our method rests on deriving a drift-diffusion equation for a particular function of the angle-averaged density and applying De Giorgi's method to show that the original equation is uniformly parabolic for positive times. We employ a Galerkin approximation to justify rigorously the passage from divergence to non-divergence form of the equation, which yields improved estimates by exploiting a cancellation. By imposing stronger constraints on the initial data, we prove the uniqueness of the weak solution, which relies on Duhamel's principle and gradient estimates for the periodic heat kernel to derive L^∞ estimates for the angle-averaged density. This is joint work with Luca Alasio (LJLL, Sorbonne Université).

Variational approximation of the heat flow of harmonic maps into non positively curved manifolds

Antonio Segatti

University of Pavia, Italy

In this talk, I will present a variational approximation of the heat flow for harmonic maps. The approach is based on the Weighted Energy Dissipation (WED) scheme, which involves a functional defined on entire trajectories and depending on a small parameter ε. As $\varepsilon \searrow 0$, minimizers of this functional are shown to converge to (weak) solutions of the heat flow. In particular, for smooth target manifolds with non-positive sectional curvature, this variational method recovers the classical Eells–Sampson theorem. This is a joint work with Fang-Hua Lin, Yannick Sire and Changyou Wang.

Optimal rate of convergence for a nonlocal-to-local limit in one dimension

Jakub Skrzeczkowski

University of Oxford, UK

We consider a nonlocal approximation of the quadratic porous medium equation where the pressure is given by a convolution with a mollification kernel. It is known that when the kernel concentrates around the origin, the nonlocal equation converges to the local one. However, the question of how fast it converges is not well-explored in the literature. In one spatial dimension, for a particular choice of the kernel, and under mere assumptions on the initial condition, we quantify the rate of convergence in the 2-Wasserstein distance. First, we reprove the recent result of Amassad and Zhou (2025) yielding the rate of $\sqrt{\varepsilon}$ using a simpler technique based on the Evolutionary Variational Inequality for both nonlocal and local equations. Next, using numerical simulations, we observe that the rate of $\sqrt{\varepsilon}$ is not optimal. Therefore, we obtain a new formula for the Wasserstein distance between two abstract gradient flows which, when applied to the problem, together with Aronson-Benilian estimates, yields the rate of ε, suggested to be optimal by numerics. This is joint work with J.A. Carrillo, S. Fronzoni (Oxford), C. Elbar (Lyon), P. Gwiazda (Warsaw).

Asymptotic behaviour and uniform-in-time approximation of nonlocal Fokker-Planck equations

Niccolò Tassi

University of Granada, Spain

We study a nonlocal approximation of the standard and fractional Fokker-Planck equation in which we can estimate the speed of convergence to equilibrium in a way which does not degenerate in the scaling limit of the equation. This uniform estimate cannot be easily obtained with standard inequalities or entropy methods, but can be obtained through the use of Harris's theorem, finding interesting links to quantitative versions of the central limit theorem in probability. As a consequence one can prove that solutions of this nonlocal approximation converge to those of the standard (fractional) Fokker-Planck equation uniformly in time. The associated equilibrium has some interesting properties, which we also study. In the last part of the talk we discuss some extensions to the non-autonomous setting, motivated by a self-similar type change of variable.

Nonlinear Anisotropic Diffusion Equations

Juan Luis Vázquez Suárez

Universidad Autónoma de Madrid, Spain

The Porous Media Equation, $u_t = \Delta u^m$, is a much studied equation in the class of parabolic equations representing nonlinear diffusion processes. In this talk, we want to contribute to the study of anisotropy in the form of a variant of the model where the exponents of the nonlinearity are different in each spatial coordinate direction, $u_t = \sum_i \partial_{ii}^2(u^{m_i})$. The existence of self-similar solutions that are asymptotic attractors is a main result.

The Stein-log-Sobolev inequality and the exponential rate of convergence for the continuous Stein variational gradient descent method

Jethro Warnett

University of Oxford, UK

The Stein Variational Gradient Descent method is a variational inference method in statistics that has recently received a lot of attention. The method provides a deterministic approximation of the target distribution, by introducing a nonlocal interaction with a kernel. Despite the significant interest, the exponential rate of convergence for the continuous method has remained an open problem, due to the difficulty of establishing the related so-called Stein-log-Sobolev inequality. Here, we prove that the inequality is satisfied for each space dimension and every kernel whose Fourier transform has a quadratic decay at infinity and is locally bounded away from zero and infinity. Moreover, we construct weak solutions to the related PDE satisfying exponential rate of decay towards the equilibrium. The main novelty in our approach is to interpret the Stein-Fisher information, also called the squared Stein discrepancy, as a duality pairing between $H^{-1}(\mathbb{R}^d)$ and $H^1(\mathbb{R}^d)$, which allows us to employ the Fourier transform. We also provide several examples of kernels for which the Stein-log-Sobolev inequality fails, partially showing the necessity of our assumptions.

Author Index