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Research Statement

1. Introduction

My research interests lie at the crossroad of symplectic geometry, gauge theory and low dimensional topology. In my research, I have explored these areas through the use of **symplectic instanton homology** (abbreviated HSI), an invariant associated to closed oriented 3-manifolds defined by Manolescu and Woodward (see below), and whose construction is motivated by the Atiyah-Floer conjecture.

I plan to continue working in these research areas, and in particular to broaden my work to other gauge and symplectic theories appearing in topology: Donaldson-Floer theory in dimensions 3 and 4, Seiberg-Witten theory, symplectic field theory, Hamiltonian actions, equivariant versions of Floer homology, Fukaya categories. To this purpose, I have constructed and am currently developping a Hamiltonian 2-category, denoted $\mathcal{H}am$, that serves as a common framework for all these theories.

I describe HSI and $\mathcal{H}am$ below, and indicate how this category relates to the other theories. I describe my future research directions in section 4.

Symplectic Instanton Homology (HSI). The starting point of Manolescu and Woodward's construction of HSI is the Atiyah-Floer conjecture. This conjecture provides a bridge between gauge theory and symplectic geometry. To a closed oriented 3-manifold Y (subject to some assumptions) is associated an instanton homology group $I_*(Y)$ ([Flo88]), whose chain complex is generated by (gauge equivalence classes of perturbed) flat connexions on the trivial $SU(2)$-bundle over Y. The differential is defined by counting instantons on $Y \times \mathbb{R}$, namely connexions whose curvature 2-form F_A satisfies the anti self-dual equation $*F_A + F_A = 0$. Let $\Sigma \subset Y$ be a Heegaard surface, separating Y into two handlebodies H_0 and H_1. Atiyah used a neck-stretching argument to suggest that $I_*(Y)$ should be computed by counting pseudo-holomorphic Whitney discs between intersection points of a pair of Lagrangians

$$\mathscr{L}(H_0), \mathscr{L}(H_1) \subset \mathscr{M}(\Sigma),$$

where $\mathscr{M}(\Sigma)$ stands for the moduli space of flat connexions over Σ, and $\mathscr{L}(H_0)$, resp. $\mathscr{L}(H_1)$ corresponds to those that can be extended flatly to H_0, resp. H_1.

Conjecture 1.1. *(Atiyah-Floer,* [Ati88]*) With HF standing for Lagrangian Floer homology,*

$$I_*(Y) \simeq HF(\mathscr{L}(H_0), \mathscr{L}(H_1)).$$

The main difficulty in this problem comes from the fact that $\mathscr{M}(\Sigma)$, $\mathscr{L}(H_0)$ and $\mathscr{L}(H_1)$ have singularities: the group $HF(\mathscr{L}(H_0), \mathscr{L}(H_1))$ is currently not defined.

Symplectic Instanton homology has been constructed by Manolescu and Woodward in [MW12] by reproducing Atiyah's construction, but replacing the (singular) symplectic manifold $\mathscr{M}(\Sigma)$ by a smooth moduli space $\mathscr{N}(\Sigma')$ associated with the surface Σ' obtained by removing a small disc from Σ. $\mathscr{N}(\Sigma')$ is an open subset of Huebschmann-Jeffrey's extended moduli space; this last moduli space consists of connexions that, seen as $\mathfrak{su}(2)$-valued 1-forms, are of the form $A = \theta ds$ in a neighborhood of the boundary, where s parametrizes the boundary circle. Smooth Lagrangian submanifolds $L(H_i) \subset \mathscr{N}(\Sigma')$ are defined analogously to $\mathscr{L}(H_0)$ and $\mathscr{L}(H_1)$.

Definition 1.2 ([MW12])**.** The isomorphism type of the group $HSI(Y) = HF(L(H_0), L(H_1))$ is well-defined for any closed oriented 3-manifold Y: it is independent of the choice of the Heegaard splitting.

An important feature of $\mathscr{N}(\Sigma')$ is that it carries a Hamiltonian action of $SU(2)$ for which the symplectic reduction is $\mathscr{M}(\Sigma)$. Therefore, Manolescu and Woodward suggested to define an **equivariant version** of HSI, as a substitute for the ill-defined $HF(\mathscr{L}(H_0), \mathscr{L}(H_1))$.

The Hamiltonian 2-category $\mathcal{H}am$ has been introduced in [Caz19], and is similar with the Moore-Tachikawa category [MT12]. A naive definition of $\mathcal{H}am$ could be the following:

- objects are Lie groups,
- 1-morphisms from G to H consist in symplectic manifolds endowed with a Hamiltonian action of $G \times H$,
- For M and N two 1-morphisms from G to H, the 2-morphisms from M to N are the $(G \times H)$-equivariant Lagrangian correspondences, namely Lagrangian submanifolds of $M^- \times N$ that are both $G \times H$-invariant for the diagonal action, and contained in the zero level of the moment map of this diagonal action.
- composition of 1-morphisms involves symplectic quotient: for M and N going respectively from G to H, and from H to K, their composition is defined as the symplectic quotient for the diagonal action:

$$M \circ_h^1 N = (M \times N) /\!\!/ H^{diag}.$$

- 2-morphisms compose as correspondences.

Unfortunately the various compositions are not always well-defined, which implies that $\mathcal{H}am$ is only a "partial" 2-category. Nevertheless it can be "completed" to a strict 2-category $\widehat{\mathcal{H}am}$, where 1-morphisms now consist in equivalence classes of finite sequences of 1-morphisms, and 2-morphisms equivalence classes of diagrams of 2-morphisms.

This definition is motivated by work of Wehrheim and Woodward [WW16, Weh16], in which they outline that Donaldson polynomials could be categorified as an extended TQFT of dimension (2+1+1), with values in a 2-category that is an enrichment of (a modified version of) Weinstein's symplectic category. Our goal is to promote $\mathcal{H}am$ to a 3-category, and extend this conjectural functoriality down to dimension 1. We hope to be able to solve the question of functoriality of Donaldson polynomials, which is a central open question in this field.

Other more algebraic approaches, including through derived algebraic geometry or shifted symplectic geometry, also address this question. While these approaches involve more elaborated geometric objects, our goal is to remain in the more classical framework of Floer theory: partial differential equations in symplectic manifolds.

2. Published papers, preprints

We now present the results contained in the following papers:

- [Caz16b]: *Symplectic instanton homology: Twisting, connected sums and Dehn surgery*, **Journal of Symplectic Geometry**,
- [Caz17]: *Symplectic Instanton Homology: naturality, and maps from cobordisms*, **Quantum topology**,
- [Caz19]: *A two-category of Hamiltonian manifolds, and a (1+1+1) field theory*, to appear in **Indiana University Mathematics Journal**.
- [CHKK20]: *The correspondence induced on the pillowcase by the earring tangle*, with Christopher M. Herald and Paul Kirk and Artem Kotelskiy, to appear in **Journal of Topology**,
- [CHK21]: *Tangles, relative character varieties, and holonomy perturbed traceless flat moduli spaces*, with Chris Herald and Paul Kirk, to appear in **The Open Book Series**.
- [Caz], *Equivariant Lagrangian Floer homology via cotangent bundles of EG_N*, preprint.

2.1. [Caz16b], Symplectic instanton homology: Twisting, connected sums and Dehn surgery.

Twisted version, functorial interpretation. In order to establish a surgery exact sequence, I defined a twisted version of HSI: a group $HSI(Y, c)$ associated with a closed oriented 3-manifold Y endowed with a class $c \in H_1(Y; \mathbb{Z}/2\mathbb{Z})$. This class corresponds to an isomorphism class of $SO(3)$-bundle. When $c = 0$ these groups correspond to $HSI(Y)$.

I also proved that these groups can be interpreted as a "(2 + 1)-Floer Field theory" in the sense of Wehrheim and Woodward, namely a functor from a category of 3-cobordisms to *Weinstein's symplectic category*, category whose objects are symplectic manifolds and morphisms are (generated by) Lagrangian correspondences. This framework is particularly well-suited for these invariants and allowed me to use Wehrheim and Woodward's theory of pseudo-holomorphic quilts to prove a connected sum and a Dehn surgery theorem.

Connected sum formula, surgery exact sequence. The following theorem, [Caz16b, Th. 1.1], allows one to compute HSI groups of connected sums.

Theorem 2.1. *The HSI groups of a connected sum satisfy a Künneth formula :*

$$HSI((Y_1\#Y_2, c_1 + c_2)) \simeq HSI(Y_1, c_1) \otimes HSI(Y_2, c_2) \oplus Tor(HSI(Y_1, c_1), HSI(Y_2, c_2))[-1].$$

The following result describes how HSI undergoes Dehn surgery.

Theorem 2.2. *Let $K \subset Y$ be a framed knot in a 3-manifold Y, and Y_0, Y_1 standing respectively for the 0 and 1 surgeries along K. The twisted HSI groups satisfy a long exact sequence*

$$\cdots \to HSI(Y, c + k) \to HSI(Y_0, c_0) \to HSI(Y_1, c_1) \to \cdots ,$$

where the classes c, c_0 and c_1 stand for the pushforward of a common class on $Y \setminus K$, and k stands for the class of K.

Computations. The exact sequence allowed me to compute the twisted HSI groups for several 3-manifolds, see [Caz16b, sec. 5.3], including:

- the boundaries of some negative definite plumbings,
- the branched double covers of quasi-alternating links,
- the integral Dehn surgeries of large slope along toric knots.

2.2. [Caz17]**, Symplectic Instanton Homology: naturality, and maps from cobordisms.**

Naturality. We show that the HSI groups with $\mathbb{Z}/2\mathbb{Z}$ coefficients are natural, in the sense that they are well-defined as (relatively $\mathbb{Z}/8\mathbb{Z}$-graded) abelian groups, rather than only up to isomorphisms. More precisely, we show that these invariants are canonically assigned to a triple (Y, P, z), where Y is a 3-manifold, P an $SO(3)$-bundle over Y, and $z \in Y$ a base point.

That enables us to define representations of the fundamental group and the mapping class group of Y, as well as an action of $H^1(Y;\mathbb{Z}/2\mathbb{Z})$ on $HSI(Y)$.

Cobordism maps. For HSI groups with $\mathbb{Z}/2\mathbb{Z}$-coefficients, given a compact oriented 4-dimensional cobordism W from Y to Y', an $SO(3)$-bundle P_W over W, and $\gamma\colon [0, 1] \to W$ an embedded path connecting two basepoints z and z', we define a map

$$F_{W,P_W,\gamma}\colon HSI(Y, P, z) \to HSI(Y', P', z'),$$

where P and P' stand for the restrictions of P_W to Y and Y' respectively. These maps provide a geometric interpretation of the morphisms appearing in the surgery exact sequence, indeed:

Theorem 2.3.

- *Among the three morphisms of the exact sequence of Theorem 2.2, two of them correspond to maps $F_{W,P_W,\gamma}$ associated with the traces of Dehn surgery (i.e. the cobordisms corresponding to the index 2 handle attachments), endowed with suitable $SO(3)$-bundles.*
- *For suitable $SO(3)$-bundles, the three maps associated with the traces form a long exact sequence.*

Furthermore, I proved that these maps satisfy a composition formula for the composition of cobordisms, as well as for the fibered connected sums along the base paths γ. I then obtained the following vanishing criterion, that can be seen as a first step towards an adjunction formula:

Theorem 2.4. *Let (W, γ) be a 4-cobordism with a base path,*

- *For a bundle P over $W\#\mathbb{C}P^2$, one has $F_{W\#\mathbb{C}P^2,P,\gamma} = 0$,*
- *For a bundle P over $W\#\overline{\mathbb{C}P}^2$ that restricts to a nontrivial bundle on $\overline{\mathbb{C}P}^2$, one has $F_{W\#\overline{\mathbb{C}P}^2,c,\gamma} = 0$.*

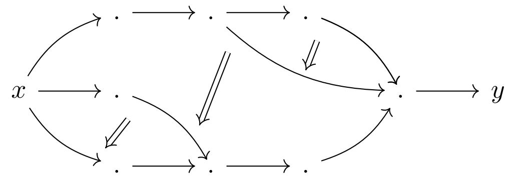
2.3. [Caz19]**, A two-category of Hamiltonian manifolds, and a (1+1+1) field theory.**

Hamiltonian category, and field theory in dimensions (1+1+1). We construct $\mathcal{H}am$, and show it is a *partial 2-category*, in a sense that we define. In particular we show that it satisfies a diagram axiom, which permits to define its completion $\widehat{\mathcal{H}am}$. This last one is a strict 2-category whose objects are the same as $\mathcal{H}am$, 1-morphisms consist in equivalence classes of finite sequence of 1-morphisms in $\mathcal{H}am$ such as:

$$\underline{\varphi} = \left(x \xrightarrow{\varphi_1} x_1 \xrightarrow{\varphi_2} \cdots \xrightarrow{\varphi_k} y \right),$$

and whose 2-morphisms consist in equivalence classes of diagrams such as:

(1)



where the cells are 2-morphisms in $\mathcal{H}am$.

We also build analogous partial 2-categories $\mathcal{L}ie_{\mathbb{R}}$ and $\mathcal{L}ie_{\mathbb{C}}$, which are non-symplectic counterparts of $\mathcal{H}am$.

Then, we build a $(1+1+1)$ field theory with values in $\widehat{\mathcal{H}am}$: to a closed 1-manifold with k components we associate $SO(3)^k$, to a surface with boundary we associate an extended moduli space, and to a 3-manifold with corners a Lagrangian correspondence corresponding to connexions that extend flatly. This defines a *quasi-functor*, in the sense that the moduli space associated with the gluing of surfaces corresponds to the composition in $\mathcal{H}am$ only up to a codimension 3 submanifold. This defect should be neglectable after taking Floer homology: generically, the pseudo-holomorphic discs involved in the differential are disjoint with such codimension 3 submanifold.

2.4. [CHKK20]**, The correspondence induced on the pillowcase by the earring tangle.** (with Chris Herald, Paul Kirk and Artem Kotelskiy)

Pillowcase homology is an Atiyah-Floer counterpart to Kronheimer-Mrowka's singular instanton homology, defined by Hedden, Herald and Kirk. It is an immersed Lagrangian Floer homology inside the pillowcase (the four-punctured two-sphere), and is combinatorial.

We study the correspondence induced by the tangle corresponding to "adding an earring". For suitable perturbations, this correspondence is a smooth genus 3 surface disjoint from the singular stratum. It permits to explain the effect of adding an earring to a tangle: the corresponding curve in the pillowcase gets doubled and misses the corners.

We also show that in some example, figure eight bubbling, a subtle degeneration phenomenon predicted by Bottman and Wehrheim, occurs in this theory. This suggests that the definition of Pillowcase Homology should incorporate bounding cochains, an algebraic tool that can capture these phenomenon.

2.5. [CHK21]**, Tangles, relative character varieties, and holonomy perturbed traceless flat moduli spaces.** (with Chris Herald and Paul Kirk)

We prove that the restriction map from the subspace of regular points of the holonomy perturbed $SU(2)$ traceless flat moduli space of a tangle in a 3-manifold to the traceless flat moduli space of its boundary marked surface is a Lagrangian immersion. A key ingredient in our proof is the use of composition in the Weinstein category, combined with the fact that $SU(2)$ holonomy perturbations in a cylinder induce Hamiltonian isotopies. In addition, we show that $(S^2, 4)$, the 2-sphere with four marked points, is its own traceless flat $SU(2)$ moduli space.

2.6. [Caz]**, Equivariant Lagrangian Floer homology via cotangent bundles of EG_N.** We provide a construction of equivariant Lagrangian Floer homology $HF_G(L_0, L_1)$, for a compact Lie group G acting on a symplectic manifold M in a Hamiltonian fashion, and a pair of G-Lagrangian submanifolds $L_0, L_1 \subset M$.

We do so by using symplectic homotopy quotients involving cotangent bundles of an approximation of EG. Our construction relies on Wehrheim and Woodward's theory of quilts, and the telescope construction.

We show that these groups are independent in the auxilliary choices involved in their construction, and are $H^*(BG)$-bimodules. In the case when $L_0 = L_1$, we show that their chain complex $CF_G(L_0, L_1)$ is homotopy equivalent to the equivariant Morse complex of L_0.

Furthermore, if zero is a regular value of the moment map μ and if G acts freely on $\mu^{-1}(0)$, we construct two "Kirwan morphisms" from $CF_G(L_0, L_1)$ to $CF(L_0/G, L_1/G)$ (respectively from $CF(L_0/G, L_1/G)$ to $CF_G(L_0, L_1)$).

Our construction applies to the exact and monotone settings, as well as in the setting of the extended moduli space of flat $SU(2)$-connections of a Riemann surface, considered in Manolescu and Woodward's work. Applied to the latter setting, our construction provides an equivariant symplectic side for the Atiyah-Floer conjecture.

3. Papers in preparation

We now present the following work:

- [CKMEY], *Equivariant Floer homology via A_∞-actions on the Floer complex*, with Paul Kirk, Mike Miller-Eismeier and Wai-Kit Yeung, in preparation.
- [CHM], *A_∞ bialgebras and Hopf algebras*, with Alex Hock and Thibaut Mazuir, in preparation.
- [CJR], *A new construction of Fukaya categories for semipositive symplectic manifolds*, with Dominic Joyce and Alex Ritter, in preparation.
- [CJLR], *Bridgeland stability for Calabi–Yau 2-folds*, with Dominic Joyce, Jason Lotay and Alex Ritter, in preparation.

3.1. [CKMEY], Equivariant Floer homology via A_∞-actions on the Floer complex. (with Paul Kirk, Mike Miller-Eismeier and Wai-Kit Yeung)

For a compact Lie group G acting on a compact smooth manifold X, both equipped with Morse functions, we endow

- the Morse complex $CM(G)$ with an A_∞-algebra structure,
- the Morse complex $CM(X)$ with an A_∞-module structure over $CM(G)$.

Furthermore, these structures are such that at homology level, they correspond to the algebra and module structures respectively induced by the multiplication $m_G \colon G \times G \to G$ and action $m_X \colon G \times X \to X$.

As a Floer theoretic counterpart, if now M is a Hamiltonian G-manifold with a pair of Lagrangians $L_0, L_1 \subset M$, we show that the Floer complex $CF(L_0, L_1)$ is an A_∞-module over $CM(G)$.

Our construction involves some kinds of "multiplicative Morse flow trees" where m_G and m_X are used as vertex conditions, and "pseudo-holomorphic foams", which are generalizations of quilts.

Using Bar constructions, this enables us to define three versions of equivariant Lagrangian Floer homology, Borel, co-Borel and Tate homologies, the last two are new in the context of Lagrangian Floer homology.

3.2. [CHM], A_∞ bialgebras and Hopf algebras. (with Alex Hock and Thibaut Mazuir)

In addition to the A_∞-algebra structure described above, $CM(G)$ can also be endowed with an A_∞-coalgebra structure, by a construction of Fukaya involving Morse flow trees. We aim to relate these two structures.

At the homology level, they induce a bialgebra structure on $H_*(G)$ (which is furthermore a Hopf algebra). We define strong homotopy versions of these two notions. Such definitions already appeared in work of Saneblidze and Umble [SU11], but our definition is different, and tailored to be applicable to Morse and Floer theory, see Section 4.1, paragraph "Lie group actions on Fukaya categories".

Our definition is based on the combinatorics of the codimension 1 boundary of some moduli spaces of graphs, which will serve as parameter spaces for defining the structure maps on $CM(G), CF(L_0, L_1), \mathcal{W}(T^*G), \mathcal{F}uk(M)$, in the same way that the associahedron is used to define the structure maps of the Fukaya category.

3.3. [CJR], A new construction of Fukaya categories for semipositive symplectic manifolds. (with Dominic Joyce and Alex Ritter)

We define a Fukaya category for a semipositive symplectic manifold, of semipositive Lagrangian immersions. The key new features of our construction are:

- a simplicial construction: our construction involves a simplicial decomposition of a Lagrangian L, and a simplicial resolution of its diagonal $\Delta_L \subset L \times L$. From this data we construct a canonical way of perturbing the boundary conditions, which allows us not to use Hamiltonian perturbations.
- the use of "map-dependent" perturbations of the almost complex structure to achieve transversality: The existence of discs with one boundary marked point does not allow domain dependent perturbations as in usual Floer theory, but we use perturbations depending both on the J-holomorphic map and its domain, which allows us to work with integer coefficients and avoid virtual regularization techniques.

3.4. [CJLR]**, Bridgeland stability for Calabi–Yau 2-folds.** (with Dominic Joyce, Jason Lotay and Alex Ritter)

Following Joyce's program [Joy15] on the Thomas-Yau conjecture, we construct a Bridgeland stability structure on the Fukaya category of a Calabi–Yau 2-fold. Very briefly, Joyce suggested that if one starts with a Lagrangian and apply to it the Lagrangian Mean Curvature Flow (LMCF), one should expect to obtain a twisted complex of special Lagrangians in the derived Fukaya category, as opposed to a special Lagrangian, as the flow may develop singularities (which might be very complicated).

We do so by using a certain elliptic flow as an alternative to the LMCF, and use a Hyper-Kähler rotation trick in order to deal with singularities: this allows us to treat Lagrangians as J-holomorphic curves, and apply Gromov compactness to them.

4. Research topics

4.1. Symplectic Topology.

A monoidal structure on the Hamiltonian 2-category. The category $\mathcal{H}am$, at least its naive version outlined above, has an obvious monoidal structure given by cartesian products (of groups, Hamiltonian manifolds, and correspondences). Defining such a structure on the completion $\widehat{\mathcal{H}am}$ is nontrivial: in order to multiply 1-morphisms (resp. 2-morphisms) one first has to find representatives of same length (resp. similar underlying diagrams), and also check the independence of the choice of representatives. With respect to this structure, the quasi-functor built in [Caz19] should be monoidal and symmetric.

Promoting $\widehat{\mathcal{H}am}$ to a 3-category. It should be possible to promote $\widehat{\mathcal{H}am}$ to a monoidal 3-category, using equivariant Floer homology as 3-morphism spaces[1]. In order to show these groups are independent of the choices of representatives (of the diagrams of 2-morphisms), we will both use the Kirwan morphisms introduced in [Caz], and and equivariant version of Wehrheim and Woodward's geometric composition theorem for quilted Floer homology. It would also be desirable to use the chain complexes instead of the homology groups. That should lead to a richer algebraic structure, namely an "A_∞ 3-category" that will have to be defined. We will inspire ourselves from Bottman-Carmeli [BC18], who defined an analogous A_∞ 2-category structure for Weinstein's symplectic category.

Relations with Fukaya and Weinstein categories. Assuming we manage to complete $\widehat{\mathcal{H}am}$ as outlined before, let M be a symplectic manifold, seen as a 1-morphism from the trivial group to itself. One should then compare $hom^2(M, pt)$ with $\mathcal{F}uk(M)$, the Fukaya category of M, or more precisely its derived category. Likewise, if M is a G-Hamiltonian manifold, seen as a 1-morphism from G to the trivial group, one could compare $hom^2(M, pt)$ with an equivariant version of $\mathcal{F}uk(M)$. In both cases we expect to define an A_∞ functor from $\mathcal{F}uk(M)$ (resp. its equivariant version) to $hom^2(M, pt)$, does this functor induce a Morita equivalence?

In the same lines, one should define an A_∞ 2-functor from $\mathcal{S}ymp$ (under construction by Bottman) to the 2-category of 1-morphisms in $\widehat{\mathcal{H}am}$, $hom^1(1, 1)$ (where 1 stands for the trivial group), that should be interesting to study. Notice that the 2-category $hom^1(1, 1)$ a priori admits more objects than $\mathcal{S}ymp$: if M is a Hamiltonian G-manifold whose action is not regular, its symplectic reduction is generally singular and doesn't define an object in $\mathcal{S}ymp$, whereas it can

[1] more precisely, a subcategory consisting in Hamiltonian manifolds and correspondences satisfying suitable conditions so that Floer homology is well-defined: monotonicity, exactness...

be seen as an object of $hom^1(1,1)$. An important manifestation of this phenomenon is the $SU(2)$-character variety of a closed surface, and will be relevant in order to define a field theory.

Relations with $\mathcal{L}ie_{\mathbb{R}}$ and $\mathcal{L}ie_{\mathbb{C}}$. In [Caz19] we define two partial 2-categories $\mathcal{L}ie_{\mathbb{R}}$ and $\mathcal{L}ie_{\mathbb{C}}$ analogous to $\mathcal{H}am$, whose objects are real (resp. complex) Lie groups, 1-morphisms smooth manifolds (resp. complex manifolds) with actions, and 2-morphisms smooth correspondences. These 2-categories are related with $\mathcal{H}am$: on the one hand, there exists a "cotangent" 2-functor $T^*\colon \mathcal{L}ie_{\mathbb{R}} \to \mathcal{H}am$ mapping a Lie group to itself, a manifold to its cotangent bundle, and a correspondence to its conormal bundle. Therefore one can think of $\mathcal{L}ie_{\mathbb{R}}$ as a toy model for $\mathcal{H}am$: it should be possible (and easier) to promote $\mathcal{L}ie_{\mathbb{R}}$ to a 3-category, via Morse theory.

On the other hand, the Kempf-Ness theorem [KN79] suggests a relation between $\mathcal{L}ie_{\mathbb{C}}$ and $\mathcal{H}am$: for M a complex projective variety endowed with a Hamiltonian G-action coming from an action of the complexification $G^{\mathbb{C}}$, then the GIT quotient corresponds to the symplectic quotient (modulo stability conditions). This suggests the existence of a 2-functor (or at least, a correspondence between 2-categories) defined on a subcategory of $\mathcal{H}am$, with values in a variation of $\mathcal{L}ie_{\mathbb{C}}$, where composition of 1-morphisms is defined via GIT quotients. It should be interesting to define and study such a functor.

(Partially) wrapped versions, and contact analogs. We will try to extend our construction of equivariant Lagrangian Floer homology to manifolds and Lagrangians with cylindrical ends, in order to get equivariant counterparts of (partially) wrapped Floer homology. This could lead to wrapped versions of $\mathcal{H}am$, that could be compared to wrapped Fukaya categories.

It would also be desirable to define contact and Legendrian analogs of equivariant homology, and study the relations with equivariant Lagrangian Floer homology. If this is possible, this could lead to a contact counterpart $\mathcal{C}ont$ of $\mathcal{H}am$, extending the augmentation categories of Bourgeois and Chantraine [BC14]. It might be related to $\mathcal{H}am$ through boundary and symplectization functors, respectively $\mathcal{H}am \to \mathcal{C}ont$ and $\mathcal{C}ont \to \mathcal{H}am$.

Extension of the cotangent functor to singular manifolds. Ongoing work from Alvarez-Gavela, Eliashberg, Nadler and Starkston, aims at assigning to a singular space S (with additional structure) a Weinstein manifold M admitting S as a Lagrangian skeleton, which can be thought as a cotangent bundle of S. If furthermore S is endowed with a Lie group action, can this action be extended to a Hamiltonian action on M? At least for simple singularities, we will try to extend such actions. That would permit us to extend the cotangent functor to a category $\mathcal{L}ie_{\mathbb{R}}^{sing}$ similar with $\mathcal{L}ie_{\mathbb{R}}$, whose 1-morphisms might have singularities. That could facilitate computations for wrapped equivariant Floer homology, in terms of Morse theory on the skeleton.

Lie group actions on Fukaya categories. (with Thibaut Mazuir)

In [Tel14], Teleman conjectured that if M is a Hamiltonian G-manifold, then G must act "topologically" on $\mathcal{F}uk(M)$. He suggested that one way to formalize this statement would be to have the wrapped Fukaya category $\mathcal{W}(T^*G)$ acts on $\mathcal{F}uk(M)$ as an E_2-algebra.

We will apply our construction in [CHM] to Morse an Floer theory, and will show that:

- The Morse complex $CM(G)$ is a $\mathcal{H}opf_\infty$-algebra, and in particular an A_∞-bialgebra,
- If G acts on X, $CM(X)$ is an A_∞-bialgebra module over $CM(G)$.
- $CF(L,L)$ is an A_∞-bialgebra module over $CM(G)$.
- The wrapped Fukaya category $\mathcal{W}(T^*G)$ is a "$\mathcal{H}opf_\infty$-category".
- If M is a Hamiltonian G-manifold, its (wrapped) Fukaya category is an "A_∞-bialgebra module category" over $\mathcal{W}(T^*G)$.

The above last statement can be seen as an alternative answer to Teleman's conjecture.

Algebraic representations of $\widehat{\mathcal{H}am}$. The constructions from the last paragraph suggest an algebraic representation of $\widehat{\mathcal{H}am}$: one should get a 2-functor (ultimately a 3-functor) from $\widehat{\mathcal{H}am}$ to a more algebraic category (to be defined) by sending

- an object G to $\mathcal{W}(T^*G)$, seen as an A_∞-bialgebra category, (or just $CM(G)$, for simplicity)
- a morphisms $M\colon G \to H$ to $\mathcal{F}uk(M)$, seen as an A_∞-bialgebra bimodule co-category,
- a 2-morphism to an equivariant A_∞-bialgebra functor.

This will in particular permit to define a cornered instanton theory in a geometric way, as detailed in the next section.

4.2. **Gauge theory and low dimensional topology.**

Atiyah-Floer conjectures. One can state analogs of the Atiyah-Floer conjecture for HSI: are the groups $HSI(Y)$ (resp. their equivariant counterparts) isomorphic to some versions of instanton homology?

Some evidence suggests that HSI should be isomorphic to a framed version of instanton homology, defined by Kronheimer and Mrowka. Indeed, they share the same Euler Characteristic, the two functionals defining them have the same critical set, have the same rank for several examples, and satisfy analogous surgery exact sequences, see [Sca15]. It has been conjectured by Manolescu and Woodward that for integral homology spheres, HSI is isomorphic to the tilde version of instanton homology. We propose to construct an isomorphism, using moduli spaces of pairs of pseudoholomorphic curves and ASD connexions satisfying a matching condition, as in [Lip14] or [Fuk15].

Another problem is to compare the equivariant version of HSI with equivariant versions of instanton homology, such as the ones defined by Miller [Mil]. We expect that our definition in [CKMEY] with A_∞-actions should be well-suited for this purpose, as we use Bar constructions similar with the ones in [Mil]. A similar program is taken by Daemi and Fukaya [DF17].

Relative Donaldson polynomials and field theory in dimensions 1+1+1+1. Functoriality of the Donaldson polynomials is a central question in instanton theory: do these invariants admit a structure similar with a TQFT, i.e. a symmetric monoidal functor from $\mathcal{C}ob_{3+1}$ (or a variant incorporating cohomology classes)? This question was partially solved by Braam and Donaldson [BD95] after Floer's introduction of instanton homology. To a 4-manifold X with boundary $\partial X = Y$, Braam and Donaldson associate (under some assumptions) a polynomial invariant with values in (Fukaya's variant of) $I_*(Y)$. However this doesn't quite define a TQFT, mostly for two reasons. First, the groups $I_*(Y)$ are only defined for integral homology spheres, and second, their construction doesn't include the trivial representation, which results in a defect in the gluing formula when reducible connexions exist.

We will try to define analogous invariants, with values in the equivariant version of HSI. It is reasonable to expect that the two problems mentioned above will not occur in this setting. Indeed, the HSI groups are defined for any closed oriented 3-manifolds, and that should also be the case for the equivariant version. Moreover, since the extended moduli space is smooth at the trivial representation, one doesn't need to remove it, and we expect a more general gluing formula.

We will try to establish a functoriality extended to dimension 1: in [Caz19], we constructed a quasi 2-functor $\mathcal{C}ob_{1+1+1} \to \widehat{\mathcal{H}am}$ that we will try to promote to a symmetric monoidal (quasi-) 3-functor. The source 3-category $\mathcal{C}ob_{1+1+1+1}$ will have to incorporate the homology classes.

This would extend the functorial structure in dimension $(2+1+1)$ suggested by Wehrheim and Woodward with values in $\mathcal{S}ymp$ [Weh16]. Notice that for trivial bundle, this structure is mostly philosophical since the character varieties are singular and are not objects of $\mathcal{S}ymp$. As indicated before, the fact that they can be seen as 1-morphisms in $\widehat{\mathcal{H}am}$ should solve this issue.

Cornered instanton theory. In [DM14, DLM19], Douglas, Lipschitz and Manolescu defined "Cornered Heegaard-Floer theory", i.e. an extension of Heegaard-Floer theory to 3-manifolds with codimension 2 corners. In particular, they assign a 2-algebra to a closed 1-manifold. By composing the quasi 2-functor $\mathcal{C}ob_{1+1+1} \to \widehat{\mathcal{H}am}$ constructed in [Caz19] with the 2-functor $\widehat{\mathcal{H}am} \to ??$ outlined in the previous section, one should get a similar theory in the instanton side.

How are these two theories related? In particular, by dualizing one of the two products of the 2-algebras associated to the circle, is it possible to get an A_∞-bialgebra of the form $CM(G)$ for some Lie group G?

Extension to dimension zero. It is natural to ask whether the functorial structure outlined before should extend to dimension zero, i.e. can one define a 4-functor

$$F\colon \mathcal{C}ob_{0+1+1+1+1} \to \mathcal{C},$$

whose restriction $\mathcal{C}ob_{1+1+1+1} \to End_{\mathcal{C}}(F(\emptyset))$ essentially corresponds to the previous conjectural functor. In dimensions smaller than 3, such a functor seems to be possible to define, by the following observation due to Donaldson: the character variety of a closed surface e Σ - seen as the symplectic quotient by $SU(2)$ of the extended moduli space $\mathscr{N}(\Sigma')$ associated with Σ', obtained by removing a disk to Σ - can as well be seen as the symplecic quotient of an infinite dimensional moduli space $\mathscr{M}(\Sigma')$, by the loop group of $SU(2)$ (seen as the gauge group of $\partial\Sigma'$). It then becomes possible to allow Σ' to have corners, ant the action of the loop groups of the boundary allows us to glue such surfaces along a part of their boundary (see [Caz19, sec. 6.2] for more details). We will first define a 3-category $\mathcal{C}$ and a 3-functor $F\colon \mathcal{C}ob_{0+1+1+1} \to \mathcal{C}$, using these moduli spaces. Extending such a theory to dimension 4 seems interesting and challenging: it would involve Floer theory in infinite dimensional symplectic manifolds, which has been very little studied so far.

4.3. **Questions related to HSI.** Among all the existing Floer theories for 3-manifolds, Heegaard Floer homology is certainly the most developed one. Due to its rich algebraic and geometric structure ($Spin^c$-structures, $\mathbb{Z}[U]$-module structure), its numerous variants (for knots and links, for sutured manifolds, for cobordisms, concordances, contact structures) and its good computability (combinatorial versions), this theory has had many applications (e.g., detection of the knot genus and the Thurston norm, of fibered knots, obstructing Dehn surgeries) and many relations with other invariants (e.g., the Casson invariant, Alexander polynomial, Khovanov homology, embedded contact homology, monopole Floer homology).

It is expected that a lot of constructions coming from this theory should extend to the HSI setting.

Absolute grading. The HSI groups are relatively graded over $\mathbb{Z}/8\mathbb{Z}$, in the sense that only relative degrees between two homogeneous elements are well-defined. One should be able to define an absolute grading, in a similar way as in [OS03].

The maps F_{W,P_W} would then have a well-defined degree with respect to this absolute grading, which should be expressed by a formula from the algebraic topology of W. One would then be able to extract more information from the exact sequence in theorem 2.2, and push our computations mentioned in section 2 further.

Adjunction formula. The adjunction formula is one of the major tools for computations in Heegaard-Floer theory. It involves an extra input: a splitting of the groups along the set of $Spin^c$-structures of the 3-manifold. Unfortunately such a splitting remains missing for HSI. At least, the same strategy as in [OS04b] do not define any splitting, as the moduli space $\mathscr{N}(\Sigma')$ is simply connected, which implies that every pair of intersection points can be connected by a Whitney disc.

However, in instanton theory, a similar splitting along the set of (real) *Spin* structures was defined by Kronheimer and Mrowka as an eigenspace decomposition for operators defined with the μ map, see [KM10]. It would be interesting to try to adapt their construction, by defining a symplectic analogue of the μ map. This would be the first step towards an adjunction formula.

Knots, links and sutured manifolds. In Heegaard-Floer theory, homology groups were defined for knots [OS04a, Ras03], and then extended to balanced sutured manifolds [Juh06]. In analogy with Juhász's definition of sutured Floer homology, the definition of a sutured version for HSI should involve variations of the extended moduli spaces associated to surfaces with several boundary components. That corresponds to viewing a sutured manifold as a 2-morphism in $\mathcal{C}ob_{1+1+1}$, apply the quasi-functor from [Caz19] and then take Floer homology (equivariant or not).

Another possible approach would be by analogy of Kronheimer and Mrowka's singular instanton homology [KM11]: their construction involves instantons with traceless holonomy along meridians of the knot. Such a construction has a simple interpretation via the category $\mathcal{H}am$: the traceless holonomy condition corresponds to compose in $\mathcal{H}am$ the extended moduli spaces with coadjoint orbits in $\mathfrak{su}(2)^*$. We will use this construction and relate it with pillowcase homology, see the last paragraph.

Concordances, and cobordisms of sutured manifolds. Heegaard-Floer theory has led to the definition of plenty of concordance invariants, such as the τ invariant, the ν invariant, the ϵ invariant, and many others; see [Hom15] for a survey. Most of them use a filtration on the Floer chain complex induced by the knot. Hence an analog of this filtration in HSI would permit to reproduce similar constructions.

Minimal manifolds. In Heegaard Floer theory, rational homology spheres with minimal $\widehat{HF}$ groups, called L-spaces, have been studied extensively, and have several interesting properties. It is for instance conjectured, and proven in many cases, see [LS04, BRW05, BGW13], that:

Conjecture 4.1. *The three following properties are equivalent:*

- *being an L-space,*
- *having a non-left orderable fundamental group,*
- *carrying a taut foliation.*

One can similarly define rational homology spheres with minimal Symplectic Instanton Homology:

Definition 4.2. A rational homology sphere Y will be called HSI-minimal if:

$$rank(HSI(Y)) = \chi(HSI(Y)) = |H_1(Y;\mathbb{Z})|.$$

If question 4.5 has a positive answer, then being HSI-minimal is equivalent to being an L-space. In that case, HSI would be a possible way to attack conjecture 4.1: for the non-left orderability, one could first try to translate this property in terms of the $SU(2)$-representation variety and its (intersection) homology, and then use the spectral sequence described in the next paragraph.

If question 4.5 has a negative answer, it would be interesting to compare the set of HSI-minimal manifolds and the set of L-spaces, and to investigate if there are similar geometric characterizations of HSI-minimal manifolds.

Relations with the $SU(2)$-representation variety. A key feature of HSI is that the unperturbed Lagrangian intersection $L(H_0) \cap L(H_1)$ defining the chain complex corresponds to the $SU(2)$-representation variety of the fundamental group of the 3-manifold Y.

Whenever this intersection is clean, i.e. $L(H_0)$ and $L(H_1)$ intersect along a submanifold in an essentially transverse fashion, there exists a spectral sequence whose first page can be identified with the Morse homology of $L(H_0) \cap L(H_1)$, and converging to $HSI(Y)$ [Poź99, Sch16]. If the intersection is not clean, a spectral sequence should still exist, but it would no longer be true that the first page is the homology of $L(H_0) \cap L(H_1)$.

Question 4.3. *Does there exist a similar spectral sequence whose E^∞-page is equal to $HSI(Y)$ for every 3-manifold? If so, is this spectral sequence a topological invariant of Y? In the non-clean intersection case, can one identify the first page of the spectral sequence with known invariants (such as intersection homology or stratified Morse homology) associated to the real algebraic variety $L(H_0) \cap L(H_1)$?*

Question 4.4. *For M a Hamiltonian G-manifold, and L_0, L_1 a pair of Lagrangians contained in the zero level of the moment map, can one define an equivariant version of this spectral sequence, that would have as first page the equivariant (intersection) homology of $L_0 \cap L_1$, and would converge to the Lagrangian Floer homology?*

Applied to HSI, this spectral sequence would relate the equivariant version to the $SU(2)$-character variety.

Relation with classical invariants. From the spectral sequence in clean intersection, one can obtain several bounds of the rank of HSI by the Casson invariant for Brieskorn spheres (see [Caz16a, sec. 7.3.3]). Since the Casson invariant corresponds to the intersection number of $\mathscr{L}(H_0)$ and $\mathscr{L}(H_1)$ inside the character variety, it should be related to the Euler characteristic of the equivariant version.

There should also be a relation with Khovanov homology. In Heegaard-Floer homology [OS05], there exists a spectral sequence (different from the clean intersection spectral sequence mentioned previously) starting from the Khovanov homology of a knot and converging to the Heegaard-Floer homology of its branched double cover. As this follows mainly from the surgery exact sequence, similar spectral sequence should be obtained in the same way.

Comparison with Seiberg-Witten theory. A deep conjecture due to Witten relates Donaldson and Seiberg-Witten invariants of a 4-manifold, [Wit94].

Just as HSI is a symplectic counterpart of (framed) instanton homology, Heegaard-Floer homology is a symplectic counterpart of monopole Floer homology [KLT10]. It should be interesting to find a symplectic analogue of a 3-dimensional version of Witten's conjecture. In these lines, Manolescu and Woodward ask the following question, whose answer is yes for all the computations mentionned in section 2.

Question 4.5. *(*[MW12, Question 7.5]*) With coefficients in $\mathbb{Q}$, are the ranks of HSI and $\widehat{HF}$ equal?*

Let the quasi 2-functor defined in [Caz19] be denoted:

$$F_{Don}\colon \mathcal{C}ob_{1+1+1} \dashrightarrow \widehat{\mathcal{H}am}.$$

One can ask whether Seiberg-Witten theory admits a similar structure. In joint work Guangbo Xu, we plan to define an analogous 2-functor (possibly the target 2-category will have to change slighly):

$$F_{SW}\colon \mathcal{C}ob_{1+1+1} \dashrightarrow \widehat{\mathcal{H}am},$$

and also a natural transformation:

$$T\colon F_{Don} \to F_{SW},$$

namely a transformation assigning to a k-morphism $W\colon X \to Y$ of $\mathcal{C}ob_{1+1+1}$, a $(k+1)$-morphism in $\widehat{\mathcal{H}am}$ relating $F_{Don}(W)$ and $F_{SW}(W)$ as in the following diagram:

$$\begin{array}{ccc} F_{Don}(X) & \xrightarrow{F_{Don}(W)} & F_{Don}(Y) \\ \Big\downarrow{\scriptstyle T(X)} & \Downarrow T(W) & \Big\downarrow{\scriptstyle T(Y)} \\ F_{SW}(X) & \xrightarrow{F_{SW}(W)} & F_{SW}(Y). \end{array}$$

We will build on work of Feehan and Leness [FL01a, FL01b].

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