Robust Portfolio Choice and Indifference Valuation

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Setting

An agent starts with an initial wealth, say x, which he can invest into a riskless bond and several risky assets. At maturity time T the agent will additionally receive a payoff H.

How can the agent determine his optimal portfolio strategy?

To answer this question one first has to address the following issues:

- How to model the payoff and the risky asset?
- How to evaluate the quality of the agent's portfolio strategy?
- Which constraints to impose on the trading strategies allowed?

Setting

For the dynamics of the assets we assume a continuous-time setting with jumps and ambiguity.

- Ambiguity: 'True' probabilistic model is unknown.
- Jumps: Economic shocks like financial crashes, unexpected announcements of the ECB, environmental disasters causing sudden movements in prices.

Setting

Consider a probability space $(\Omega, \mathcal{F}, P)$ with two independent stochastic processes:

- A standard d-dimensional Brownian Motion W.
- A Poisson counting measure $N(ds, dx)$ on $[0, T] \times \mathbb{R} \setminus \{0\}$ with compensator

$$\hat{N}(ds, \omega, dx) = n(s, \omega, dx)ds.$$

We assume that the measure $n(s, dx)$ is predictable and satisfies

$$\left\| \sup_s \int_{\mathbb{R}\setminus\{0\}} (|x|^2 \wedge 1) n(s, dx) \right\|_\infty < \infty.$$

Define

$$\tilde{N}(ds, dx) = N(ds, dx) - n(s, dx)ds.$$

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We assume that the financial market consists of one bond with interest rate zero and $n \leq d$ stocks. The return of stock i under a reference measure P evolves according to

$$\frac{dS_t^i}{S_{t-}^i} = b_t^i dt + \sigma_t^i dW_t + \int_{\mathbb{R}\setminus\{0\}} \beta_t^i(x)\tilde{N}(dt,dx), \quad i = 1,\dots,n,$$

where b^i, σ^i, β^i are $\mathbb{R}$, $\mathbb{R}^d$, $\mathbb{R}$-valued, predictable, uniformly bounded, stochastic processes.
Assume $\beta^i > -1$ for $i = 1,\dots,n$. Set $b = (b^i)_{i=1,\dots,n}$ $\sigma_t = (\sigma_t^i)_{i=1,\dots,n}$, and $\beta = (\beta^i)_{i=1,\dots,n}$. Further suppose σ has full rank and is uniformly elliptic, and

$$\left\| \sup_s \int_{\mathbb{R}\setminus\{0\}} |\beta_s(x)|^2 n(s,dx) \right\|_\infty < \infty.$$

Write $\beta \in L^{\infty,2}$.

- Denote by π_t^i the amount of money invested in the i-th risky asset at time t.
- Denote by $(X_t^{(\pi)})$ the wealth process of a trading strategy π with initial capital x. In other words $X_t^{(\pi)}$ is the total value of the portfolio at time t.

Definition

Let U be a compact set in $\mathbb{R}^{1\times n}$. The set of admissible trading strategies $\mathcal{A}$ consists of all n-dimensional predictable processes $\pi = (\pi_t)_{0\leq t\leq T}$ which satisfy $\pi_t \in U$ $dP \times ds$ a.s.

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Choice under uncertainty

Specifying the measure P implies estimating σ_t, b_t, and $\beta_t(x)n(t, dx)$. However, since the trader does not know these quantities he faces ambiguity.

Many approaches in the literature to make choices under uncertainty are based on axiomatic foundations of preferences:

Decision criteria for a payoff H:

- Subjective expected utility: $U(H) = \mathbb{E}_Q[u(H)]$, Savage (1954).
- Multiple priors: $U(H) = \min_{Q \in M} \mathbb{E}_Q[u(H)]$, Gilboa and Schmeidler (1989).
- Variational preferences: $U(H) = \min_Q\{\mathbb{E}_Q[u(H)] + c(Q)\}$, Maccheroni, Marinacci and Rustichini (2006).

The portfolio selection problem

Let H be a bounded contingent claim. We start with a probabilistic reference model P.
The class of all alternative models considered will be given by $\mathcal{Q} = \{Q | Q \ll P\}$. The robust portfolio selection problem is given by

$$V(H) = \max_{\pi \in \mathcal{A}} U(H + X_T^{(\pi)}),$$

where $X^{(\pi)}$ is the wealth process arising from an portfolio strategy π. U is an evaluation based on variational preference.

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Different probabilistic models

What does a different model $Q \in \mathcal{Q}$ entail for the evolution of the asset return?

Let $\mathcal{P}$ be the predictable σ-algebra. One can show that every model Q is uniquely characterized by a predictable drift (q_t), a $\mathcal{P} \otimes \mathbb{B}(\mathbb{R} \setminus \{0\})$-measurable $\psi_s(x)$ such that under the model Q:

- $W_t - \int_0^t q_s ds$ is a Brownian motion.
- $N(ds, dx)$ has a compensator given by $n^Q(s, dx) = (1 + \psi_s(x))n(s, dx)$.

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The choice of the penalty function

A standard example for the penalty function is the relative entropy, i.e.,

$$c(Q) = \gamma H(Q|P) = \gamma \mathrm{E}_Q \left[\log \left(\frac{dQ}{dP} \right) \right], \quad \gamma > 0$$

see for instance Hansen and Sargent (1995, 2000, 2001). In our setting it may be seen that

$$H(Q|P) = \mathrm{E}_Q \Big[\int_0^T \Big\{ r_1(q_s) + \int_{\mathbb{R}\setminus\{0\}} r_2(\psi_s(x)) n(s, dx) \Big\} ds \Big],$$

with $r_1(q) = \frac{|q|^2}{2}$,

$$r_2(y) = \begin{cases} (1+y)\log(1+y) - y, & \text{if } y \geq -1; \\ \infty, & \text{otherwise.} \end{cases}$$

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Assumptions

- The plausibility index c is of the form

$$c(Q) = \mathrm{E}_Q\Big[\int_0^T \Big\{r_1(s, q_s) + \int_{\mathbb{R}\setminus\{0\}} r_2(s, x, \psi_s(x))n(s, dx)\Big\}ds\Big],$$

for convex non-negative functions r_1 and r_2 which are continuous on their domain with $r_1(t, 0) = r_2(t, x, 0) = 0$.

Assumptions

- There exist $K_1, K_2 > 0$ such that

$$c(Q) \geq -K_1 + K_2 H(Q|P)).$$

- There exist a $\hat{K}_1, \hat{K}_2$,

$$|\partial_q r_1(t,q)| \geq -\hat{K}_1 + \hat{K}_2 |q|.$$

Furthermore, for every $C > 0$ there exist $\hat{K}_3 > 0$ and a process $\hat{K}_4(x) \in L^{\infty,2}$ such that

$$|\partial_y r_2(t,x,y)| \geq -\hat{K}_4(x) + \hat{K}_3 |\log(1+y)| \text{ for } y \in [-1, C].$$

- u is linear, exponential, or logarithmic.

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Relation to previous works

- Static Duality methods: Biagini and Frittelli (2004), Schachermayer (2004).
- BSDEs have been used in utility maximization problems
  - in a Brownian framework by Skiadas (2003), Hu, Imkeller and Müller (2005), Cheridito and Hu (2010), or Horst et al. (2011).
  - in a framework with continuous or non-continuous filtrations by Mania and Schweizer (2005), Becherer (2006), Bordigoni et al. (2007), or Morlais (2009a),
  - in a framework with unpredictable jumps in the asset price by Jeanblanc et al. (2009), or Morlais (2009b), (2010).
  - in a Brownian framework for evaluations given by BSDEs by Klöppel and Schweizer (2005) and Sturm and Sircar (2011)
  - in utility maximization with ambiguity by Müller (2005), Delong (2011) and Øksendal and Sulem (2011).

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u linear

The optimization problem is $V(H) = \max_\pi U(H + X_T^{(\pi)})$. Assume first that u is linear. Define

$$\begin{aligned} g_1(t,z) : &= \sup_{q \in \mathbb{R}^d} \left\{ zq - r_1(t,q) \right\}; \\ g_2(t,x,\tilde{z}) : &= \sup_{y \in \mathbb{R}} \left\{ y\tilde{z} - r_2(t,x,y) \right\} \end{aligned}$$

Note that and $g_i \geq 0$ are convex functions with minimum $g_1(t,0) = g_2(t,x,0) = 0$.

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Variational preferences with a linear u

If u is linear the dynamic evaluation according to variational preferences is given by

$$U_t(H) = \min_{Q \in \mathcal{Q}} \left\{ \mathrm{E}_Q \left[H \middle| \mathcal{F}_t \right] - c_t(Q) \right\}.$$

We can show that there exist unique suitably integrable processes Z and $\tilde{Z}$ such that

$$U_t(H) = H - \int_t^T \left[g_1(s, Z_s) + \int_{\mathbb{R}\setminus\{0\}} g_2(s, x, \tilde{Z}_s(x)) n(s, dx) \right] ds + \int_t^T Z_s dW_s + \int_t^T \int_{\mathbb{R}\setminus\{0\}} \tilde{Z}_s(x) \tilde{N}(ds, dx)$$

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$$+ \int_t^T Z_s dW_s + \int_t^T \int_{\mathbb{R}\setminus\{0\}} \tilde{Z}_s(x) \tilde{N}(ds, dx)$$

Theorem

Suppose that we start with functions $g_1, g_2 \geq 0$ *with* $g_1(t,0) = g_2(t,x,0) = 0$. *Assume further:*

(a) *There exists* $K' > 0$ *such that*

$$g_1(t,z) \quad \leq \quad K'(1+|z|^2).$$

For every $C > 0$ *there exists* $K'' > 0$ *and* $\tilde{K}' \in L^{2,\infty}$ *such that*

$$g_2(t,x,\tilde{z}) \quad \leq \quad \tilde{K}'(x) + K''|\tilde{z}|^2 \text{ for all } |\tilde{z}| \leq C.$$

(b) $|\partial_z g_1(t,z)| \leq \bar{K}(1+|z|)$ *for* $z_1, z_2 \in \mathbb{R}^d$

(c) *For every* $\tilde{C} > 0$ *there exists* $\hat{K} > 0$ *and* $\tilde{H} \in L^{\infty,2}$ *such that*

$$|\partial_y g_2(t,x,y)| \leq \tilde{H}(x) + \hat{K}|y| \text{ for } x \in \mathbb{R} \text{ and } y \in [-1, \tilde{C}].$$

Then for every bounded terminal condition F *the corresponding BSDE with driver* $g(t,z,\tilde{z}) = g_1(t,z) + \int_{\mathbb{R}\setminus\{0\}} g_2(t,\tilde{z}(x))n(t,dx)$ *has a unique bounded solution.*

Define

$$f(t,z,\tilde{z}) := \min_{\pi \in U}\{-\pi b_t + g_1(t, z - \pi\sigma_t) + \int_{\mathbb{R}\setminus\{0\}} g_2(t,x,\tilde{z}(x) - \pi\beta(x))n(t,dx)\}.$$

Theorem

Let $(Y_t, Z_t, \tilde{Z}_t)$ be the unique solution of the BSDE with terminal condition H and driver function f. Then we have

$$V(H) = Y_0 + x.$$

Furthermore, the optimal strategy is given by the strategy π^* which attains the minimum in $f(t, Z_t, \tilde{Z}_t)$.

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Interpretation

$$f(t,z,\tilde{z}) = \min_{\pi\in U}\{-\pi b_t + g_1(t, z-\pi\sigma_t) + \int_{\mathbb{R}\setminus\{0\}} g_2(t,x,\tilde{z}(x)-\pi\beta(x))n(t,dx)\}.$$

When choosing π the trader faces a tradeoff between:

(a) Getting the excess return $-\pi_s b_s$.

(b) Diminishing the fluctuation of the future payoff coming from the locally Gaussian part, this means choosing π such that $|Z_s - \pi_s\sigma_s|$ is small.

(c) Diminishing the fluctuation of the future payoff coming from the jumps, this means choosing π such that $|\tilde{Z}_s - \pi_s\beta_s|$ is small.

Relationship of the optimal portfolio selection problem and the excess return

The KKT conditions yields that there exists Lagrange multiplier $\mu^*, \zeta^* \in \mathbb{R}^n$ with $\mu^*, \zeta^* \geq 0$ such that

$$\begin{aligned} b_s &= (\mu_s^* - \zeta_s^*) - \sigma_s \partial_z g_1(s, z - \pi\sigma_s) \\ &\qquad - \int_{\mathbb{R}\setminus\{0\}} \partial_{\tilde{z}} g_2(s, x, \tilde{z}_s(x) - \pi\beta_s(x))\beta_s(x) n(s, dx) \\ &= A + B + C, \end{aligned}$$

where:

- A: Sensitivity of f with respect to the constraints.
- B : Sensitivity of f with respect to Z, the fluctuation of the evaluation due to the Brownian motion.
- C : Sensitivity of f with respect to $\tilde{Z}$, the fluctuation of the evaluation due to the jumps.

Multiple priors with a CARA utility function

Start again with a reference model P. Let M be the set of all models which are 'close' to P. Specifically choose $\lambda \geq 0$ and $\mathcal{P} \otimes \mathbb{B}(\mathbb{R} \setminus \{0\})$-measurable processes $d^-(x), d^+(x) \in L^{\infty,2}$ Denote

$$M := \Big\{ Q \ll P \Big| ||q||_\infty \leq \lambda, \text{ and } d_s^-(x) \leq \psi_s(x) \leq d_s^+(x) \Big\}.$$

With a CARA utility function the problem becomes

$$V(H) = \max_{\pi} \min_{Q \in M} -\mathrm{E}_Q \left[\exp\{-\alpha(H + X_T^{(\pi)})\} \right] \text{ for } \alpha > 0.$$

Ambiguity with a CARA utility function

Theorem

We have $V(F) = -\exp\{-\alpha(x + Y_0)\}$ *where* Y *is the unique solution of the backward stochastic equation with terminal payoff* H *and driver function*

$$\min_{\pi \in U} \Bigg\{ -\pi b_t + \frac{\alpha}{2}|Z_t - \pi\sigma_t|^2 + \lambda|Z_t - \pi\sigma_t| + \frac{1}{\alpha}\Big(\exp\{\alpha(\tilde{Z}_t(x) - \pi\beta_t(x))\} - \alpha(\tilde{Z}_t(x) - \pi\beta_t(x)) - 1 \Big) + \Big(d_s^{+}(x) I_{\{\pi\beta_t(x) \le \tilde{Z}_t(x)\}} + d_s^{-}(x) I_{\{\pi\beta_t(x) \ge \tilde{Z}_t(x)\}} \Big) \times \frac{\exp\Big(\alpha(\tilde{Z}_t(x) - \pi\beta_t(x)\Big) - 1}{\alpha} \Bigg\},$$

Furthermore, the optimal portfolio strategy is given by π^* *which minimizes the expression above.*

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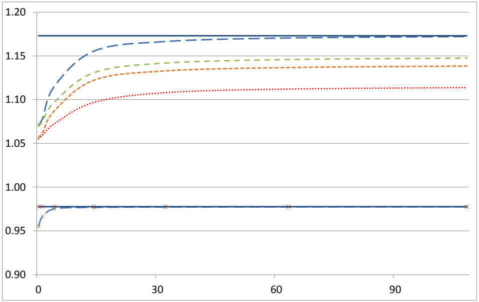
Numerical Results

Assume a degenerate one point jump distribution with intensity 1. We consider a European put option with strike price 2 and time-to-maturity of 0.5 years. We take
$b = 0.04,\ \sigma = 0.2,\ a = 1,\ \beta = 0.03,\ u_{upper} = 10$ and $u_{lower} = 0$.
The number of simulations is 10,000.



- (i) no ambiguity, no hedge (long dashes with cross);
- (ii) no ambiguity, with hedge (long dashes);
- (iii) Brownian ambiguity only ($\lambda = 0.25$), with hedge (dashes);
- (iv) jump ambiguity only ($d^- = -0.25$ and $d^+ = 0.5$), with hedge (short dashes);
- (v) both Brownian ambiguity and jump ambiguity ($\lambda = 0.25$, $d^- = -0.25$ and $d^+ = 0.5$) with hedge. (dots)

The KKT conditions of the optimization problem yield

$$b_t = A + B + C + D + E$$

- A: Due to the hedging constraints.
- B : Due to the risk coming from the Brownian part. Vanishes if $\alpha \downarrow 0$, or if there is no Gaussian part.
- C : Due to the risk coming from the jumps. Vanishes if $\alpha \downarrow 0$, or if there are no jumps.
- D : Due to the ambiguity coming from the Brownian motion. Vanishes as $\lambda \downarrow 0$.
- E : Due to the ambiguity coming from the jumps. Vanishes if $d^+, d^- \to 0$, or if there are no jumps.

Variational preferences with a logarithmic utility

We will consider trading strategies ρ which denote the part of wealth invested in stock i. The admissible trading strategies are supposed to take values in a compact set C and $\rho_s \beta_s \geq -1 + \epsilon$. We denote the wealth process corresponding to a trading strategy ρ with initial capital x by $X^{(\rho)}$.

Portfolio selection problem with a logarithmic utility

We want to maximize

$$\inf_{Q\in\mathcal{Q}} \mathrm{E}_Q\left[\log\left(X_T^{(\rho)}\right) + \int_t^T \left\{ r_1(s,q_s) + \int_{\mathbb{R}\setminus\{0\}} r_2(s,x,\psi(x)n(s,dx)\right\} ds\right],$$

over all admissible strategies ρ. Let

$$\begin{aligned} f(s,z,\tilde{z}) &: \\ &= \inf_{\rho\in C}\left\{ -\rho b_s + \int_{\mathbb{R}\setminus\{0\}} g_2(s,x,\tilde{z}(x) - \log(1+\rho\beta_s(x)))n(s,dx) \right. \\ & \left. g_1(t, z-\rho\sigma_s) + \frac{|\rho|^2}{2} - \int_{\mathbb{R}\setminus\{0\}} [\log(1+\rho\beta_s(x)) + \rho\beta_s(x)]n(s,dx)\right\}. \end{aligned}$$

Denote by Y the solution of the BSDE

$$Y_t = 0 + \int_t^T f(s, Z_s, \tilde{Z}_s)ds - \int_t^T Z_s dW_s - \int_t^T \int_{\mathbb{R}\setminus\{0\}} \tilde{Z}_t(x)\tilde{N}(ds, dx).$$

Theorem

The BSDE has a unique solution and the value of the portfolio selection problem under ambiguity with a logarithmic utility is given by

$$V(x) = Y_0 + \log(x).$$

Furthermore, the optimal strategy is given by the trading strategy ρ^* which attains the minimum in the driver function $f(t, Z_t, \tilde{Z}_t)$.

Robust portfolio selection with a logarithmic utility

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