BSDEs, Numerics and Finance

Oxford, 2 – 4 July 2012

Supported by:

Oxford Man Institute
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St John's College, Oxford

With the cooperation of SMAI

Timetable

Monday

2:00	Welcome	
2:15	Daniel Lacker	*Weak formulation of mean field games*
2:45	Kevin Webster	*Order book model with heterogeneous, competitive agents*
3:15	Jianing Zhang	*Dual representations for general multiple stopping problems*
3:45	Afternoon Tea	
4:15	Weinning Wei	*Representation of Dynamic Time-Consistent Convex Risk Measures with Jumps*
4:45	Zhiyong Yu	*Probabilistic interpretation for systems of quasilinear parabolic PDEs combined with algebra equations*
5:15	Samuel Drapeau	*Minimal Supersolutions of BSDEs and Robust Hedging*
5:45	Reception	

Tuesday

9:00	Arrive	
9:15	Xiaolu Tan	*A numerical scheme for a class of non-Markovian stochastic control problem: A probabilistic approach*
9:45	Polynice Oyono	*Convolution Method for BSDEs*
10:15	Plamen Turkedjiev	*Approximating BSDEs using least-squares regression and Malliavin weights*
10:45	Morning Tea	
11:15	Shuntai Hu	*The utility indiference price of the defaultable bond in a jump-diffusion model*
11:45	Mitja Stadje	*Portfolio Selection and Indifference Valuation*
12:15	Anthony Réveillac	*FBSDEs for the utility maximization problem*
12:45	Lunch	
2:00	Jinniao Qiu	*American Option Pricing Problems and Reflected BSPDEs*
2:30	Jia Zhuo	*A probabilistic Numerical Method for Fully Nonlinear Parabolic PDEs*
3:00	Salvador Ortiz-Latorre	*Optimal simulation schemes for Levy driven SDEs*
3:30	Afternoon Tea	
4:00	Dylan Possamai	*Second order BSDEs with quadratic growth and applications to utility maximization under volatility uncertainty*
4:30	Nabil Kazi-Tani	*Existence, Uniqueness and Representation of Second Order Backward SDEs With Jumps*
5:00	Romuald Elie	*Exact replication under portfolio constraint: a viability approach for BSDE*

Wednesday

9:00	Arrive	
9:15	Quan Yuan	*Valuation and Optimal Decision for Perpetual American ESOs*
9:45	Sébastien Choukroun	*Mean variance hedging in a multiple default optimal investment problem using default-density modeling*
10:15	Thomas Lim	*Mean-variance hedging when there is a default time*
10:45	Morning Tea	
11:15	Hao Xing	*On BSDEs and strict local martingales*
11:45	Kihun Nam	*BSDEs with terminal conditions with bounded Malliavin derivative*
12:15	Idris Kharroubi	*BSDEs with partially constrained jumps and nonlinear second order IPDEs*
12:45	Lunch	
2:00	Xinpeng Li	*Limit Theorems under Sublinear Expectations and Probabilities*
2:30	Yongsheng Song	*Backward Stochastic Differential Equations Driven by G-Brownian Motion*
3:00	Kai Du	*On the long-term asymptotic exponential arbitrage*
3:30	Afternoon Tea	
4:00	Julien Azzaz	*Numerical scheme for quadratic BSDEs and Dynamic Risk Measures.*
4:30	Adrien Richou	*Time discretization of Markovian quadratic and superquadratic BSDEs with an unbounded terminal condition*
5:00	Jean-François Chassagneux	*High order discrete-time approximations for BSDEs*
6:45	Conference Dinner (St Giles House)	

Abstracts

Julien Azzaz (Université de Lyon 1)

Numerical scheme for quadratic Backward Stochastic Differential Equations and Dynamic Risk Measures

We consider stability properties on sets of probability measures. Those properties may be related to dynamic consistency of some specific Dynamic Risk Measures, which are expressed as solutions of quadratic Backward Stochastic Differential Equations. Those operators can also be interpreted in a dual representation using sets of probability measures. At this point, it is possible to apply stability properties in order to construct efficient numerical schemes. Next, we consider a Forward Backward framework to tackle some financial and insurance problems. The goal of numerical schemes is to provide approximation of Dynamic Risk Index which can be used by agents to compare risky positions. Finally we provide a discussion on implications of this method in some specific examples, especially with regards to the approximations of the numerical schemes and in case of presence of jumps in the underlying process.

Jean-François Chassagneux (Imperial College London)

High order discrete-time approximations for BSDEs

We study the convergence of Runge-Kutta type schemes and linear multistep schemes for BSDEs in a Markovian framework. The study of the convergence is done by rst proving some kind of stability for the scheme and then analysing the local truncation error. For this last part, we assume that the solution of the BSDE is given by a smooth function solution of a semilinear PDE and use Ito-Taylor expansion. Our methods allows theoretically to build a scheme with any given order of convergence.

Sébastien Choukroun (Paris 7)

Mean variance hedging in a multiple default optimal investment problem using default-density modeling

We solve an optimal investment problem using Mean-Variance Hedging approach in an incomplete market where multiple default can appear. For this, we use a default-density modeling approach. The global market information is formulated as progressive enlargement of a default-free Brownian ltration and the dependence of default times is modeled by a conditional density hypothesis. We prove the quadratic form of each value processes between each defaults times and solve the recursively system of backward stochastic dierential equations which give the coecients of the quadratic representation. We illustrate our results with some specic cases.

Samuel Drapeau (Humboldt Universität Berlin)

Minimal Supersolutions of BSDEs and Robust Hedging

We study minimal supersolutions of BSDEs – related to Peng's g-expectation – which can be seen as superhedging functionals. We prove existence, uniqueness, monotone convergence, Fatou's Lemma and lower semicontinuity of our functional. Unlike usual BSDE methods, based on xed point theorems, the existence relies on compactness methods. We then study some robust extensions which correspond to the problem of superhedging under volatility uncertainty. The talk is based on joint works with Gregor Heyne and Michael Kupper.

Kai Du (ETH Zürich)

On the long-term asymptotic exponential arbitrage

The main goal of this talk is to give an extension and a proof of the conjecture in Föllmer and Schachermayer [Math. Financ. Econ., 2007] that in the case where the stock price process S is a continuous semi-martingale and satisfies a large deviations estimate, S allows asymptotic exponential arbitrage with exponentially decaying failure probability.

Romuald Elie (Université Paris-Dauphine)

Exact replication under portfolio constraint: a viability approach for BSDE

In this talk, we consider the problem of super-replicating a given contingent claim, whenever the incompleteness of the market is due to the presence of closed convex constraints on the portfolio strategies, written in terms of number of shares. In the dimension 1 Black Scholes model, Broadie, Shreve and Soner observed that the price under constraint of a given claim is simply the unconstrained price of a more expensive claim, defined as the facelift transform of the one of interest. For a given model and convex constraint set in dimension d, we exhibit a necessary and sufficient condition under which the latter is true for a large class of European options. Our argumentation relies on the use of viability arguments for BSDEs together with localization procedures. Several financial examples will be considered in this talk. This is a joint work with Jean-François Chassagneux, Imperial College and Idris Kharroubi, University Paris-Dauphine.

Shuntai Hu (Tongji University, Shanghai)

The utility indifference price of the defaultable bond in a jump-diffusion model

This talk discusses utility indifference pricing model for a defaultable bond. We characterize default by intensity-based model, in which the first jump of an exogenous Poisson process is regarded as the default time. A jump-diffusion process is introduced to drive the firms stock price, so as to more accurately reflect the real market. The firms asset is nontradable and credit risk cant be hedged, therefore utility indifference pricing model becomes our choice. By virtual of dynamic programming principle, we derive the integro-differential HJB equations for two optimal portfolio problems, and prove that the value functions are the viscosity solutions of the HJB equations. Furthermore, comparison principle is also proved. Finally, some numerical results are given to illustrate the model.

Nabil Kazi-Tani (École Polytechnique)

Existence, Uniqueness and Representation of Second Order Backward SDEs with Jumps

We define a notion of second order backward stochastic differential equation with jumps, for which we prove existence and uniqueness in appropriate spaces. We can interpret these equations as standard BSDEs with jumps, under both volatility and jump measure uncertainty. We also prove a representation of the Y part of a second order BSDE with jumps as an essential supremum of solutions of classic BSDEs with jumps. By definition, these equations must hold $\mathbb{P}$-a.s., where $\mathbb{P}$ lies in a wide family of probabilty measures, corresponding to laws on the Skorohod space $\mathbb{D}$ of some cadlag local martingales. These equations are the natural candidates for the probabilistic interpretation of fully non linear partial integro-differential equations.

An important issue is the possibility to aggregate, in the sense of [Soner, Touzi and Zhang (2010)] and [Cohen (2011)], both the quadratic variation and the jump measure of the canonical process on $\mathbb{D}$. For this purpose, we extend in this paper the aggregation result obtained in [Soner et al. (2010)] to our context with jumps, the aggregation being valid for a family of probabilty measures obtained as solutions of martingale problems on $\mathbb{D}$.

As an application of these results, we treat a robust exponential utility maximization problem under model uncertainty. The uncertainty affects both the volatility process and the jump measure. We prove existence of an optimal strategy, and that the value function of the problem is the unique solution of a particular second order BSDE with jumps.

Idris Kharroubi (Université Paris-Dauphine)

BSDEs with partially constrained jumps and nonlinear second order IPDEs

We consider a class of BSDE where the jumps component of the solution is subject to a partial constraint. After proving existence and uniqueness of a minimal solution under mild assumptions, we provide a dual representation of this solution as an essential supremum process over some specific change of probability. We then concentrate on the Markovian case and show that this class of BSDEs provides a probabilistic representation for solutions of second order integral PDEs of HJB type.

Daniel Lacker (Princeton University)

Weak formulations of mean field Games

The theory of mean field games provides a convenient methodology for finding approximate Nash equilibria for large-population stochastic differential games. We present a probabilistic approach to mean field games based on the weak formulation of stochastic optimal control problems, a crucial ingredient of which is a BSDE representation for the value function in the case that volatility is uncontrolled. General conditions are given which guarantee the existence and uniqueness of a solution to the mean field game problem; an advantage of the weak formulation is that minimal regularity assumptions are required on the drift of the state process SDE. Furthermore, the "mean field game value function" turns out to be an interesting type of filtration-consistent nonlinear expectation which typically fails to be monotone. Using g-expectation results we prove that the value function is monotone if and only if the Hamiltonian has no mean field term.

Xinpeng Li (Shandong University / Paris 1)

Limit Theorems under Sublinear Expectations and Probabilities

We will give the limit theorems under sublinear expectations and probabilities. One is the law of large numbers. The concept of independence is essential in this case. We give three definitions of independence and prove the corresponding law of larger numbers in weak and strong senses. Another limit theorem is central limit theorem under sublinear expectation framework, which can be regard as a central limit theorem for martingales with variance uncertainty. As an application, we use it to calculate the maximal L_p-variation for martingales.

Thomas Lim (University d'Evry)

Mean-variance hedging when there is a default time

In this work, we consider a financial market composed by assets subject to default. We study in this market the problem of mean-variance hedging when the terminal time is the default time. This problem is formulated as a stochastic control problem, and we combine decomposition results coming from the filtration enlargement theory and Brownian BSDEs technics to solve it.

Kihun Nam (Princeton University)

BSDEs with terminal conditions with bounded Malliavin derivative

We show the existence and uniqueness of a solution to BSDEs in the case where the terminal condition is bounded with a bounded Malliavin derivative and the driver F is of the form $F(s, y, z) = f(s, y, z) + zg(s, y, z)$. We consider the case when $f(s, y, z)$ is Lipschitz in y and locally Lipschitz in z and $zg(s, y, z)$ is locally Lipschitz with arbitary growth rate in y and z. In particular, $f(s, y, z)$ is allowed to grow superquadratically in z. Then, using the Markovian BSDE result, we present the corresponding result on semilinear parabolic PDE.

Salvador Ortiz-Latorre (Imperial College London)

Optimal simulation schemes for Levy driven SDEs

Abstract: In this talk we consider a general class of high order weak approximation schemes for stochastic differential equations driven by Levy processes with innite activity. These schemes combine a compound Poisson approximation for the jump part of the Levy process with a high order scheme for the Brownian driven component, applied between the jump times. The overall approximation is analyzed using a stochastic splitting argument. The resulting error bound involves separate contributions of the compound Poisson approximation and of the discretization scheme for the Brownian part, and allows, on one hand, to balance the two contributions in order to minimize the computational time, and on the other hand, to study the optimal design of the approximating compound Poisson process.

Polynice Oyono (Concordia University, Montreal)

Convolution Method for BSDEs

This article deals with numerical solution to backward stochastic differential equations. A numerical method which finds its roots in Fourier analysis is proposed. The method consists in solving the underlying PDE using the Euler time discretization of the backward stochastic differential equation. A Fourier analysis then allows to compute the conditional expectations that appear in the Euler scheme with the FFT algorithm. The problem of error control is briefly addressed and we give some numerical examples including the extension of the method to reflected backward stochastic differential equations.

Dylan Possamai (École Polytechnique)

Second order BSDEs with quadratic growth and applications to utility maximization under volatility uncertainty

Recently, motivated by applications in financial mathematics and probabilistic numerical methods for PDEs, Cheredito, Soner, Touzi and Victoir introduced the notion of Second order BSDEs (2BSDEs), which are connected to the class of fully nonlinear PDEs. Then, Soner, Touzi and Zhang provided a complete theory of existence and uniqueness for 2BSDEs under uniform Lipschitz conditions similar to those of the original article of Pardoux and Peng. Their key idea was to reinforce the condition that the 2BSDE must hold $\mathbb{P}-a.s.$ for every probability measure $\mathbb{P}$ in a non-dominated class of mutually singular probability measures.

Our first aim is to relax the Lipschitz-type hypotheses on the driver of the 2BSDE, in order to extend the existence and uniqueness result to the quadratic case. Indeed, we prove existence and uniqueness for a driver which is Lipschitz in y, satisfy some locally Lipschitz condition in z and has quadratic growth in z, provided that the terminal condition is bounded.

Then, it is now commonly known that an application of the BSDE theory is the problem of utility maximization. Indeed, El Karoui and Rouge, followed by Hu, Imkeller and Muller studied exponential, power and logarithmic utility functions in incomplete markets and proved that the maximization problem was linked to BSDEs with quadratic generator. We generalize their results and prove that the problem of exponential, power and logarithmic utility maximization under volatility uncertainty in incomplete markets is linked to 2BSDEs with quadratic growth. Moreover, we give specific examples which can be solved explicitely, giving further insight in the influence of the volatility uncertainty.

Jinniao Qiu (Fudan University, Shanghai)

American Option Pricing Problems and Reflected BSPDEs

In this talk, we mainly consider the American option pricing problems with random coefficients. Throng investigating a class of reflected backward stochastic partial differential equations (BSPDEs) under a more general structure, we solve the corresponding American option pricing problems.

Anthony Réveillac (Université Paris-Dauphine)

FBSDEs for the utility maximization problem

In this talk, we deal with the utility maximization problem with a general utility function. We derive a new approach in which we reduce the utility maximization problem with general utility to the study of a fully-coupled Forward-Backward Stochastic Differential Equation (FBSDE). This is a joint work with Ulrich Horst, Ying Hu, Peter Imkeller and Jianing Zhang

Adrien Richou (Université de Bordeaux 1)

Time discretization of Markovian quadratic and superquadratic BSDEs with an unbounded terminal condition

My presentation will deal with the existence and the uniqueness of solutions to quadratic and superquadratic Markovian backward stochastic differential equations (BSDEs for short) with an unbounded terminal condition. Our results are deeply linked with a strong a priori estimate on Z that takes advantage of the Markovian framework. This estimate allows us to give explicit convergence rates for time approximation of quadratic or superquadratic Markovian BSDEs.

Yongsheng Song (Chinese Academy of Sciences, Beijing)

Backward Stochastic Differential Equations Driven by G-Brownian Motion

In this paper, we study the backward stochastic differential equations driven by G-Brownian motion in the following form:

$$Y_t = \xi + \int_t^T f(s, Y_s, Z_s)ds + \int_t^T g(s, Y_s, Z_s)d\langle B\rangle_s - \int_t^T Z_s dB_s - K_T + K_t.$$

Under a Lipschitz condition on f and g in Y and Z, the existence and uniqueness of the solution (Y, Z, K) is proved, where K is a decreasing G-martingale.

Mitja Stadje (Tilburg University)

Portfolio Selection and Indifference Valuation

We solve, theoretically and numerically, the two canonical optimization problems of portfolio choice and indifference valuation, under ambiguity and ambiguity averse preferences in a general continuous-time framework. We also allow for constraints on the trading strategies. Using dynamic programming principles we characterize the optimal solution in terms of certain backward stochastic differential equations in a fairly general setting. We prove new existence, uniqueness, and comparison results for the associated BSDEs and also provide some numerical examples using Monte Carlo least-square regression. The talk is based on joint work with Roger Laeven.

Xiaolu Tan (École Polytechnique)

A numerical scheme for a class of non-Markovian stochastic control problem: A probabilistic approach.

We give a probabilistic interpretation of the scheme proposed by Fahim, Touzi and Warin for fully nonlinear parabolic PDEs, and hence generalize it to the non-Markovian case. A rate of convergence is obtained using Dolinsky's method, which is better than that obtained in the Markovian case by the viscosity solution method. By approximating the conditional expectations with simulation-regression method, we get an implementable scheme. In particular, our scheme can solve a class of second order backward stochastic differential equations (2BSDE) proposed by Soner, Touzi and Zhang.

Plamen Turkedjiev (Humboldt Universität Berlin)

Approximating BSDEs using least-squares regression and Malliavin weights

In this talk, I present joint work with Emmanuel Gobet (Ecole Polytechnique, Paris). Two algorithms for approximating forward-backward SDEs are discussed.

In the first part of the talk, we consider the numerical approximation multi-step forward dynamical programming equations in the spirit of Bender/Denk. No Picard iterations are required. Least-squares regression is used to approximate conditional expectations. We demonstrate convergence in an L^2 sense. Moreover, we allow local (in time) Lipschitz continuity and boundedness assumptions on the driver of the BSDE. This extension allows us to treat a wider class of BSDEs than previously considered with least-squares algorithms, including certain classes of quadratic BSDEs. It also allows the numerical analysis of an interesting variance reduction method recently suggested by Bender/Steiner. The new assumptions lead to some problems in the analysis of the L^2 error; we handle these problems and demonstrate that the order of convergence is similar to the uniformly Lipschitz and bounded driver case if some care is taken in the selection of the time-grid.

In the second part of the talk, we consider a numerical algorithm based on the Malliavin weights representation of Z given by Ma/Zhang (Representation theorems for BSDEs). We work with the same local assumptions on the driver as for the first part of the talk. We demonstrate that the time-discretized version of this representation converges to the continuous-time BSDE; assuming that the terminal condition has fractional smoothness in the sense of Gobet/Makhlouf, the order of convergence is the optimal $O(N^{-1/2})$. Moreover, with the help of stability results, we present similar results to Bender/Denk when the conditional expectation is replaced by projection on a finite basis; our results, however, are on a stronger norm. It is also interesting to note that the results depend on much weaker restrictions on the time-grid than the first part of the talk.

Kevin Webster (Princeton University)

Order book model with heterogeneous, competitive agents

Since they were authorized by the U.S. Security and Exchange Commission in 1998, electronic exchanges have boomed. By 2010 high frequency trading accounted for over 70% of equity trades in the US. We explore an equilibrium model where agents with heterogeneous beliefs interact competitively on an electronic limit order book. The Pontryagin maximum principle allows us to derive properties on the trade and order microstructure of the market, as well as a consistency equation for market impact models. The model also explains some empirically known features of the limit order book.

Weinning Wei (Fudan University, Shanghai)

Representation of Dynamic Time-Consistent Convex Risk Measures with Jumps

For the natural filtration generated by a Brownian motion and a Poisson random measure, the representation of the generator of backward stochastic differential equations and a converse comparison theorem are proved. Moreover, the relation is discussed between g- expectations and dynamic convex and coherent risk measures. The integral representation is discussed for the minimal penalty term of a dynamic convex risk measure.

Hao Xing (London School of Economics)

On BSDEs and strict local martingales

We study a backward stochastic differential equation whose terminal condition is an integrable function of a local martingale and generator has bounded growth in z. When the local martingale is a strict local martingale, the BSDE admits at least two different solutions. Other than a solution whose first component is of class D, there exists another solution whose first component is not of class D and strictly dominates the class D solution. Both solutions are $\mathbb{L}^p$ integrable for any $0 < p < 1$. These two different BSDE solutions generate different viscosity solutions to the associated quasi-linear partial differential equation. On the contrary, when a Lyapunov function exists, the local martingale is a martingale and the quasi-linear equation admits a unique viscosity solution of at most linear growth.

Zhiyong Yu (Shandong University, Jinan)

Probabilistic interpretation for systems of quasilinear parabolic partial differential equations combined with algebra equations.

The well known Feynman–Kac formula gives a probabilistic interpretation for the linear second order partial differential equations (PDEs) of elliptic or parabolic types, and has been generalized to the case of semilinear second order PDEs with the help of the theory of backward stochastic differential equations (BSDEs). In this talk, we consider a new kind of quasilinear parabolic PDEs systems combined with algebra equations. By introducing a family of coupled forward-backward stochastic differential equations (FBSDEs), we give a probabilistic interpretation for the new PDEs system. We discuss the existence and uniqueness of the solution for the PDEs system both in classical sense and viscosity sense.

Quan Yuan (Tongji University, Shanghai)

Valuation and Optimal Decision for Perpetual American Employee Stock Options under a Constrained Viscosity Solution Framework

This paper is concerned with the valuation of a block of perpetual employee stock options (ESOs) and the optimal exercise decision for an employee endowed with them. Treating the number of options as continuous, we adopt a fluid model to characterize the exercise process by imposing an upper bound on the exercise rate. The objective is to maximize the overall discount returns for the employee through exercising the options over time. We define this optimum value as the grant-date value of these options. By virtue of stochastic control approach, we derive the Hamilton–Jacobi–Bellman (HJB) equation governing the value function with state constraint. Then the value function is shown to be the constrained viscosity solution of the associated HJB equation, with the uniqueness verified by the comparison principle. The approximation of the value function is obtained through numerical simulation, leading to a threshold-style optimal strategy. Lastly, more numerical examples indicate the impact of parameters on the exercise decision. The obtained results provide an appropriate estimated cost of the ESOs for the company and meanwhile offer favorable suggestions on the exercise decision for the employee.

Jianing Zhang (Humboldt Universität Berlin)

Dual representations for general multiple stopping problems

In this talk, we study the dual representation for generalized multiple stopping problems, hence the pricing problem of general multiple exercise options. We derive a dual representation which allows for cash ows which are subject to volume constraints modeled by integer valued adapted processes and refraction periods modeled by stopping times. As such, this extends the works by Schoenmakers [2010], Bender [2011a], Bender [2011b], Aleksandrov and Hambly [2010] and Meinshausen and Hambly [2004] on multiple exercise options, which either take into consideration a refraction period or volume constraints, but not both simultaneously. We also allow more exible cash ow structures than the additive structure in the above references. We supplement the theoretical results with an explicit Monte Carlo algorithm for constructing confidence intervals for the price of multiple exercise options and exemplify it by a numerical study on the pricing of a swing option in an electricity market. Time permitting, we also point out a relationship to reflected BSDEs which may serve as a stepping stone for further discussion. This talk is based on a joint work with Christian Bender (Universität des Saarlandes) and John Schoenmakers (WIAS Berlin).

Jia Zhuo (University of Southern California, Los Angeles)

A probabilistic Numerical Method for Fully Nonlinear Parabolic PDEs

Motivated by the remarkable work by Fahim, Touzi, and Warin (2010), we introduce a probabilistic numerical method for fully nonlinear parabolic PDEs in this talk. By using certain trinomial tree instead of Brownian Motion, we remove a serious constraint imposed in Fahim, Touzi, and Warin (2010). Our scheme works well for high dimensional PDEs with a diagonal dominant Hessian matrix, and it is comparable with finite difference method when the dimension is low ($d \leq 3$). As a special case, our scheme can be applied to solve high dimensional coupled FBSDEs, especially when the forward diffusion is diagonal. We will show several numerical examples, with dimension up to 12. The talk is based on a joint work with Wenjie Guo (Fudan University) and Jianfeng Zhang (University of Southern California).