Stochastic Processes and Applications (2nd Ed.): Errata–Corrigenda.

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Please email further errors to samuel.cohen@maths.ox.ac.uk. Thanks to the following people, who have kindly pointed out errors: Pietro Siorpaes, Gonçalo Simões, Victor Fedyashov, Martin Tegnér, Nicola Doninelli, Alexandros Saplaouras, Vladimir Vovk, Andi Wang, Carlo Cabrera, David Prömel, Frank Seifried, John Enyang, Simone Pasquini, Mark Owen, Martin L Jones, Jonathan Durston, Ivo Tavares

Mathematical errors

- p23: The sums in the final displayed equation should be over $i = 0$ to $i = N(n) - 1$, rather than to $N(n)$.

- p30, Lemma 1.5.9: The proof is incorrect. An alternative is to take $T : (y, z) \to y + z = x$, and apply the bounded inverse theorem to see that

  $$\|x\|_{\oplus} = \|y\|_X + \|z\|_X = \|T^{-1}(x)\|_{Y\times Z} \leq C\|x\|_X$$

  for some constant C. This and the triangle inequality prove the equivalence of the norms.

- p32, Lemma 1.5.18: The proof of this standard result is incorrect (in particular, the claim that $F^n(x)$ forms a Cauchy sequence). A correct proof can be found in [160], p216.

- p43, lines 15–23: A better proof that g is in L^q is as follows. Let ϕ_n be a sequence of nonnegative simple functions increasing pointwise to $gI_{\{g\geq 0\}}$. As ϕ_n is simple, we know that

  $$\int_S \phi_n^q d\mu = \int_S \phi_n^{1+q/p} d\mu \leq \int_S \phi_n^{q/p} g\, d\mu = F(\phi_n^{q/p}).$$

  By boundedness of F, for some $c \in \mathbb{R}$,

  $$\int_S \phi_n^q d\mu \leq F(\phi_n^{q/p}) \leq c\Big(\int_S \phi_n^q d\mu\Big)^{1/p}.$$

  Rearranging, we obtain $\big(\int_S \phi_n^q d\mu\big)^{1/q} \leq c$, that is, ϕ_n is a sequence uniformly bounded in L^q. By the monotone convergence theorem, this implies that $gI_{\{g\geq 0\}} \in L^q$.

We can now repeat this argument for a sequence increasing to $(-g)I_{\{g<0\}}$, and so conclude that $(-g)I_{\{g<0\}} \in L^q$ and, therefore, $g \in L^q$.

- p52, Definition 2.1.11: The definition given is for 'pairwise independence'. In general a finite collection of events (one per random variable) should be allowed to appear simultaneously.

- p85, Problem 3.4.3: "evanescent sets" should be "evanescent sets and their complements"

- p101, Proof of Theorem 4.5.6: The last step in the proof (application of Lemma 4.5.5 and letting $k \to \infty$) needs to be done in the opposite order. We should have that the inequality on line 14 allows us to apply Lemma 4.5.5 to obtain (for each k)

$$\Big\| \sup_{n\leq k} |X_n| \Big\|_p \leq q\|X_k^+\|_p \leq q\|X_k\|_p$$

and by using monotone convergence, we conclude

$$\lim_{k\to\infty} \Big\| \sup_{n\leq k} |X_n| \Big\|_p = \Big\| \sup_{n} |X_n| \Big\|_p \leq q \sup_k \|X_k\|_p.$$

The argument as stated can then be used to establish the (tighter) bound

$$\Big\| \sup_{n} |X_n| \Big\|_p \leq q\|X_\infty^+\|_p.$$

- p102, Remark 4.6.4: This remark is false as stated (Fatou's inequality does not show the stated relationship, which is generally false). However, it is true if we restrict our attention to *uniformly integrable* nonnegative supermartingales, in which case the result follows from the Vitali convergence theorem.

- p103, Lemma 4.6.6: The final term in the statement of the Lemma should be $I_{\{m\geq S\}}E[X|\mathcal{F}_{S\wedge m}]$.

- p103, final line: "martingale" should be "uniformly integrable martingale".

- p104, proof of Theorem 4.6.7: Throughout this proof, we should only say that Z is a nonnegative supermartingale (rather than a potential, given the requirement of uniform integrability in Remark 4.6.4).

- p112, Theorem 5.1.8: We should be more precise here, as X_{t+} and X_{t-} are being used to indicate the right and left limits of X taken using sequences in the rationals, as in Lemma 5.1.5 (though these could be replaced with some other dense countable set). We do not guarantee the existence of right and left limits for X when taken over the reals.

- p116, Remark 5.4.2: As on p102, this should only apply to uniformly integrable processes.

- p120, Equation (5.1), the scaling is incorrect, it should read

$$X_t^n = X_t^{n-1} + 2^{-(n/2+1)}Z_t.$$

- p122, line 16ff, The scaling of the right hand side here is incorrect, and the uniform convergence is more delicate than indicated. Starting at this line, the argument should read:

$$P\big(\sup_{s\in[0,t]}\|X^n_s - X^{n+1}_s\| > \epsilon\big) = P\Big(\max_{\{s\in D_{n+1}\setminus D_n: s<t\}}\|Z_s\| > 2^{n/2+1}\epsilon\Big)$$
$$\leq \sum_{\substack{\{s\in D_{n+1}\setminus D_n,\\ s<t\}}} P\big(\|Z_s\| > 2^{n/2+1}\epsilon\big) = t2^n(1-F(2^{n+2}\epsilon^2)).$$

By changing into polar coordinates, it is easy to show that $F(x) = 1-e^{-x/2}$ (this simple form is the reason we chose $d=2$). Therefore,

$$P\big(\sup_{s\in[0,t]}\|X^n_s - X^{n+1}_s\| > \epsilon\big) \leq t2^n\exp(-2^{n+1}\epsilon^2).$$

In particular,

$$P\big(\sup_{s\in[0,t]}\|X^n_s - X^{n+1}_s\| > n^{-3}\big) \leq t2^n\exp(-2^{n+1}n^{-6}).$$

Taking N large enough that $N\log(2) - 2^{N+1}N^{-6} < -N$, for all $n > N$ we have

$$P\big(\sup_{s\in[0,t]}\|X^n_s - X^{n+1}_s\| > n^{-3}\big) \leq te^{-n}.$$

By the Borel–Cantelli Lemma, as this sequence is summable we have

$$P\big(\sup_{s\in[0,t]}\|X^n_s - X^{n+1}_s\| > n^{-3} \text{ for infinitely many } n\big) = 0.$$

In particular, with probability one, taking N sufficiently large, for all $n \geq N$,

$$\sup_{s\in[0,t]}\|X^n_s - X^{n+1}_s\| \leq n^{-3}$$

and by the triangle inequality, recalling that $\sum_n n^{-2} = \pi^2/6$, for $N < n < m$,

$$\sup_{s\in[0,t]}\|X^n_s - X^m_s\| \leq \sum_{j=n}^{m-1}\Big(\sup_{s\in[0,t]}\|X^j_s - X^{j+1}_s\|\Big) \leq \frac{\pi^2}{6n}.$$

Therefore, with probability one, the processes X^n are converging uniformly on the interval $[0,t]$.

- p123, The scalings and indexing on this page are incorrect. In particular, line 9 should read

$$X_u - X_{\lfloor u\rfloor_n} = \frac{X_{\lceil u\rceil_n} - X_{\lfloor u\rfloor_n}}{2} + 2^{-(n/2+1)}Z_u \sim N(0, 2^{-(n+1)})$$

(which holds by induction), line 10 can be omitted, line 12 should just read $X_{\lceil u\rceil_n} - X_u \sim N(0, 2^{-(n+1)})$ and line 20 should read

$$E\big[(X_{\lceil s\rceil_n} - X_s)(X_s - X_{\lfloor s\rfloor})^\top\big] = 0.$$

(which holds by calculation using line 9)

- p130, proof of Lemma 5.5.20: The use of Fatou's inequality (on line 3 of the page) does not give the stated result. An alternative approach is to argue as follows:

  Let τ_1 be the first jump of N after time T, and $\tau = (T+\delta)\wedge\tau_1$, so that τ is a stopping time. Then

  $$0 \le E[(N_{T+\delta} - N_T - 1)^+|\mathcal{F}_S] = E[(N_{T+\delta} - N_\tau)(N_\tau - N_T)|\mathcal{F}_S]$$

  and we observe that $0 \le N_\tau - N_T \le 1$. Therefore, applying the optional stopping theorem,

  $$\begin{aligned} E[(N_{T+\delta} - N_\tau)(N_\tau - N_T)|\mathcal{F}_S] &= E[E[(N_{T+\delta} - N_\tau)|\mathcal{F}_\tau](N_\tau - N_T)|\mathcal{F}_S] \\ &= E[\lambda(T+\delta-\tau)(N_\tau - N_T)|\mathcal{F}_S] \\ &\le \lambda\delta E[N_\tau - N_T|\mathcal{F}_S] \\ &\le \lambda\delta E[N_{T+\delta} - N_T|\mathcal{F}_S] = (\lambda\delta)^2. \end{aligned}$$

  It follows that
  $$\lim_{\delta\to 0} \frac{E[(N_{T+\delta} - N_T - 1)^+|\mathcal{F}_S]}{\lambda\delta} = 0$$
  as desired.

- p132, Definition 5.6.2: This definition should be for a general right-continuous process X, rather than for a right-continuous uniformly integrable supermartingale (otherwise Lemma 5.6.6 becomes unclear).

- p132, Lemma 5.6.4: The proof claims that M^n is a uniformly integrable martingale, which is not generally true. We know that M^{T_n} is a martingale, so $M^{T_n\wedge n}$ is a martingale with $M_\infty^{T_n\wedge n} = M_n^{T_n} = M_{T_n\wedge n} \in L^1$. The optional stopping theorem then implies that $M_t^{T_n\wedge n} = E[M_{T_n\wedge n}|\mathcal{F}_t]$, and the stated uniform integrability follows, with localizing sequence $\{T_n \wedge n\}_{n\in\mathbb{N}}$.

- p132, Lemma 5.6.5: Should state that "Every càdlàg martingale is in $\mathcal{M}_{\text{loc}}$, that is, is locally a uniformly integrable martingale."

- p144, Theorem 6.2.9. The uniqueness of the proof is only in the case $T < \infty$ a.s. In general, A and B are unique up to subsets of $\{T = \infty\}$, and our construction gives the case where A and $\{T=\infty\}$ are disjoint. In particular, we know that the graphs $[\![T_A]\!]$ and $[\![T_B]\!]$ are uniquely defined.

- p146, The comment is made in Remark 6.2.17 that $T-\epsilon$ is generally not a stopping time, so $X_{T-\epsilon}$ is only $\mathcal{F}_{T-}$-measurable, not $\mathcal{F}_{T-\epsilon}$-measurable. The point is that $\mathcal{F}_{T-\epsilon}$ is not well defined (we don't have a notion of the σ-algebra at a random time), which should be made more clearly.

- p147, line 17: The equation $\{T_A \le T\} = \{T = 0\} \cup \big(\bigcap_n \{S_n < T\}\big)$ is incorrect, as $\{T=0\}$ should be $\{T_A = 0\} = \{T=0\}\cap A$. However, $\{T_A = 0\} \in \mathcal{F}_0 \subset \mathcal{F}_{T-}$, so the argument then proceeds as written.

- p246, Lemma 11.5.2: The assumption of the lemma should be that $Z \ge Y$, rather than that $X - Y + Z \ge 0$. Without this we cannot guarantee that $R = T_1 < \infty$ on the desired set in the second line of the proof.

- p377, Theorem 15.2.8: The assumption of the theorem that $\langle Y \rangle$ exists under P is unnecessary, as the changes to the proof on the following page will make clear.

- p378, In the second displayed equation on the page, we should have that, using the BDG inequality, as $n \to \infty$ we have

$$cE^Q\Big[\Big(\int_{[0,\infty[}(H^n - H^m)^2 d[\tilde{Y}]\Big)^{1/2}\Big] \le \|X^n - X^m\|_{\mathcal{H}^1(Q)} \to 0.$$

  Therefore, at least for a subsequence, H^n converges pointwise $d[\tilde{Y}] \times dP$-a.e. As H^n is predictable, this implies that H^n converges pointwise almost everywhere on the predictable support of $d[\tilde{Y}] \times dP$ (that is, the support of $d[\tilde{Y}] \times dP$ considered as a measure on the predictable σ-algebra, which agrees with the support of $d\langle \tilde{Y} \rangle \times dP$ if $\langle \tilde{Y} \rangle$ exists). Taking a limit, we have a predictable process H with the desired properties, and the proof follows.

- p407, Theorem 16.2.6: The proof of this theorem, as stated, requires us to know that the space of semimartingales under the stated operator norm is a Banach space, otherwise Lemma 1.5.9 does not apply. The difficulty is in verifying that the space is complete, and agrees with $\mathcal{H}^p_{\mathcal{S}}$. The following argument can be used to check this.

  Let $\|X\|_{op} = \sup_{H:|H|\le 1}\{\|H \bullet X\|_{S^p}\}$ be the stated operator norm. Suppose X^n is a Cauchy sequence under this norm. It is clear that X^n is Cauchy in the semimartingale topology, so converges (in semimartingale topology) to some semimartingale X. Let $Y^{k,n} = X^{n+k} - X^k$. Fatou's lemma shows that for any simple H with $|H| \le 1$,

$$\|H \bullet (X - X^k)\|_{S^p} \le \liminf_n \|H \bullet (X^n - X^k)\|_{S^p} \le \liminf_n \|Y^{k,n}\|_{op}.$$

  Taking the supremum over such H we have

$$\|X - X^k\|_{op} \le \liminf_n \|Y^{k,n}\|_{op}.$$

  As X^k is Cauchy, $\liminf_n \|Y^{k,n}\|_{op} \to 0$ as $k \to 0$, so we have $\|X - X^k\|_{op} \to 0$. Therefore, X^k converges to X in the operator norm, and the completeness is proven.

  Whenever $\|X\|_{op} < \infty$, we know that X is a special semimartingale, so we can write $X = M + A$, where M is a local martingale and A is a predictable process. Taking $H = \mathrm{sign}(dA)$, we know that $H^2 \equiv 1$, so $\|H \bullet X\|_{op} = \|X\|_{op}$, and $H \bullet X = H \bullet M + H \bullet A$. However, $H \bullet A$ is increasing, so after localizing, by Garsia's inequality, $\|A\|_{op} = \|H \bullet A\|_{op} \le \|X\|_{op} < \infty$. The proof as stated then shows that $A \in \mathcal{H}^p_{\mathcal{S}}$, and hence $X \in \mathcal{H}^p_{\mathcal{S}}$, so our spaces agree, and the equivalence of the norms follows as stated.

  (Thanks to Pietro Siorpaes for pointing out this difficulty, and for assistance in finding a simple proof of the missing steps.)

- p590, on the last line, the mean variation should be defined as

$$\mathrm{MV}(X, \pi) = E\Big[|X_0| + \sum_{i=1}^n \big|E[X_{t_i} - X_{t_{i-1}}|\mathcal{F}_{t_{i-1}}]\big|\Big].$$

(the conditioning on the right is wrong).

- p608, Theorem A.8.14 (i). The term $E[M_\infty N_\infty]$ in the equality should be omitted, as there is no guarantee (for general $M \in \mathcal{H}^1$) that $M_\infty N_\infty$ is integrable. The proof of this theorem should also be more precise: after we have constructed N using the reflexive structure of $\mathcal{H}^2$, we then show that N has bounded jumps (as written). We then should consider the martingale $M := I_{]\!]T,T']\!]} \bullet N$, for any stopping times $T \leq T'$ such that $M \in \mathcal{H}^2$. Using the stated argument, we can show that

$$E\big[[N]_{T'} - [N]_T | \mathcal{F}_T\big] \leq c$$

and by monotone convergence send $T' \to \infty$ to guarantee that $N \in \mathcal{H}^{\mathrm{BMO}}$ (this can be done while ensuring $M \in \mathcal{H}^2$, as we know that N has bounded jumps). Finally, using Fefferman's inequality, we can see that both sides of the equality (for $M \in \mathcal{H}^2$)

$$\phi(M) = E\big[[M,N]_\infty\big]$$

are continuous in $M \in \mathcal{H}^1$, and so the density of $\mathcal{H}^2$ in $\mathcal{H}^1$ gives the desired statement.

Typos,etc...

- p4, Remark 1.1.3: (iii) should be (iii'), as countable additivity is used.
- p5, Definition 1.1.12: the indices for the union and intersection should begin with $n = 1$, to be consistent with the choice of $\mathbb{N}$.
- p9, Theorem 1.2.7: this should read $\mu : \Sigma \to [0, +\infty]$ to ensure extension to a (positive) measure.
- p14, line 5: the line should begin "Next assume g is a"
- p14, line 8: it should be clear that we assume g is $\sigma(f)$-measurable
- p16, Lemma 1.3.28: in the final sentence of the proof, it should be "Set $\tilde{f} = \limsup f_n$"
- p20, line 15: A should be defined as "$A = \cap_{k=1}^\infty \cup_{i=k}^\infty B_i$"
- p20, line 19: The sets E_i shoudl be B_i
- p23, final equation: The sums should all be taken for i up to $N(n) - 1$, rather than $N(n)$
- p24, Corollary 1.3.44: The integrals and sums should all have subscript u, rather than s
- p24, Theorem 1.3.45: In the second line of the theorem, it should read "For $s \in [0, \infty[$"
- p27, Theorem 1.4.6: f should be a nonnegative measurable function
- p28, line 14: "topology" should be "topologies"

- p29, Definition 1.5.3: It should eb assumed that $c, c' > 0$ (the inequality is strict)
- p29, line -1: this should read 'contains more than one point in K' (however, the definition as stated can be used if the space satisfies the T_1 axiom, or more generally is a Hausdorff space, which all the spaces we will consider are).
- p30, line 19: the operator norm should be defined by $\sup_{\{x:\|x\|>0\}}\{\|T(x)\|/\|x\|\}$
- p30, line -10: it should be clarified that the norm of (y, z) is $\|y\| + \|z\|$.
- p31, Lemma 1.5.16: in [160] (p.303), this result is proven only for the closed unit ball. The result as stated can be found in Whitely, R. *An elementary proof of the Eberlein–Šmulian theorem*, Mathematische Annalen, 172(2):116–118, 1967
- p33, line 12: $\mathcal{B}([0, \infty])$ should be $\mathcal{B}(\overline{\mathbb{R}})$.
- p36, Theorem 1.5.35: Each $\wedge$ in the proof should be a $\vee$
- p37, Defininition 1.6.1: (S, Σ) is a measurable space, not a measure space
- p42, line 1 and line 6: "measure space" should be "measurable space"
- p64, line 5: "$A \in \mathcal{F}$" should be "$A \in \mathcal{G}$".
- p65, line -5: "$\mu_{\mathcal{G}}$" should be "$\mu|_{\mathcal{G}}$"
- p67, line -5: $\mu|_{\mathcal{G}}(\omega, A)$ should be $\mu|_{\mathcal{G}}(A, \omega)$
- p68, line 6 and line 8: "$k \in \mathbb{R}$" should be "$k > 0$"
- p77, line 15: In the first line of the proof, $I_{\{S<T\}}$ should be $I_{\{S\leq T\}}$
- p86, Exercise 3.4.9(iii): $\int_{[0,t[} X_t dt$ should be $\int_{[0,t[} X_s ds$
- p97, Theorem 4.4.6, the filtration should be indexed by $\mathbb{Z}^+$, that is $\{\mathcal{F}_n\}_{n\in\mathbb{T}}$ should be $\{\mathcal{F}_n\}_{n\in\mathbb{Z}^+}$
- p100, line 12: the right hand side of the first line of this equation should be $-\int_{[0,\infty]} \lambda^p d\tilde{F}(\lambda)$, (not dF)
- p101, line 6: this line should read

$$\sup_n E[(-X_n)^-] = \sup_n E[X_n^+] \leq \sup_n E[|X_n|^p]^{1/p} = \sup_n \|X_n\|_p < \infty$$

- p102, line -10, the inequality should end $E[Y_0'] > -\infty$
- p103, line 17: Z^*_{n+p} should be Z'_{n+p}
- p103, in the statement of Lemma 4.6.6, $\mathcal{F}_{\{S\wedge m\}}$ should be $\mathcal{F}_{S\wedge m}$
- p105, Exercise 4.7.3, the sequence of random variables should be $X_1, X_2, ...$ (X_0 is not needed). In part (iv), X_n should be X_i and in the statement of Kronecker's lemma, in the last line, a_n should be a_i.

- p106, line 21: the "to" before the closing parenthesis is redundant
- p107, line 7: to clarify, $n^{-\alpha}$ is the conditional variance of $Y_n|\mathcal{F}_{n-1}$, not its standard deviation.
- p111, line -5: to clarify, this condition is satisfied for any nonincreasing sequence of stopping times, provided X satisfies some integrability conditions (as given in Theorem 5.3.1)
- p125, line 14: "$\mathcal{F}_t = \sigma(X_s : s < t)$" should be "$\mathcal{F}_t = \sigma(X_s : s \leq t)$" for consistency (although it makes no difference, as our processes here are continuous)
- p125, line -10: "For a X" should be "For X a"
- p136, line 8: "$\{\tilde{\mathcal{F}}_t = \mathcal{F}_{c^2t}\}_{t\geq 0}$" should be "$\{\tilde{\mathcal{F}}_t = \mathcal{F}_{c^{-2}t}\}_{t\geq 0}$"
- p140, line 13: $S(\omega)$ should be $T(\omega)$.
- p146, line 1: the final vertical bar should be on the next line.
- p148, line 9: The reference to "Theorem 3.1.13 and Theorem 6.1.4(iv)" should be simply a reference to "Theorem 3.1.15".
- p148, line -12 and -8: "Theorem 6.3.1" should be "Theorem 6.3.2". In case of confusion, it should also be clarified that, except in the first line of this proof, we are assuming that $\{S = T\} \in \mathcal{F}_{T-}$ for all predictable T.
- p149, line 21: when proving (i)⇒(iii), we consider T to be any predictable stopping time.
- p153, line 2 of main text: the process Y should be X
- p155, line -9 and -3: stochastic intervals in this example should be open on the right, that is, $[\![S, T[\![$ not $[\![S, T]\!]$.
- p159, line 13: "a complete filtration $\{F_t\}_{t\in[0,\infty[}$" should be "a filtration $\{F_t\}_{t\in[0,\infty[}$ satisfying the usual conditions"
- p165, line -7: "Corollary 7.2.5" should be "Corollary 7.1.9"
- p165, line -5: "$\phi(x) = \frac{\pi}{4}\arctan(x)$" should be "$\phi(x) = \frac{2}{\pi}\arctan(x)$"
- p167, Definition 7.6.1: the definition should require that "$\{(t,\omega) : |X_t(\omega)| > k\}$ is evanescent".
- p167, line 12 and 13: "$I_{]\!]S,T[\![}$" should be "$I_{]\!]S,T]\!]}$" while "only on $[\![S]\!]$" should be "only on $[\![S]\!] \cup [\![T]\!]$"
- p167, line -1: it should be made clear the equality should hold for all $T \in \mathcal{T}_x$
- p170, line 3 and elsewhere on this page: "From Theorem 7.6.5" should be "From Theorem 7.6.5 and Remark 7.6.4"
- p171, line 6: "$A \subseteq [0, t[\times \Omega$" should be "$A \subseteq [0, \infty[\times \Omega$"

- p171, line 9: "Theorem 7.4.1 or otherwise" should be "Theorem 7.4.1, Theorem 7.3.3 or otherwise"
- p178, line -7: The integrals should be with respect to a free variable, rather than with respect to s (i.e. in both places where it appears, dA_s should be dA_u).
- p176, line 17: V should be $\mathcal{V}$.
- p180, lemma 8.1.16: The final equality in the statement of the lemma should be "$= \mu_A\big([0,\infty[\times\pi(C)\big)$,"
- p186, line 14: the final equation has an excess parenthesis, it should read "$E[(X \bullet A)_\infty] = E[(Y \bullet A)_\infty]$"
- p188, line -13: The central two terms in this equation should be $E[((\mathbf{\Pi}_x X)\bullet A)_\infty] = E[[(\mathbf{\Pi}_x Y) \bullet A)_\infty]$. It should also be made clearer that the final equality is due to the linearity and monotonicty of $\mathbf{\Pi}_x$, so we have the general equality

$$E[((\mathbf{\Pi}_x X) \bullet A)_\infty] \equiv \int_{[0,\infty[\times\Omega} X(t,\omega) d\mu_x.$$

- p194, line -3: $X^{T_n} \in \mathcal{A}_{\text{loc}}$ should be $X^{T_n} \in \mathcal{A}$
- p211, Definition 10.1.1: the index of X on the right of the displayed equations should be s in the first definition, and u in the second
- p214, lines 10 and 11: The sets considered are uniformly integrable with respect to n, not k. That is, it should read "$\{|X^n_{T_k} - X_{T_k}|\}_{n\in\mathbb{N}}$ is uniformly integrable. Therefore, $\{(Y^n)^*_{T_k}\}_{n\in\mathbb{N}}$ is uniformly integrable."
- p229, line -6 and p230 lines 4 and 9: On each occasion, $\big(X_s^2 I_{\{|X_s|\leq a\}} + |X_s| I_{\{|X_s|\leq a\}}\big)$ should be $\big(X_s^2 I_{\{|X_s|\leq a\}} + |X_s| I_{\{|X_s|>a\}}\big)$ (or similarly with stopping times)
- p247, line -1: ψ should be given by $\psi(\delta) = (4\delta/(\beta^2 - 1 - \delta^2))^2$
- p250, line -3: In the displayed equation of Remark 11.5.8, $\sup_t(\Delta M_t)^p$ should be $\sup_t |\Delta M_t|^p$
- p295, line 20: the integral on the right should be $\int_{]0,t]} \lambda(A,s)dF_s$ (rather than being taken over $]0,t[$
- p296, line 12: a minus sign is missing on the right hand side, so this line should read:

$$E[\mu_p(t,A) - \mu_s(t,A)|\mathcal{F}_s] = -I_{\{T>s\}} E\Big[\int_{]s,t]} \lambda(A,u) I_{\{T\geq u\}} \frac{dF_u}{F_{u-}}\Big|\mathcal{F}_s\Big]$$

- p304, line 13: $\tilde{\Sigma}_x = \Sigma_x \otimes \mathcal{Z}$ should be $\tilde{\Sigma}_x = \Sigma_x \otimes \mathfrak{Z}$
- p318, line 14: the left hand side of the equation should be $M_\mu[\Delta X|\tilde{\Sigma}_p]$

- p326, in the Lévy–Khintchine Formula, the left hand side of the first equation should read $\int_{\mathbb{R}^d} e^{i\langle x,y\rangle}\mu(dy)$
- p346, line -1: the integral $\int_{]0,t]} X_{s-}dY$ should be $\int_{]0,t]} X_{s-}dY_s$
- p373, line 4: $(\Omega, \mathcal{F}, P)$ should be $(\Omega, \mathcal{F})$
- p468, line 3: ξ should be Y_T.
- p470, line 12: Y^* should be Y_t^*.
- p473, equation (19.3), the term $f(\omega, t, Y_t, Z_t, \Theta_t) - \tilde{f}(\omega, t, \tilde{Y}_t, \tilde{Z}_t, \tilde{\Theta}_t)$ should be $f^1(\omega, t, Y_t^1, Z_t^1, \Theta_t^1) - f^2(\omega, t, Y_t^2, Z_t^2, \Theta_t^2)$
- p477, line 21: $[Y, \Gamma]_t$ should be $d[Y, \Gamma]_t$
- p484, Remark 19.4.2: the domain of g (in the first bullet point) is $\mathcal{Z} \times [0,T] \times \mathbb{R}^d$, not $\mathbb{R}^n \times [0,T] \times \mathbb{R}^d$
- p486, line 26: $t \geq x$ should be $t' \geq t$.
- p487, line -2: $\frac{\partial v}{\partial t}(t, X_s^{(t,x)}) + \mathcal{L}_t(t, X_s^{(t,x)})$ should be $\frac{\partial v}{\partial s}(s, X_s^{(t,x)}) + \mathcal{L}_s(s, X_s^{(t,x)})$
- p488, line 2: the right hand side of this line should be

  $$-f\big(s, X_s^{(t,x)}, v(s, X_s^{(t,x)}), \partial_x v(s, X_s^{(t,x)})\sigma(s, X_s^{(t,x)}), \tilde{v}(s, X_s^{(t,x)})\big)ds$$

  and the final line of the equation should also have t replaced by s (except in the superscript of X)
- p488, equation (19.9) should read

  $$0 = \frac{\partial v}{\partial t}(t,x) + \mathcal{L}_t v(t,x) + f\big(t, x, v(t,x), \partial_x v(t,x)\sigma(t,x)\big).$$

  and in the following equation, the final (s, x) should be (t, x).
- p511, Theorem 20.3.5: in the proof, read (i) for (ii) and (ii) for (i).
- p522, line 8, the equation should be $M_t^{u^*} = E_{u^*}[M_\tau^{u^*} | \mathcal{F}_t]$.
- p523, Lemma 21.3.1: in the definition of J, the integral on the right should be $\int_{]t,T]} c(\omega, s, u_s)ds$, not $\int_{]t,T]} c(\omega, t, u_t)dt$
- p528, equation (21.2): X_t should be X_{t-} on the right hand side (this is particularly important in the final term)
- p528, equation (21.4), p529 line 8: there should be no dt at the end of the line, and the integral term should be

  $$\int_{\mathcal{Z}} g(\zeta, t, X_t)\big(\beta(\zeta; t, X_t, u_t) - 1\big)\nu(d\zeta).$$

- p530, Theorem 21.4.7: as f doesn't depend on ω directly, the conditions should hold dt almost everywhere (not $dP \times dt$ almost everywhere).

- p531, Definition 21.4.9: Whenever it does not have an argument, v should be evaluated at either (t,x) or (s,x), as appropriate.
- p553, equation (22.26), the final integral should be with respect to $d\tilde{Y}_u$, not $d\tilde{Y}_t$
- p555 lines 6–8: N_t^{ij} should be $\mathcal{J}_t^i$, J_t^i should be $\mathcal{O}_t^i$ and G_t^i should be $\mathcal{T}_t^i$, for consistency with later sections.
- p562, line 9: This equation should read

$$\sigma_t(\langle X_s, e_i\rangle X) = E_Q[\Lambda_t\langle X_s, e_i\rangle X|\mathcal{Y}_t] \propto E[\langle X_s, e_i\rangle X|\mathcal{Y}_t].$$

- p578, line -16: it should read "Take an arbitrary $c_m = (c_m^1, c_m^2, ..., c_m^{k(m)}) \in \tilde{C}_m$"
- p591, lines -3 to -4 (the final displayed equation on the page) should read

$$\begin{aligned}\sum_{i=1}^n \big|E[X_{t_i} - X_{t_{i-1}}|\mathcal{F}_{t_{i-1}}]\big| \le \sum_{i=1}^n \Big(E\big[|B_{t_i} - B_{t_{i-1}}|\,\big|\mathcal{F}_{t_{i-1}}\big] + E\big[|C_{t_i} - C_{t_{i-1}}|\,\big|\mathcal{F}_{t_{i-1}}\big]\Big)\\ = E\big[B_{t_n}|\mathcal{F}_{t_{n-1}}\big] + E\big[C_{t_n}|\mathcal{F}_{t_{n-1}}\big]\end{aligned}$$

  (the conditioning on the right is wrong).
- p593, lines 16, 19 and 20: these equations all have expectations conditional on $\mathcal{F}_{t_i}$, which should be conditional on $\mathcal{F}_{s_i}$.
- p594, line -7: the limit should be as $n \to \infty$, not $n \to 0$.
- p605, Lemma A.8.7: the stated inequality should be weak, that is, $|\Delta X| \le \|X\|_{\mathrm{BMO}}$
- p608, line 6: isomporphic should be isomorphic
- p612, line 9: Strook should read Stroock
- p618, line 14: the interval should be $[-1+\epsilon, \epsilon^{-1}]$.
- p620, line 14: on the last line of the second displayed equation, $f(Z_t, \Theta_t)$ should be $|f(Z_t, \Theta_t)|$, so that the use of the monotone convergence theorem on line 15 is valid.
- p658, the entry $[\![T[\![$ should be $[\![T]\!]$