Toric Varieties RG - Tentative Plan

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Hilary Term 2022

All section references are for Fulton's book.

Week 2: Refinements of fans give rise to toric morphisms (§1.4). The polytope construction for toric varieties (§1.5). Describing when toric varieties are non-singular in terms of their fans, toric varieties are always Cohen-Macaulay (§2.1).

Week 3: One-parameter subgroups and limit points (§2.3). When morphisms between toric varieties are proper in terms of fans, toric blowups (§2.4). Toric resolutions of singularities (§2.6).

Week 4: The orbit-cone correspondence (§3.1). Fundamental groups of toric varieties, the integral cohomology of U_σ and computing $\chi(X_\Sigma)$ (§3.2).

Week 5: T-Weil and T-Cartier divisors (§3.3). Computing $\mathrm{Pic}(X_\Sigma)$, relating line bundles on X_Σ to continuous piecewise linear functions on $|\Sigma|$ and convex polyhedrons and consequences of this in terms of computing global sections and amplitude (§3.4).

Week 6: Computing $H^i(X_\Sigma, \mathcal{O}(D))$ for D a T-Cartier divisor, the higher pushforwards of the structure sheaf of a refinement morphism $f : X_{\Sigma'} \to X_\Sigma$ all vanish and $f_*(\mathcal{O}_{X_{\Sigma'}}) = \mathcal{O}_{X_\Sigma}$ (§3.5). Canonical divisors and sheaves of differentials on toric varieties (§4.3).

Week 7: The Chow group of a toric variety, the intersection product on a toric variety (§5.1). The intersection ring on a toric variety, giving bases for $A_*(X_\Sigma)$ and $H_*(X_\Sigma, \mathbb{Z})$ (§5.2).

Week 8: Using toric intersection theory and HRR to prove Pick's theorem (§5.3). Using toric intersection theory to prove Stanley's theorem (§5.6).