Honour Moderations: Linear Algebra

Michaelmas Term 2005

Axioms for Vector Spaces over the Real Numbers

A real vector space is a set V together with two laws of composition:

(a) Addition on V is a binary operation from $V \times V \to V$ written as $(u, v) \to u + v$.

(b) Scalar multiplication V is a binary operation from $\mathbb{R} \times V \to V$ written as $(\lambda, u) \to \lambda u$.

The set V has a distinguished element 0.

The following conditions are satisfied by the laws of composition:

(1)	$u + v = v + u$	$+$ is commutative.
(2)	$(u + v) + w = u + (v + w)$	$+$ is associative.
(3)	$0 + v = v + 0 = v.$	
(4)	$(\forall v \in V)(\exists - v \in V)(v + (-v) = 0).$	
(5)	$\lambda(u + v) = (\lambda u + \lambda v)$	Scalar action is distributive over $+$ in V.
(6)	$(\lambda + \mu)v = \lambda v + \mu v$	Addition in $\mathbb{R}$ is distributive over scalar action.
(7)	$\lambda(\mu v) = (\lambda\mu)v$	Scalar multiplication is associative with multiplication of real numbers.
(8)	$1v = v$	Scalar multiplication by the real number 1 is the identity operation.

All the axioms should be universally quantified: that is, they hold for all $\lambda, \mu \in \mathbb{R}$ and all $u, v \in V$.

G.A.S.