Proc. 6th European Conference on Mathematics in Industry, ed. F. Hodnett, B.G. Teubner, Leipzig, pp. 75-78, (1992).

MATHEMATICAL MODELLING OF THERMOSYPHONS IN CRYOGENIC AIR SEPARATION PLANTS

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ABSTRACT: A simple mathematical model for the flow in a thermosyphon is described, and some preliminary analysis is reported.

1. Introduction

A thermosyphon is a circulating fluid system in which the flow is driven by thermal buoyancy forces. Thermosyphons are used in cryogenic air separation processes. Specifically, a fluid loop of liquid nitrogen is heated by the external condensation of oxygen at cryogenic temperatures. The nitrogen boils, and the production of vapour leads to a flow around the loop.

The system under consideration is an experimental facility designed to test the flow, and is illustrated in Figure 1. A plate-fin heat exchanger is employed, which basically consists of a series of parallel channels through which the nitrogen flows as it is heated.

The aim of the work reported here is to model the flow in the thermosyphon, particularly the heated two-phase flow, incorporating a single-channel description of the heat exchanger. A complete description of the model is given by Aldridge and Fowler (1991).

2. Model Summary

We model the system in terms of its separate components: reservoir, feeder pipe, heat exchanger and exit pipe. The heat exchanger is modelled using one-dimensional conservation equations, while in the other sections algebraic balances are used.

The reservoir of liquid nitrogen is assumed to be so large that its state is effectively decoupled from the flow in the heat exchanger. We assume that the level of the reservoir is constant, so providing a constant pressure head for the flow. The enthalpy in the reservoir is also assumed to be constant, with thermodynamic equilibrium at the surface and the liquid nitrogen at saturation enthalpy there. As the pressure in the fluid increases towards

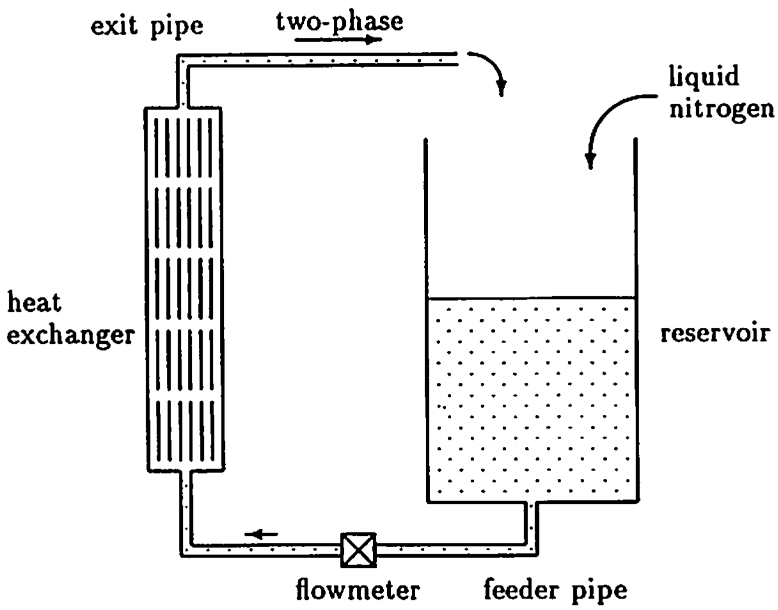

Figure 1: Schematic of thermosyphon system

the base, the saturation temperature increases due to the Clapeyron effect, and hence at the reservoir base the fluid is subcooled.

The flow in the feeder pipe is of single-phase unheated nitrogen. Bernoulli's law is applied at each end, and we use an appropriate friction correlation to give the pressure drop in the feeder pipe. In the heat exchanger, subcooled liquid nitrogen enters and is heated, causing boiling to occur. The heat exchanger contains regions of subcooled single-phase and evaporating two-phase flow. We describe the model in this region below. The exit pipe contains an unheated two-phase mixture; again an appropriate pressure drop correlation is used, with Bernoulli's law applied at the heat exchanger exit.

3. Heat Exchanger Model

In a channel, suitable dimensionless reduced equations for the subcooled region $0 < z < r(t)$ are those of mass, momentum and enthalpy:

$$\begin{aligned} u &= u_0(t), \\ p_z &= -1 - f_{lw} u_0^2, \\ h_t + u h_z &= q_{lw}, \end{aligned} \tag{1}$$

where $u_0(t)$ is the inlet velocity, f_{lw} is a turbulent friction factor, and q_{lw} is the dimensionless heat input. The boiling boundary is at $z = r(t)$, and in the two-phase region $r < z < 1$, a drift-flux model (Zuber 1967, Ishii and Zuber 1979) can be reduced to

$$-\alpha_t + [(1-\alpha)u]_z = 0,$$

$$[\ \delta(1-\alpha)(u_t+uu_z)+\ ]\ p_z=-(1-\alpha)-\tau_{mw},$$
$$H_t+(uH)_z=q_{mw}-\{\alpha V_j/(1-\alpha)\}_z,$$
$$H=a_1(1-\alpha)p+\alpha, \tag{2}$$

where α is the void fraction, H is the enthalpy per unit volume, τ_{mw} is the two-phase friction, q_{mw} is the heat input, and V_j is the drift velocity, which must be constituted as a function of α. The $O(\delta)$ acceleration terms in $(2)_2$, where $\delta=\rho_g/\rho_l\ll 1$, are neglected, but are included for the purposes of computing characteristics (below).

Appropriate boundary conditions are

$$p=a_2-a_7u_0^{1.6},\ h=0 \text{ at } z=0;$$
$$H=a_1p,\ \alpha=0,\ u=u_0,\ [p]_-^+=0 \text{ at } z=r;$$
$$p=0 \text{ at } z=1. \tag{3}$$

Values $a_1\sim 3$, $a_2\sim .75$, $a_7\sim 3.7$ are typical.

For prescribed heat flux q_{lw}, we can solve (1) to obtain

$$r=\int_{t-a_1p_r/q_{lw}}^{t}u_0(s)ds, \tag{4}$$

where $p_r=p|_{z=r}$, and if f_{lw} is a constant which depends on u_0, then

$$\begin{aligned} p_r &= a_2+a_7u_0^{1.6}-[1+f_{lw}u_0^2]r \\ &= p_0(u_0)-f_l(u_0)r. \end{aligned} \tag{5}$$

4. Analysis

If we write $\alpha=1-\rho$, $\nu=1/\rho$, $h=a_1p+\nu-1$, $W(\rho)=\alpha V_j(\alpha)/\rho$, and $q(\nu)=-W'(\rho)/\nu$, then (2) can be written as

$$A\underline{\psi}_t+B\underline{\psi}_z=\underline{c}, \tag{6}$$

where

$$A=\begin{pmatrix}-1&0&0\\0&a_1\delta&0\\0&0&1\end{pmatrix},\quad B=\begin{pmatrix}-u&\nu&0\\-\nu&a_1\delta u&\nu\\q&0&u\end{pmatrix}, \tag{7}$$

and $\underline{\psi}=(\nu,u,h)^T$. For $\delta\ll 1$, the characteristics are $dz/dt=\lambda$, where $\lambda\approx u+q$, $\pm(\nu^2/a_1\delta)^{1/2}$, and are all real. By putting $\delta=0$, we neglect rapidly propagating 'sound' or density waves. The system is hyperbolic, and thus well-posed.

If u_0, r vary slowly, then a quasi-static solution in the two-phase region is appropriate. For example, if q_{mw} is constant, we have

$$u=u_0/(1-\alpha),$$
$$1-\alpha=\frac{u_0+V_j}{u_0a_1(p_r-p)+(u_0+V_j)+q_{mw}(z-r)}, \tag{8}$$

thus $\alpha = \alpha(p, z; p_r, u_0, r)$, and a quadrature gives

$$p_r = \int_r^1 (1 - \alpha + \tau_{mw}) dz. \tag{9}$$

In principle, then, p_r can be determined as the solution of an equation $p_r = F(u_0, p_r, r)$, whence $p_r = p_r(u_0, r)$, and the model reduces to the delay-integral equation

$$r(u_0) = \int_{t-\tau(u_0)}^{t} u_0(s) ds, \tag{10}$$

where $r(u_0)$ is determined from solving (5) with $p_r = p_r(u_0, r)$.

It appears that the dependence of r on u_0 may admit the existence of multiple steady states, with more than one value of u_0 corresponding to a given value of r over a certain range. Further investigations are currently being made in this area.

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