Module 6: Monte Carlo question (Mike Giles)

The aim in this question is to investigate the use of both finite differences ("bumping") and pathwise sensitivity analysis to estimate the Delta of a discretely-monitored down-and-out barrier call option.

Suppose the evolution of a single underlying asset S_t is modelled by the standard Geometric Brownian Motion model, with risk-free interest rate $r=0.02$, volatility $\sigma=0.2$, and initial asset price $S_0=100$.

Let the discounted payoff function be

$$f = \exp(-rT)\ \max(0, S_T-K)\ \mathbf{1}_{\min_n S_{t_n}>B}$$

with strike $K=100$, barrier $B=90$, maturity $T=1$ and discrete measurement dates $t_1=0.2, t_2=0.4, t_3=0.6, t_4=0.8$.

1. Implement a standard Euler-Maruyama approximation to the GBM model, using timestep $\Delta t = 0.05$, and use this to obtain an estimate of the risk-neutral value of the option.

2. Estimate the Delta by using "bumping". Plot how the variance of the estimator changes with the bump size, and comment on the reasons for this.

   (Remember to use the same random numbers for the two sets of path calculations because this will greatly reduce the variance.)

3. Explain very briefly why there is a problem in using the pathwise sensitivity approach with this Euler-Maruyama approximation.

4. Change the Euler-Maruyama approximation to "jump over" the time points at which the barrier is applied. i.e. the first few time points should be $t = 0, 0.05, 0.1, 0.15, 0.25, 0.3, 0.35, 0.45, \ldots$.

5. Use the Brownian Bridge construction from Module 4, to approximate the probability that the option is not knocked out at any of the measurement dates, and use this to construct a new estimator for the value of the option.

6. Apply the pathwise sensitivity analysis approach to this new estimator to generate an estimate for Delta, and compare the computed value to what you obtained using "bumping".

You should hand in the Monte Carlo codes you develop and a short writeup presenting your results.