MATHEMATICAL GAMES

Reflections on Newcomb's problem: a prediction and free-will dilemma

by Martin Gardner

This department's topic for July, 1973, Newcomb's paradox, produced an enormous outpouring of letters. Robert Nozick, who first wrote about the paradox in a paper published in 1970, agreed to look over the correspondence and put down his reactions. Nozick is a philosopher at Harvard University and the author of *Anarchy, State and Utopia*, a book that will be published this summer by Basic Books. William A. Newcomb, the man who discovered the paradox, is a theoretical physicist at the Lawrence Livermore Laboratory of the University of California.

What follows is the communication I received from Nozick in October. May I urge readers who wish to write again not to do so until they have read Nozick's original paper? It goes into considerably more technical detail than my first article or Nozick's present comments.

Newcomb's problem involves a Being who has the ability to predict the choices you will make. You have enormous confidence in the Being's predictive ability. He has already correctly predicted your choices in many other situations and the choices of many other people in the situation to be described. We may imagine that the Being is a graduate student from another planet, checking a theory of terrestrial psychology, who first takes measurements of the state of our brains before making his predictions. (Or we may imagine that the Being is God.) There are two boxes. Box 1 contains $1,000. Box 2 contains either $1 million or no money.

You have a choice between two actions: taking what is in both boxes or taking only what is in the second box. If the Being predicts you will take what is in both boxes, he does not put the $1 million in the second box. If he predicts you will take only what is in the second box, he puts the million in the second box. (If he predicts you will base your choice on some random event, he does not put the money in the second box.) You know these facts, he knows you know them and so on. The Being makes his prediction of your choice, puts the $1 million in the second box or not, and then you choose. What do you do?

There are plausible arguments for reaching two different decisions:

1. The expected-utility argument. If you take what is in both boxes, the Being almost certainly will have predicted this and will not have put the $1 million in the second box. Almost certainly you will get only $1,000. If you take only what is in the second box, the Being almost certainly will have predicted this and put the money there. Almost certainly you will get $1 million. Therefore (on plausible assumptions about the utility of the money for you) you should take only what is in the second box [*see illustration on opposite page*].

2. The dominance argument. The Being has already made his prediction and has either put the $1 million in the second box or has not. The money is either sitting in the second box or it is not. The situation, whichever it is, is fixed and determined. If the Being put the million in the second box, you will get $1,001,000 if you take both boxes and only $1 million if you take only the second box. If the Being did not put the money in the second box, you will get $1,000 if you take both boxes and no money if you take only the second box. In either case you will do better by $1,000 if you take what is in both boxes rather than only what is in the second box [*see illustration on page 104*].

Each argument is powerful. The problem is to explain why one is defective. Of the first 148 letters to *Scientific American* from readers who tried to resolve the paradox, a large majority accepted the problem as being meaningful and favored one of the two alternatives. Eighty-nine believed one should take only what is in the second box, 37 believed one should take what is in both boxes—a proportion of about 2.5 to one. Five people recommended cheating in one way or another, 13 believed the problem's conditions to be impossible or inconsistent and four maintained that the predictor cannot exist because the assumption that he does leads to a logical contradiction.

Those who favored taking only the second box tried in various ways to undercut the force of the dominance argument. Many pointed out that if you thought of that argument and were convinced by it, the predictor would (almost certainly) have predicted it and you would end up with only $1,000. They interpreted the dominance argument as an attempt to outwit the predictor. This position makes things too simple. The proponent of the dominance argument does believe he will end up with only $1,000, yet nevertheless he thinks it is best to take both boxes. Several proponents of the dominance principle bemoaned the fact that rational individuals would do worse than irrational ones, but that did not sway them.

Stephen E. Weiss of Morgantown, W.Va., tried to reconcile the two views. He suggested that following the expected-utility argument maximizes expectation, whereas following the dominance argument maximizes correct decision. Unfortunately that leaves unexplained why the correct decision is not the one that maximizes expectation.

The assumptions underlying the dominance argument, that the $1 million is already in the second box or it is not and that the situation is fixed and determined, were questioned by Mohan S. Kalelkar, a physicist at the Nevis Laboratories of Columbia University, who wrote: "Perhaps it is false to say that the Being has definitely made one choice or the other, just as it is false to say that the electron [in the two-slit experiment] went through one slit or the other. Perhaps we can only say that there is some amplitude that B2 [second box] has $1 million and some other amplitude that it is empty. These amplitudes interfere unless and until we make our move and open up the box. . . . To assert that 'either B2 contains $1 million or else it is empty' is an intuitive argument for which there is no evidence unless we open the box. Admittedly the intuitive evidence is strong, but as in the case of the double-slit electron diffraction our intuition can sometimes prove to be wrong."

Kalelkar's argument makes a version of the problem, in which the second box is transparent on the other side and someone has been staring into it for a week before we make our choice, a significantly different decision problem. It seems not to be. Erwin Schrödinger, in a

famous thought experiment, imagined a cat left alone in a closed room with a vial of cyanide that breaks if a radioactive atom in a detector decays. Must a disciple of Niels Bohr's assert that the cat is neither alive nor dead, Schrödinger asked, until measurements have been made to decide the case? Even if one accepts the Bohr interpretation of quantum mechanics, however, what choice *does* one make, in Newcomb's problem, when one knows that others can see into the box from the other side and observe whether it is filled or empty?

Many who wrote asserted that the dominance argument assumes the states to be probabilistically independent of the actions and pointed out that this is not true for the two states "The $1 million is in Box 2" and "The $1 million is not in Box 2." The states would be probabilistically independent of the actions (let us assume) in the matrix for the utility argument, which has the states "He predicts correctly" and "He predicts incorrectly." Here, however, there is no longer dominance. Therefore it appears that the force of dominance principles is undercut. "It is legitimate to apply dominance principles if and only if the states are probabilistically independent of the actions. If the states are not probabilistically independent of the actions, then apply the expected-utility principle, using as the probability-weights the conditional probabilities of the states given in the actions." The quotation is from my original 1970 essay, which formulated this position, then went on to reject it as unsatisfactory for the following reasons.

Suppose a person knows that either man *S* or man *T* is his father but he does not know which. *S* died of some very painful inherited disease that strikes in one's middle thirties and *T* did not. The disease is genetically dominant. *S* carried only the dominant gene. *T* did not have the gene. If *S* is his father, the person will die of the dread disease. If *T* is his father, he will not. Furthermore, suppose there is a well-confirmed theory that states a person who inherits this gene will also inherit a tendency toward behavior that is characteristic of intellectuals and scholars. S had this tendency. Neither *T* nor the person's mother had such a tendency. The person is now deciding whether to go to graduate school or to become a professional baseball player. He prefers (although not enormously) the life of an academic to that of a professional athlete. Regardless of whether or not he will die in his middle thirties, he would be happier as an academic. The choice of the academic life would thus appear to be his best choice.

Now suppose he reasons that if he decides to be an academic, the decision will show that he has such a tendency and therefore it will be likely that he carries the gene for the disease and so will die in his middle thirties, whereas if he chooses to become a baseball player, it will be likely that *T* is his father, therefore he is not likely to die of the disease. Since he very much prefers not dying of the disease (as a baseball player) to dying early from the disease (as an academic), he decides to pursue the career of an athlete. Surely everyone would agree that this reasoning is perfectly wild. It is true that the conditional probabilities of the states "S is his father" and "*T* is his father" are not independent of the actions "becoming an academic" and "becoming a professional athlete." If he does the first, it is very likely that S is his father and that he will die of the disease; if he does the second, it is very likely that *T* is his father and therefore unlikely that he will die of the disease. But who his father is cannot be changed. It is fixed and determined and has been for a long time. His choice of how to act legitimately affects our (and his) estimate of the probabilities of the two states, but which state obtains (which person is his father) does not depend on his action at all. By becoming a professional baseball player he is not making it less likely that S is his father, therefore he is not making it less likely that he will die of the disease.

This case, and others more clearly including a self-reference that this case may seem to lack, led me to think probabilistic nonindependence was not sufficient to reject the dominance principle. It depends on whether the actions influence or affect the states; it is not enough merely that they affect our judgments about whether the states obtain. How do those who reject the dominance principle for Newcomb's problem distinguish it from those other cases where dominance principles obviously apply even though there is probabilistic nonindependence?

But one must move carefully here. One cannot force a decision in a difficult case merely by finding another similar case where the decision is clear, then challenging someone to show why the decision should be different in the two cases. There is always the possibility that whatever makes one case difficult and the other clear will also make a difference as to how they should be decided. The person who produces the parallel example must not only issue his challenge; he must also offer an explanation of why the difficult case is less clear, an explanation that does not involve any reason why the cases might diverge in how they should be decided. Interested readers can find my additional parallel examples where dominance is appropriate, plus an attempt to explain why Newcomb's case, although less clear, is still subject to dominance principles, in my original essay, "Newcomb's Problem and Two Principles of Choice," in *Essays in Honor of Carl G. Hempel*, edited by Nicholas Rescher, Humanities Press, 1970.

This obligation to explain differences in the clarity of parallel examples in order to show that no different decision should be made also rests on those who argued in their letters for taking only what is in the second box. For example, it rests on Robert Heppe of Fairfax, Va., who said that the situation "is isomorphic with one in which the human moves first and openly," and on A. S. Gilbert of the National Research Council of Canada, who called the Newcomb case "effectually the same as" one where you act first and an observer attempts to communicate with a "mindreader" in the next room who then guesses your choice, using a payoff matrix identical with Newcomb's.

A large number of those who recommended taking only the second box performed the expected value calculation and concluded that, provided the probability that the Being was correct was at least .5005, they would take only the second box. Not only did they see no problem at all; they either maximized expected monetary value or made utility linear with money in the range of the problem. Otherwise the cutoff probability would be different. William H. Riker

	HE PREDICTS YOUR CHOICE CORRECTLY	HE PREDICTS YOUR CHOICE INCORRECTLY
TAKE BOTH	$1,000	$1,001,000
TAKE ONLY SECOND	$1,000,000	$0

Payoff matrix for expected-utility argument

of the department of political science at the University of Rochester suggested that people making different decisions merely differed in their utility curves for money. Such persons, however, need not differ in their choices among probability mixtures of monetary amounts in the standard situations in order to calibrate their utilities.

Those who favored taking both boxes made almost no attempt to diagnose the mistakes of the others. An exception is William Bamberger, an economist at Wayne State University. He wrote that the proponent of choosing only the second box "computes not the alternative payoffs of choosing one or two boxes for a given individual, but the average payoff of those who choose two as opposed to the average observed payoff of those who choose one." The problem, of course, is how to compute the probability for a given individual of his payoff for each choice. Should one use the differing conditional probabilities, or ignore them because dominance applies only when the states are probabilistically independent of the action (and so when for each state its conditional probabilities on each act is the same), or ignore them since the conditional probabilities of the state on the acts are to be used only when they represent some process of the act's influencing or affecting which state obtains?

A number of respondents said their choice would depend on whether the predictor made his prediction after they had at least started to consider the problem. If so, they would do their best to decide to take only the second box (so that this data would be available to the predictor), and some added that they hoped they would change their mind at the last minute and take both boxes. (They gave the predictor too little credit.) On the other hand, if the predictor made his prediction before they even considered the problem, these writers believed they would take both boxes, since there was no possibility of their deliberations affecting the prediction that had been made.

Several respondents maintained that if the conditions of the problem could be realized, we might be forced to revise our views about the impossibility of backward causality. Newcomb himself seems to think that special difficulties arise for proponents of backward causality if the predictor writes some term designating an integer on a slip of paper in the second box, with the understanding that you get $1 million only if that integer is a prime. Of course, the predictor writes a prime if, and only if, he predicts that you will take the second box. How can your choice determine whether a number is prime or composite? The advocate of backward causality need not think it does. What your choice affects, in his view, is what term the predictor writes down (or wrote down earlier), not whether the integer it designates is prime or composite.

The reasoning of some of the letters indicates it would be useful to specify precisely the conditions whereby we could discover in which time-direction causality operates. Might one even say that some conditions universally preceding certain decisions are part of the effects of the decision (by backward causality) rather than part of the cause?

Not everyone was willing to choose one or the other action. Among the five respondents who suggested some form of cheating, Robert B. Pitkin, editor of *American Legion Magazine,* speculated that Dr. Matrix, the numerologist, would walk in with a device to scan the contents of the boxes, take the boxes with the money in them and never open an empty box. "He quite naturally succeeded in getting all the money, for the rule of bridge that one peek is worth two finesses applies here too.... By introducing a choice which the Being has not anticipated, and is not permitted to take into account, he achieves a stunning victory for free will." (What prevents the Being from taking this into account?)

Other letter writers also struck blows for free will. Nathan Whiting of New York would take both boxes but would open only the first one, leaving the second box unopened. Ralph D. Goodrich, Jr., of Castle Rock, Colo., would take only the first box. Richard B. Miles of Los Altos, Calif., also recommended a "creative" solution: Turn to another person before you make your choice and offer to sell him for $10,000 the contents of whatever box or boxes you choose.

Isaac Asimov wrote: "I would, without hesitation, take both boxes.... I am myself a determinist but it is perfectly clear to me that any human being worthy of being considered a human being (including most certainly myself) would prefer free will, if such a thing could exist.... Now, then, suppose you take both boxes and it turns out (as it almost certainly will) that God has foreseen this and placed nothing in the second box. You will then, at least, have expressed your willingness to gamble on his nonomniscience and on your own free will and will have willingly given up a million dollars for the sake of that willingness—itself a snap of the finger in the face of the Almighty and a vote, however futile, for free will.... And, of course, if God has muffed and left a million dollars in the box, then not only will you have gained that million but *far more important* you will have demonstrated God's nonomniscience. If you take only the second box, however, you get your damned million and not only are you a slave but also you have demonstrated your willingness to be a slave for that million and you are not someone I recognize as human." (No one wrote to argue for taking only the second box on the grounds that either it results in getting $1 million or it demonstrates the Being's fallibility, either of which is desirable.)

Those who held that the conditions of the problem could not be realized were of two types. There were those who believed the situation to be physically impossible because the Being could not predict all the information input of every light signal that would arrive at your eyes in the appropriate time interval. ("To gain such knowledge the Being must have a physical agency for collecting information that travels faster than the speed of light," wrote George Fredericks, a physicist at the University of Texas.) And there were those who argued that if the room is closed, the problem reduces to that of Maxwell's demon—a suggestion made by Fredericks and by John A. Ball of the Harvard College Observatory.

Those who believed the conditions of the problem to be inconsistent as well as physically impossible said that the almost certain predictability of decisions was inconsistent with free will, and therefore with making choices, yet the problem assumed that genuine choices could be made. This is a hard argument

	HE PUT $1,000,000 INTO BOX 2	HE DID NOT PUT $1,000,000 INTO BOX 2
TAKE BOTH	$1,001,000	$1,000
TAKE ONLY SECOND	$1,000,000	$0

Payoff matrix for dominance argument

FAR EAST & AUSTRALIA

10 Express flights a week by SAS, Thai International and Aeroflot.

connections from all over Europe to Copenhagen/ Frankfurt	Monday	Tuesday	Wednesday	Thursday Friday Saturday	Saturday	Sunday	TRANS-SIBERIAN EXPRESS	
							Thursday	Saturday
	SAS	SAS	THAI	SAS	THAI	Aeroflot	Aeroflot	SAS
departures:								
LONDON	11.20	15.10	10.05	8.15	10.50	11.20	8.15	8.15
COPENHAGEN	15.00	16.25	12.05	12.05		15.00	12.05	12.00
FRANKFURT	12.50	18.30	14.05	9.30		12.50	9.30	9.30
	Tashkent	NONSTOP	NONSTOP	Tashkent	NONSTOP	Moscow	Moscow	Moscow
arrivals, following day:								
DELHI		7.00				5.05		
BANGKOK	9.20	12.50	6.55	6.25	6.00	11.30		
HONG KONG[1]	14.10	17.25	11.25	11.25	11.25	19.40		
KUALA LUMPUR		15.50						
SINGAPORE	12.55	17.10	10.35[3]	13.15/11.05/10.35	10.35[3]	15.00		
JAKARTA	14.25							
SYDNEY			22.20[3]	22.20[2]	22.20[3]			
TOKYO							9.35	9.10

[1]) Connecting flights from Bangkok. [2]) Thai International from Bangkok, Sunday.
[3]) Connecting flight from Bangkok, through-going aircraft Europe – Sydney.

FIRST AND ONLY NONSTOP SERVICES
FRANKFURT—DELHI (SAS)
FRANKFURT—BANGKOK (THAI)
LONDON—BANGKOK (THAI)

ONLY SERVICES WITH NEXT-DAY ARRIVAL IN SYDNEY— JUST ONE NIGHT EN ROUTE!

SAS SCANDINAVIAN AIRLINES
Representing THAI International

to drive through because it appears to be the choices that are predicted. The relevant connections are difficult to get straight. Predictability of decisions does not logically imply determinism under which the decisions are caused (for example the possibility of backward causality where an uncaused decision causes an earlier prediction, or "seeing ahead" in time in a block universe).

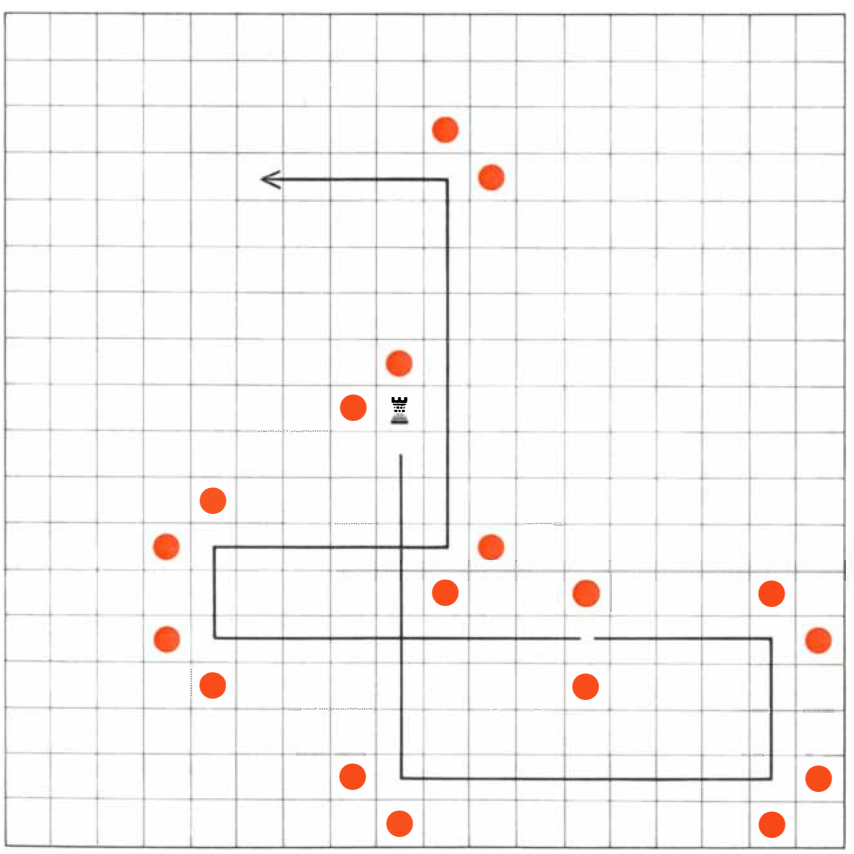
Nor, we should note in passing, does determinism entail predictability, even in principle. Events could be fixed in accordance with scientific laws that are not recursive. Is determinism incompatible with free will? It seems to many to be so, yet the argument that determinism is incompatible with responsibility for action, which free will implies, depends on a notion of responsibility insufficiently worked out to show precisely how the connections go. Some say merely that a free act is an uncaused one. Yet being uncaused obviously is not sufficient for an act to be free; one surely would not be responsible for such an action. What other conditions, then, must be satisfied by an uncaused act if it is to be a free one? The literature on free will lacks a satisfactory specification of what a free action would be like (given that "uncaused" is not enough). Perhaps if we were given this specification of additional conditions, they would turn out to be sufficient apart from the action's being uncaused.

Another problem will help to exhibit some complicated relations between free will and determinism. It has been asserted (by C. S. Lewis, for instance) that no determinist rationally can believe in determinism, for if determinism is true, his beliefs were caused, including his belief in determinism. The idea seems to be that the causes of belief, perhaps chemical happenings in the brain, might be unconnected with any reasons for thinking determinism true. They might be, but they need not be. The causes might "go through" reasons and be effective only to the extent that they are good reasons. In the same way it might be a causal truth about someone that he is convinced only by arguments that constitute specified types of good reasons (deductive, inductive, explanatory and so on).

Some philosophers have argued recently that we know some statement p only if part of the cause (or more broadly the explanation) of our believing p is, if we pursue the story far enough, the fact that p is true. You know this magazine is before you now only if its being there is part of the explanation of why you believe it is there. If psychologists are stimulating your brain to create the illusion that you are seeing a magazine, you would not really know there is a magazine before you even if a psychologist happened to have left one on the table in front of you. The magazine's being there would not play the proper causal role in the story of your belief. If we do not mind our beliefs being caused by the facts, and indeed find it somewhat plausible to think we have knowledge only to the extent that they are, then we may also find it less disturbing that our actions are caused by certain types of facts holding in the world, for example, the fact that it would be better to do one thing rather than another. To say this, of course, is not to present a theory of free action; it is merely to hint that it may be possible to remove the sting of determinism. This approach is a comfortable one when we act correctly, but it is difficult to see how it can be extended plausibly to wrong acts where questions of responsibility are particularly pressing.

Proponents of the C. S. Lewis position might reply that the determinist should not feel so comfortable. Even though he says he is caused to believe in determinism (and anything else) by what are good reasons, he must also maintain that he is caused to believe that such reasons are good reasons. He may have a second set of reasons for believing the first set of reasons are good. Now, however, his opponent can raise the same question as before. Why does he believe the second set of reasons? The determinist must end either by finding self-supporting reasons (which say of themselves that they are good reasons) or by admitting that the best explanation of why he believes they are good reasons is that they are. This surely leaves his opponent unsatisfied, and the match seems to be a draw.

Those who believe in free will find themselves in similar dilemmas. Kurt Rosenwald of Washington wrote: "When I was 19 or 20, I thought about the free-will problem . . . and I came to this conclusion: If we make an exhaustive study of that problem, and finally arrive at the result that our will is free, we still will not know whether our will is indeed free or our mind is of such a nature that we have to find our will to be free, although it is not, in fact, free. This became one of my reasons for studying not philosophy but the natural sciences. Thinking about it now, 50+ years later, it still seems to me that I was right." But does not the possibility that we are caused to



How to trap a rook with two quads per move

Fly Smithsonian.

Have you ever been to a place where they teach eagles to be wild?
Have you ever journeyed deep into the mind of a man
who single-handedly revolutionized Man's entire perception of the universe?
Have you ever penetrated to the heart of the Cretan Maze, to confront the Minotaur?

Visit some of the world's most enlightening destinations.
Fly with SMITHSONIAN Magazine for the next 12 months... for less than one dollar a month.

We invite you to spread the wings of your imagination every month, and come with us into many surprising worlds of nature, art, history and science. And we invite you to bring your entire family along.

When did Man first set foot on the North American continent—and think about when Man is likely to first set foot on Mars. Is rainmaking becoming respectable? Why is it essential to save the prairie dog from extinction? What *really* happened the night that Paul Revere made his famous ride?

What happens when the gold-bedecked chieftains of the Akan pay a state visit to their new King? What is currently the rarest stamp in the world? Were dinosaurs the only warm-blooded reptiles that ever lived? What causes the brilliant colors of butterfly wings? How close are we to directly harnessing the power of the Sun?

If these questions intrigue you, you'll be intrigued and pleased with SMITHSONIAN. And if you are attracted and moved by beauty, you'll love SMITHSONIAN the more... because it is one of the handful of magazines in the world today that is almost a work of art in itself. Here every month you will find a dazzling display of beauties both ancient and modern—from the most recently discovered Pompeiian frescoes to the finest works of modern masters (many of them never seen outside private collections before).

What's more, as beautiful, informative, intelligent and entertaining as SMITHSONIAN Magazine is, it's *only one* of the knowledge-enhancing benefits you and your family receive when you become a National Member of the Smithsonian Associates.

Much more than a magazine

In addition to SMITHSONIAN every month, your membership in the Smithsonian Associates entitles you and your family to:

■ **Special travel programs** organized for Members only—nearly 50 opportunities each year to visit historic sites, art centers and scientific installations (many of which are not open to the general public).

■ **Discounts on quality books** dealing with history, art, science and nature.

■ **Discounts on unusual gifts** from the museum shops of the Smithsonian Institution... fascinating arts and crafts from all corners of the globe.

■ **Special welcome** when you and your family visit the Smithsonian in Washington, D.C. including use of the Member's Lounge and the privilege of dining with the Smithsonian staff.

The contribution you are making to knowledge

Beyond the extensive, almost unlimited, resources and opportunities for learning that your Membership will bring to you and your family, a substantial portion of your annual $10 Membership Fee goes *to help increase the fund of Man's knowledge.*

It goes to help support the many research and educational programs of the Smithsonian Institution's private sector which depends on the private support of individuals for its activities.

Join today— return the coupon below

If you are interested in exploring many new regions of the world and of the mind, if you respond to the historic and the beautiful and the innovative, if you would like to help increase the range of Man's mind, join the Smithsonian today. All it takes is a few brief moments to fill out and mail the coupon below to spread the wings of your mind.

Mail this coupon to:

SMITHSONIAN Membership Service
P.O. Box 2606 • Greenwich, Conn. 06830

Please enroll me as a National Member of the Smithsonian Associates, and enter my subscription for the next year (12 issues) of SMITHSONIAN.

Bill me for the annual dues of $10.

Name *(please print clearly)*

Address

City

State Zip

MEMBERSHIP BENEFITS
- SMITHSONIAN Magazine — 12 issues
- Travel Program — Domestic and Foreign
- Book and Gift Discounts
- Reception Center at Smithsonian Institution in Washington, D.C.

P6-34

believe in false conclusions apply also to conclusions in the natural sciences? And to the verdict of 50+ years later?

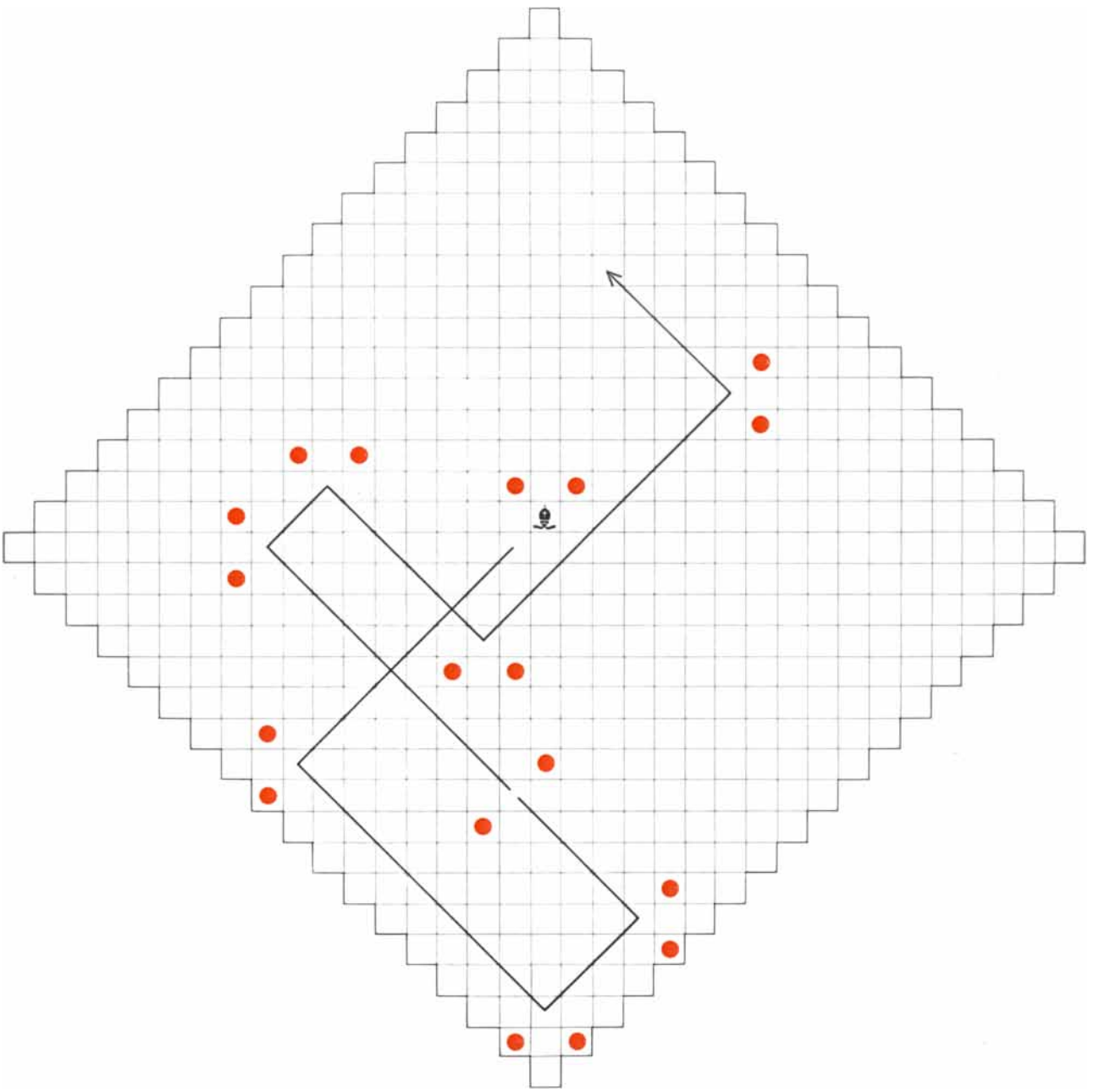
I published my original essay after thinking about Newcomb's problem intermittently for five years. In that essay I expressed the hope that someone would come forth with a solution to the problem that would enable me to stop returning to it. It is not surprising that no one did, yet it is surprising (to me) that the mere act of publishing Newcomb's problem, and sending my thoughts on it into the world, rid me of it. That is, I was rid of it until the problem was presented in *Scientific American* and I was invited to read more than 650 pages of letters about it. Unfortunately the letters do not, in my opinion, lay the problem to rest. And they have started me thinking about it again! You can't win.

Answers to some of the questions raised last month follow:

If a rook's maximum move in a game of quadraphage is n cells, it can be trapped by two quads per move on a board of side $2n + 2$. The strategy is to consider the unobstructed paths from the rook to sides of the board and place the quads adjacent to the rook to block its movement to the two nearest sides. (If two borders are the same distance away, choose either one.) The illustration on page 106 shows the strategy on a go board, the rook limited to maximum moves of eight cells. The rook clearly can never reach the edge. Eventually it must head toward a quad. When this happens, quads on each side confine it to a segment of the path and entrapment quickly follows.

The same procedure will trap a bishop within the borders of a sawtooth board, $2n + 2$ on a side, where n is the bishop's longest move. The illustration below shows how the strategy operates when the bishop can move no more than eight cells.

I conjecture that a similar strategy, using four quads per move, will trap the



How to trap a bishop with two quads per move

queen. For n greater than 2 a board of at least $4n + 2$ on a side seems necessary. (Thus a queen with maximum move 8 can be trapped on a 34-sided field.) The first move is to put quads on the four corner cells. Thereafter use the "nearest sides" strategy. When there is a choice between blocking equal paths on an orthogonal and a diagonal, block the orthogonal.

Some of the questions raised about "spirolaterals," in last November's column, have been answered by readers. The main question—How can you determine from a spirolateral's formula whether it closes and, if it does, in how many repetitions?—was answered by James Thomas, William Laubenheimer, Steven Wolfson and E. Lawrence McMahon. It turns out (as other readers also reported) that a spirolateral results whenever the angle of turn is a rational number. If the angle is irrational, the spirolateral remains within a bounded region but never closes.

Thomas gave the following procedure: First determine the angle's supplement (its difference from 180 degrees). Multiply this by the difference between the number of left turns and number of right turns. (The difference is equal to the spirolateral's order minus twice the number of left turns.)

From the above result, keep subtracting 360 until the remainder is between -180 and 180. Take the absolute value and call it x. This represents the net angular change after each cycle. If x equals 0, there is either no closure (and the spirolateral is infinite) or it closes after the first cycle.

If x is not zero, divide it into the lowest multiple of 360 that it will go into evenly. The result is the number of cycles required to close the spirolateral.

To express this procedure by a compact formula, McMahon proposed letting n equal the spirolateral's order, k equal the number of left turns and m equal 360 divided by the rational angle. Write the following fraction,

$$\frac{(m-2)(n-2k)}{2m},$$

and reduce to lowest terms. If the result is an integer, the spirolateral either does not close or closes after one cycle. If the result is an integral fraction, a/b, the figure closes after b cycles.

Correction: In January's column it was inadvertently stated that six coins are used in consulting the *I Ching*. As the description of the tossing procedure made clear, only three coins are used.