Beyond $\mathcal{FOUR}$:
Representations of Non-interlaced Bilattices Using Natural Duality

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A bilattice **B** is said to be *interlaced* if the truth lattice operations are monotonic with respect to the knowledge order, and the knowledge lattice operations are monotonic with respect to the truth order. As the existing product representations only apply to interlaced bilattices, we look at natural dualities for quasivarieties generated by non-interlaced bilattices. Examples include the seven-element bilattice which was used by Ginsberg [1] for applications in default logic.

Given an algebra **M** with M its underlying set, an n-ary relation on M is said to be *algebraic* over **M** if it forms a subalgebra of $\mathbf{M}^n$. The theory of natural dualities uses topological spaces with additional algebraic relations (and operations) as the dual structures. As the bilattices we consider are not interlaced, the knowledge order is no longer an algebraic binary relation on the bilattice and thus it cannot be used as a relation in the dual structures given by the theory of natural dualities.

However, the knowledge order is still intrinsic to these dual structures, and we show how it is encoded in a relational structure which yields a duality. The nature of the obtained dual structures suggest an alternative algebraic semantics which may include a change in the signature.

[1] M. L. Ginsberg. Multivalued logics: A uniform approach to inference in artificial intelligence. Computational Intelligence, 4, (1988), 265–316.