

# The categorical duality between complete (semi)lattices with operators and contexts with relations

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Joint work with and M. A. Moshier

Since the seminal papers of Jónsson and Tarski in the early 1950s, the categorical duality between complete and atomic Boolean algebras with operators and complete homomorphisms, and Kripke frames with bounded morphisms has been a corner stone of algebraic logic. In a paper with N. Galatos [1], we introduced residuated frames as objects dual to complete residuated lattices, but did not dualize complete residuated lattice homomorphisms. In a residuated frame the underlying lattice-order of a complete residuated lattice is represented by a context (in the terminology of formal concept analysis) or a polarity (in Birkhoff's terminology). Related work appears in [2,3] under the additional assumption that the contexts are reduced.

Recently M. A. Moshier [4] defined morphisms for contexts to obtain a category  $\mathbf{Cxt}$  that is dual to the category  $\mathbf{INF}$  of complete meet semilattices with completely meet-preserving homomorphisms. In this talk we define the category of contexts with relations by showing how the context morphisms, with suitable restrictions to bounded morphisms, correspond to homomorphisms between complete lattices with operators. Hence we obtain a categorical duality between complete (semi)lattices with operators and contexts with relations. This setting includes for example an equivalence between complete residuated lattices and residuated frames, which can be viewed as a basic duality of substructural logic.

Considering the subcategory of algebraic contexts with relations gives a duality to all semilattices with operators, while adding topology to the contexts leads to a bigger category that is dual to all lattices with operators, extending dualities of Urquhart, Hartung and Hartonas. One of the advantages of working on the context side is that objects can be logarithmically smaller and are simpler to construct. The notion of context morphism can also be loosened to a pair of Chu-like relations which compose with ordinary relation composition. This allows further freedom and efficiencies in the construction of operators and morphisms.

As application one has, for example, that the duality maps products of complete lattices to certain disjoint unions of contexts, and other constructions like ordinal sums and poset products can also be obtained by combinatorial means on the context side.

- [1] N. Galatos and P. Jipsen, Residuated frames with applications to decidability, to appear in Transactions of the American Math. Soc.
- [2] J. M. Dunn, M. Gehrke and A. Palmigiano, Canonical extensions and relational completeness of some substructural logics, J. Symbolic Logic, 70(3) 2005, 713–740.
- [3] M. Gehrke, Generalized Kripke frames, Studia Logica, 84, 2006, 241–275.
- [4] M. A. Moshier, A relational category of formal contexts, preprint.