Generalised Gelfand Spectra
for Noncommutative Operator Algebras
and Multi-Valued Logic for Quantum Systems

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Gelfand duality provides an enormously useful bridge between commutative C^*-algebras algebras and (locally) compact Hausdorff spaces. Yet, in quantum theory noncommutative C^*-algebras play a key role. For these, a suitable notion of spectrum is largely lacking. I will show that with each unital C^*-algebras one can associate a presheaf topos with a distiguished object, the spectral presheaf, that generalises the Gelfand spectrum of the commutative case. This assignment is shown to be functorial, providing a step towards noncommutative Gelfand duality. In the case of von Neumann algebras, the spectral presheaf can also be seen as a generalised Stone spectrum. I will discuss some topologies on the spectral presheaf, in particular the daseinisation topology, and how they relate to propositions about the quantum system at hand. This leads to a new, multi-valued and intuitionistic form of logic for quantum systems.