A General Duality Theory for Clones

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When it comes to dualising clones, the usual approach is to consider a clone as the set of term functions of a suitable algebra and then try to dualise this algebra, which may, or may not, be possible. Another approach was introduced by D. Mašulović in 2006, where clones are dualised by treating them as sets of homomorphisms in a quasivariety that is then understood and dualised as a category. Although new results were obtained by using this duality, the approach is somewhat limited. It only works for a tiny fraction of clones (the so-called centralizer clones on finite sets), and it does not give us any information about what happens to the Galois connection Pol-Inv, that is set up between operations and relations and is possibly the single most effective item in a clone theorist's toolbox.

The aim of the talk is to extend this approach such that both of these drawbacks are overcome. In other words, we want to build a duality theory that is general enough to work for any clone and also dualises all parts of the aforementioned Galois connection Pol-Inv. To do so, we will look at clones as sets of morphisms in categories, obtaining a notion essentially identical to that of a model of a Lawvere theory. We will then outline how to generalize relations to categories and construct a Galois theory that is similar to Pol-Inv but applicable in any (possibly abstract) category. With these notions, we will present a general duality theory for clones, allowing us to dualise any given clone together with its relational counterpart and the relationship between them.

Finally, we will put the approach to work and illustrate its application by producing some specific results for concrete examples as well as some general results that come from studying the duals of clones in a rather abstract fashion.