Introduction to Integration: Corrections and amendments to 1st printing

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4_{11} $\sqrt{\pi/2} \quad \to \quad \sqrt{2\pi}$

12^{1} $\operatorname{dom} g \subseteq \operatorname{im} f \quad \to \quad \operatorname{dom} g \supseteq \operatorname{im} f$

15^{12} infinite subset $\quad \to \quad$ bounded infinite subset

15^{16} Replace **[16]** by **[1]**

21_{4} Add 'or I_1 and I_2 have a common endpoint a with $a \in I_1 \cup I_2$' after 'only if'

28_{10} Add condition $\varphi(0) = 0$ in (a)

28_{2} Add condition $\operatorname{im} \varphi$ finite in (b)

29_{10-3} There is a flaw in this proof. A fix is:—

Drop condition (ii) and refine the representation $\varphi = \sum_{t=1}^{q} a_t \ell(K_t)$ further at the end of Stage 2 by removing all K_t with $a_t = 0$

38_{1} replace 'the antiderivative of cos is *sin*, that is, $\mathrm{d}/\mathrm{d}x(\sin x) = \cos x$' [to fit with line 3 on p.39]

39^{13} $= y(0) \quad \to \quad -y(0) =$

48^{9} answer should be $\left[\frac{-4}{\sqrt{2}(1+t^2)^{1/2}}\right]_0^1 = 2\sqrt{2} - 2$

41^{11} property (P) and 5.2(d) $\quad \to \quad$ property (P), 5.2(d) and 5.3(a).

41_{8} Replace 'choose n by 'consider n

$48^{9}+$ answer should be $\left[\frac{-4}{\sqrt{2}(1+t^2)^{1/2}}\right]_0^1 = 2\sqrt{2} - 2$

42^{9} $\int_a^b (g \pm h)^2, \quad \to \quad \int_a^b (g \pm th)^2$, for $t \in \mathbb{R}$,

43^{3} 2.26(c)(a) $\quad \to \quad$ 2.26(3)(a)

$44_{1}1$ Delete $-$ sign

45_{5} $x + 2\log|x+2| \quad \to \quad x - 2\log|x+2|$

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48_{5} Substitute integral for (iv): $\int_0^{\pi/2} \frac{1}{5+4\cos x}\,\mathrm{d}x$ [more typical example for $\tan\frac{1}{2}t$ substitution; the given integral can be more easily handled using trigonometric formulae]

48_1 Answer should be $\log 2$

$49_{10-8}+$ Replace by:

$$I = [\mathrm{e}^x \sin \pi x]_0^1 - \pi \int_0^1 \mathrm{e}^x \cos \pi x \,\mathrm{d}x$$
$$= -\pi\,[\mathrm{e}^x \cos(\pi x)]_0^1 - \pi^2 \int_0^1 \mathrm{e}^x \sin \pi x \,\mathrm{d}x = \pi(\mathrm{e}+1) - \pi^2 I.$$

Hence $I = \pi(\mathrm{e}+1)/(\pi^2+1)$.

49_{14} $(1 - 3\mathrm{e}^{-1/2}) \quad \to \quad (2 - 3\mathrm{e}^{-1/2})$

50^3 Integrand in (iv) should be $\frac{x}{(1+x)^3}$

50_{10} $I_{2n} = \cdots = \frac{(2n)!}{(2^n n!)^2} . \frac{\pi}{2}$,

50_9 $I_{2n+1} = \cdots = \frac{(2^n n!)^2}{(2n+1)(2n)!}$.

51^3 $I_n = -I_{n-2} \quad \to \quad I_n = (-\pi)^n/n! - I_{n-2}$

52^9 $\mathrm{e}^{-\lambda t} \quad \to \quad \mathrm{e}^{-\lambda t^2/2}$

52^{10-11} should read: … we have, for $x \in [0, b]$,

$$f(x)\mathrm{e}^{-\lambda x^2/2} - f(x) = g(x) + \lambda \int_0^x \mathrm{e}^{-\lambda t^2)/2} g'(t)\,\mathrm{d}t$$
$$= \mathrm{e}^{-\lambda x^2)/2} g(x) - g(0) + \lambda \int_0^x t\mathrm{e}^{\lambda(-t^2} g(t)\,\mathrm{d}t,$$

whence, integrating by parts,

$$f(x) = g(x) + \lambda \int_0^x t\mathrm{e}^{\lambda(x^2-t^2)/2} g(t)\,\mathrm{d}t.$$

54^4 $\lim_{n\to\infty} \frac{1}{2n+1} . \frac{2^{4n+1}(n!)^4}{(2n)!^4} \quad \to \quad \lim_{n\to\infty} \frac{1}{2n+1} . \frac{2^{4n+1}(n!)^4}{(2n)!^2}$

54^{13} $(m+n)I_{m,n} = (-1)^{m+n} - 1 + mI_{m-1,n-1} \quad \to \quad (m+n)I_{m,n} = -(-1)^{m+n} + 1 + mI_{m-1,n-1}$

56_2 Replace ‘shaded’ by ‘indicated’

56_1 Replace $f'(a)$ by $\frac{f(b)-f(a)}{b-a}$

61

64_8 $[x \log x]_1^n - \int_1^n \frac{1}{x}\,\mathrm{d}x \quad \to \quad [x \log x]_1^n - \int_1^n x . \frac{1}{x}\,\mathrm{d}x$

83–84 Lemma 9.8 requires $\varphi \geqslant 0$; assume this, and at start of proof of Theorem 9.9 add ‘Without loss of generality we may assume that $\varphi_n \geqslant 0$ for all n since we may replace $\{\varphi_n\}$ by $\{\varphi_n - \varphi_1\}$ if this is not initially satisfied.’

86 Add after the proof of Technical Theorem II:

[We remark that in Technical Theorem II we cannot assert that $\{\varphi_n\}$ diverges *precisely* on the given null set E. This comes from facts about Borel sets given in Exercise 22.3. The set on which the sequence diverges is $F := \bigcap_k \bigcup \{ x \mid \varphi_n(x) \geqslant k \}$ and this is a Borel set, whereas the null set E need not be a Borel set.]

86_{8-5} Replace by: Let $\varphi_n := \chi_{I_n}$, where $\{I_n\}$ is the sequence of intervals

$$[0, \tfrac{1}{2}],\ (\tfrac{1}{2}, \tfrac{2}{2}],\ [0, \tfrac{1}{3}],\ (\tfrac{1}{3}, \tfrac{2}{3}],\ (\tfrac{2}{3}, \tfrac{3}{3}],\ \ldots.$$

Prove that

(i) $\{\varphi_n(x)\}$ does not converge for any $x \in [0,1]$, and

92_{14} Add at end: (This is hard; an example can be found in 24.11.)

98_8 Then $\rightarrow$ Prove that

100_6 Assume f, g are non-negative.

100_4 is bounded $\rightarrow$ is bounded and $g \geqslant 0$

101_7 $F(x) = xF(x) - \frac{1}{3}\{A(x)\}^2 \quad \rightarrow \quad F(x) = xA(x) - \frac{1}{3}\{A(x)\}^2$

101_1 $\displaystyle\int_0^x f(t)\,\mathrm{d}t = 1 - \sum_{q_n \leqslant x} q_n 2^{-n} \quad \rightarrow \quad \int_0^x f(t)\,\mathrm{d}t = xf(x) - \sum_{q_n \leqslant x} q_n 2^{-n}$

108^{19} Delete i before Im —?? but line 18 improved

113^2 16.5 $\rightarrow$ 16.9

114_{17} $\lim_{X\to\infty} \int_0^X f(x)\,\mathrm{d}x \quad \rightarrow \quad \lim_{X\to\infty} \int_{\pi/2}^X f(x)\,\mathrm{d}x$

115^{1-8} Should read

$$\begin{aligned} I_k &:= \int_{\sqrt{2/(2k+1)\pi}}^{\sqrt{2/(2k-1)\pi}} \left| x\cos(x^{-2})\right| \mathrm{d}x \\ &= \frac{(-1)^k}{2} \int_{\frac{(2k-1)\pi}{2}}^{\frac{(2k+1)\pi}{2}} \frac{\cos u}{u}\,\mathrm{d}u \qquad \text{(by 6.6, putting } u = x^{-2}\text{)} \\ &= \frac{1}{2}\int_0^{\pi} \frac{\sin y}{y + (2k-1)\pi/2}\,\mathrm{d}y \qquad \text{(by 6.6, putting } y = u - \frac{(2k-1)\pi}{2}\text{)} \\ &\geqslant \frac{2}{(2k+1)\pi}. \end{aligned}$$

Hence $\sum I_k$ diverges by comparison with $\sum 1/k$. ...

119_{13} $\varphi_n \rightarrow \psi_n$

124_9 where f is integrable $\rightarrow$ where f is integrable and $\lim \int f_n = \int f$

133_{6-5} Replace by: Integration by parts cannot be used to **prove** integrability; see Exercise 16.9. It is sometimes possible to use a substitution to prove that a given function is integrable, but we warn that it is very easy to get the logic wrong in arguments of this sort.

135^1 $x^{-r}e^{-x} \quad\to\quad x^r e^{-x}$

135_2 $?\,(n) = n! \quad\to\quad ?\,(n) = (n-1)!$

139^5 $\frac{1}{3\sqrt{3}} \quad\to\quad \frac{\pi}{3\sqrt{3}}$

141_6 Should read '... the right-hand side = 0.'

142^{14} $(-1 \leqslant t \leqslant 1) \quad\to\quad (-1 < x \leqslant 1)$

142_1 $\frac{\sqrt{\pi}}{2} \quad\to\quad \frac{\sqrt{\pi}}{4}$

147_2 is integrable $\quad\to\quad$ is not integrable

154_9 $-\frac{\pi}{2t^{3/2}} \quad\to\quad -\frac{\pi}{4t^{3/2}}$

156_1 $-\mathrm{i}xf(x) \quad\to\quad -\mathrm{i}xf(x)e^{-\mathrm{i}yx}$

157_{11-8} Line 157_8 is wrong. Replace by:

We can prove these assertions by the Continuous DCT. Define

$$f_t(x) = \frac{e^{-(1+x^2)t^2}}{2(1+x^2)} \quad (t > 0).$$

Then each f_t is integrable and we have the integrable dominating function $G(x) := 1/(2(1+x^2))$. Hence

$$\lim_{t\to\infty} \int_0^\infty f_t(x)\,\mathrm{d}x = \int_0^\infty \lim_{t\to\infty} f_t(x)\,\mathrm{d}x = 0 \quad \text{and}$$
$$\lim_{t\to 0+} \int_0^\infty f_t(x)\,\mathrm{d}x = \int_0^\infty \lim_{t\to 0+} f_t(x)\,\mathrm{d}x = \tfrac{\pi}{4}.$$

161 Fig. 21.1: $N \quad\to\quad k$

161 Delete the given proof of the 'if' direction (flawed) and note instead that this follows from 21.5(b) (this correction does not result in any arguments in 21.2-21.5 becoming circular).

162_{12-7} Replace by: We now prove (c). Define f_n by

$$f_n := \frac{nf}{n\,|f| + 1}.$$

Then f_n is measurable, by (a). Also, $f_n \to \operatorname{sgn} f$ as $n \to \infty$. Hence $\operatorname{sgn} f$ is measurable, by (b).

171_{20} $E_1, E_2, \ldots$ disjoint $\to m(\bigcap_{j\in J} E_j) = \prod_{j\in J} m(E_j)$ for every non-empty finite subset J of $\mathbb{N}$.

173^5 $(f \pm g)^2 \quad\to\quad (f \pm tg)^2 \ (t \in \mathbb{R})$

182_6 Add: and maps the null set C onto the non-null set $[0, 1]$

192^6 Delete $E^{[y]}$ (not needed)

194^2 $f^{\square n} \quad\to\quad |f|^{\square n}$

203 Figure 27.1: radius labels should be r_1, r_2

207^4 $(|g| \wedge n) \quad \to \quad ((g \wedge n) \vee (-n))$

207^{12} (see 27.2) $\quad \to \quad$ (see Fig. 27.2)

209^7 $(\alpha > 1) \quad \to \quad (\alpha > 2)$

209^8 Replace by: By using the Cauchy–Schwarz inequality, or otherwise, prove that, for $\beta > 1$ and $\alpha - \beta > 1$,

211_{18} L^1 as the elements of L $\quad \to \quad$ $\mathcal{L}^1$ as the elements of L^1

217^6 $\int f_n \quad \to \quad \left(\int f_n\right)^{1/2}$

224^2 Delete the initial factor of 2

229_3 In denominator, replace n by $2n - 1$

233^{10} formula should be $c_n = \frac{(-1)^n}{2\pi} \int_{-\infty}^{\infty} \mathrm{e}^{-|x| - \mathrm{i}nx}\,\mathrm{d}x$

233_8 $\beta_n = a_n/n \quad \to \quad n\beta_n = a_n - (-1)^n a_0$

243^2 2π on wrong side of equation

244_7 I. Kaplansky $\quad \to \quad$ A. Kolmogorov

245^8 $\cos n \quad \to \quad \cos nx$

245_7 $\sum_{m=-n+1}^{n-1} \left(1 - \frac{|m|}{n}\right) \mathrm{e}^{\mathrm{i}mx} \quad \to \quad \sum_{m=-n+1}^{n-1} d_m \left(1 - \frac{|m|}{n}\right) \mathrm{ei}mx$

$247_{8,7,3}$ Replace $u - v = 0$ by $u - v$

249^9 $\langle \psi_m, \psi_m \rangle \quad \to \quad \langle \psi_n, \psi_m \rangle$

250 Fig. 31.1: Add label 0

253^{15} $u_n(n) \quad \to \quad u_n(x)$

267_{10} These lie in $\mathcal{S}$ $\quad \to \quad$ (These lie in $\mathcal{S}$, the Schwartz space defined in 33.20.)

287^4 1904 $\to$ 1902

288^{12} $\varepsilon\, m(S_k) \quad \to \quad k\varepsilon\, m(S_k)$

287^4 19014 $\to$ 1902

289^{16} ‘Lebesgue’ $\quad \to \quad$ ‘Riemann’

289^{17} (and 289_1+) $\chi_{\mathbb{Q}} \cap [0,1] \quad \to \quad \chi_{\mathbb{Q} \cap [0,1]}$

291_5 Formula for $\sin A \pm \sin B$: $2\sin\left(\frac{1}{2}(A \pm B)\right)\sin\left(\frac{1}{2}(A \mp B)\right) \to 2\sin\left(\frac{1}{2}(A \pm B)\right)\cos\left(\frac{1}{2}(A \mp B)\right)$

294 Expansion for $(1 - x)^{-1}$ for $|x| > 1$: there is a missing minus sign

295 **[13]** replace ‘Revised edition 1990’ by ‘2nd edition 2003’

Fonts and other minor typos (punctuation, etc.)

12_9 $sin\, x$ $\to$ $\sin x$

29_4 Delete 2nd .

30_{15} An $\to$ As

37^{11} .] $\to$).]

38_5 amtiderivative $\to$ antiderivative

39^8 it $\to$ It

47_5 Missing)

57_1 f" $\to$ f''

61_1+? Replace $\frac{\mathrm{a+b}}{2}$ by $\frac{a+b}{2}$

79^7 Remove bracket after 'FTC'

$110-5$ Replace , by .

134^3 $\frac{p}{a}$ $\to$ $\frac{p}{a}$

148^{12} f(x,t) $\to$ $f(x,t)$

149_2 Replace (by ,

150_7 == $\to$ =

170_{13} exbibited $\to$ exhibited

170_1 Reword as: Deduce that not every null set, and so not every Lebesgue measurable set, is a Borel set.

186_7 Fubini's Theorem) $\to$ (Fubini's Theorem)

195^2 $[1,\infty]$ $\to$ $[1,\infty)$

197^9 This a $\to$ This is a

197^{11} Interchange dx and dt on left-hand side.

197_9 delete 'enough'

205_5 Delete first ,

222^1 asssumption $\to$ assumption

222^3 Missing)

234_9 Bernouilli $\to$ Bernoulli

236_2 hte $\to$ the

249^1 Missing)

251_7 $\langle w-v.w-v\rangle$ $\to$ $\langle w-v,w-v\rangle$

285_{18} confition $\to$ condition

287_9 it $\to$ It

287_5 motably $\to$ notably

289^{15} Replace , by .