

## a2: Complex Analysis and Geometry

### §1 THE COMPLEX PLANE: EXERCISES

I hope that students taking this course will already be familiar with the material covered in the following exercises.

1. Calculate the real and imaginary parts of the following complex numbers:

$$(i) \frac{2 + 3i}{1 + 2i}; \quad (ii) \frac{e^{i\lambda}}{\alpha + i\beta}, \quad \text{where } \alpha, \beta \text{ and } \lambda \text{ are real.}$$

2. Write down  $-i$  and  $-\sqrt{3} + i$  in polar form, and hence find a square root for each of them.

3. Write down a formula for each of  $\arg(-z)$ ,  $\arg \bar{z}$ ,  $\arg z^2$  and  $\arg(e^{i\lambda}z)$ , where the constant  $\lambda$  is real, in terms of  $\arg z \pmod{2\pi}$ . Describe or draw diagrams to show the images of

$$(i) \text{ the half-line } \{z : \arg z = \frac{1}{4}\pi\} \text{ and}$$

$$(ii) \text{ the set } \{z : -\frac{1}{4}\pi < \arg z < \frac{1}{4}\pi\}$$

under the mappings  $z \mapsto -z$ ,  $z \mapsto \bar{z}$ ,  $z \mapsto z^2$ , and  $z \mapsto e^{i\lambda}z$ .

4. Find all the complex solutions of

$$(i) z^6 + 1 = 0,$$

$$(ii) (z - 1)^6 = (z + 1)^6,$$

$$(iii) z^4 + z^3 + z^2 + z + 1 = 0,$$

$$(iv) z^6 + z^4 + z^2 + 1 = 0.$$

Additional exercises: Priestley, Ch 1, Ex 1-8. Supplementary exercises to Chapter 1, Ex 1-4.