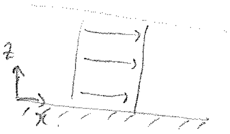
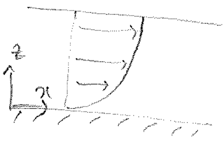
1. (i) $\underline{u} = \begin{pmatrix} U \\ 0 \\ 0 \end{pmatrix} \Rightarrow \boxed{\nabla \cdot \underline{u} = 0}$ $\quad \dfrac{\partial u_i}{\partial x_j} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \dot{\varepsilon}_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$



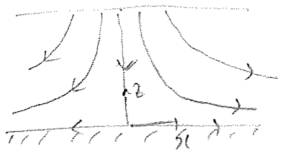
plug flow - found in rapidly sliding regions, eg. ice streams.

(ii) $\underline{u} = \lambda \begin{pmatrix} H^4 - (H-z)^4 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \boxed{\nabla \cdot \underline{u} = 0}$ $\quad \dfrac{\partial u_i}{\partial x_j} = \begin{pmatrix} 0 & 0 & 4\lambda (H-z)^3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \dot{\varepsilon}_{ij} = 2\lambda (H-z)^3 \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$



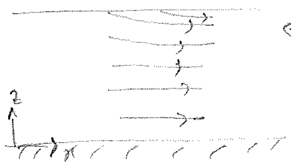
shear flow with no sliding (shallow ice approx.)
- found, eg. in a region with frozen bed.

(iii) $\underline{u} = \dfrac{a}{H} \begin{pmatrix} x/2 \\ y/2 \\ -z \end{pmatrix} \Rightarrow \boxed{\nabla \cdot \underline{u} = \tfrac{1}{2} + \tfrac{1}{2} - 1 = 0}$ $\quad \dfrac{\partial u_i}{\partial x_j} = \dfrac{a}{H} \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & -1 \end{pmatrix} = \dot{\varepsilon}_{ij}$



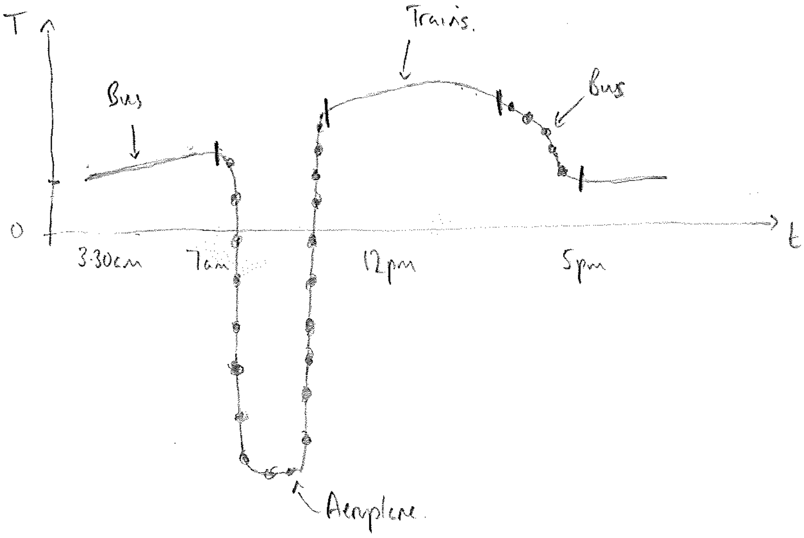
ice dome (centre of an ice sheet).

(iv) $\underline{u} = \begin{pmatrix} U \\ 0 \\ -\kappa e^{\frac{z-H}{d}} \end{pmatrix} \Rightarrow \boxed{\nabla \cdot \underline{u} = -\dfrac{\kappa}{d} e^{\frac{z-H}{d}}}$ $\quad \dfrac{\partial u_i}{\partial x_j} = -\dfrac{\kappa}{d} e^{\frac{z-H}{d}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \dot{\varepsilon}_{ij}$



compacting firn layer near surface

2. From U.K.



dotted periods dominated by advection $\underline{u} \cdot \nabla T$
other periods dominated by $\dfrac{\partial T}{\partial t}$.

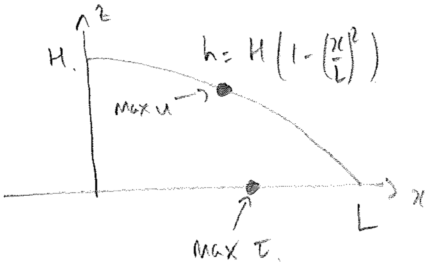
3. (i) $\boxed{p=0,\ \tau=0 \text{ at } z=h}$ (atmospheric pressure taken as zero)

$$0 = -\frac{\partial p}{\partial z} - \rho g \Rightarrow \boxed{p = \rho g(h-z)}$$

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial \tau}{\partial z} \Rightarrow \frac{\partial \tau}{\partial z} = -\rho g \frac{\partial h}{\partial x} \Rightarrow \boxed{\tau = \left(-\rho g \frac{\partial h}{\partial x}\right)(h-z)}$$

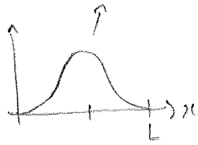
(ii) $$\frac{\partial u}{\partial z} = 2A\tau^n = 2A(\rho g)^n\left(-\frac{\partial h}{\partial x}\right)^n (h-z)^n \Rightarrow u = \frac{2A(\rho g)^n}{n+1}\left(-\frac{\partial h}{\partial x}\right)^n\left[h^{n+1}-(h-z)^{n+1}\right].$$

(iii)



Highest velocity for each x occurs at $z=h$.
where $u \propto \left(-\frac{\partial h}{\partial x}\right)^n h^{n+1} = \frac{2^n H^{2n+1}}{L^n}\left(\frac{x}{L}\right)^n\left(1-\left(\frac{x}{L}\right)^2\right)^{n+1}$

which has max where.

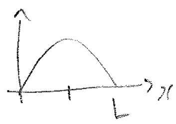


$$2(n+1)\left(\frac{x}{L}\right)^2 = n\left(1-\left(\frac{x}{L}\right)^2\right)$$

$$\Rightarrow \left(\frac{x}{L}\right)^2 = \frac{n}{3n+2} = \frac{3}{11} \text{ for } n=3.$$

Highest strain rate is also where highest shear stress occurs, which is at bed, where

$$\tau \propto \left(-\frac{\partial h}{\partial x}\right)h = \frac{2H^2}{L}\left(\frac{x}{L}\right)\left(1-\left(\frac{x}{L}\right)^2\right)$$



max is where $1 = 3\left(\frac{x}{L}\right)^2 \Rightarrow \left(\frac{x}{L}\right)^2 = \frac{1}{3}$

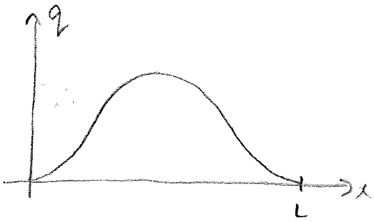
(iv) $\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0$ with $w=0$ at $z=0$, $w + \overset{\text{accumulation}}{a} = \frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x}$ at $z=h$.

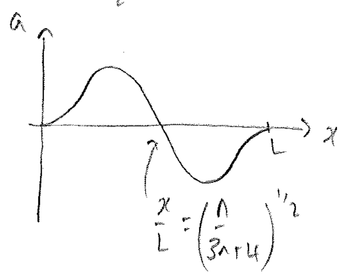
$\Rightarrow$ (integrating from 0 to h) $$\int_0^h \frac{\partial u}{\partial x}\,dz = -[w]_0^h = -\frac{\partial h}{\partial t} - \cancel{u\big|_h\frac{\partial h}{\partial x}} + a \Rightarrow \boxed{\frac{\partial h}{\partial t}+\frac{\partial q}{\partial x}=a}$$

$$= \frac{\partial}{\partial x}\underbrace{\int_0^h u\,dz}_{q} - \cancel{u\big|_h \frac{\partial h}{\partial x}}.$$

From above $$q = \int_0^h u\,dz = \frac{2A(\rho g)^n}{n+2}\left(-\frac{\partial h}{\partial x}\right)^n h^{n+2} = \frac{2A(\rho g)^n 2^n H^{2n+2}}{(n+2)\,L^n}\left(\frac{x}{L}\right)^n\left(1-\left(\frac{x}{L}\right)^2\right)^{n+2}$$

$$\Rightarrow a = \frac{\partial q}{\partial x} = \frac{2A(\rho g)^n 2^n H^{2n+2}}{(n+2)L^{n+1}}\left(1-\left(\frac{x}{L}\right)^2\right)^{n+1}\left(\frac{x}{L}\right)^{n-1}\left[n\left(1-\left(\frac{x}{L}\right)^2\right) - 2(n+2)\left(\frac{x}{L}\right)^2\right] = 0 \text{ at } \left(\frac{x}{L}\right)^2 = \frac{n}{3n+4} = \frac{3}{13}$$





Height at ELA

$$H\left(1-\frac{n}{3n+4}\right) = \frac{2n+4}{3n+4}H.$$