Modelling the spread of coronavirus

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Governments around the world are taking quite different approaches to dealing with the coronavirus pandemic. Studying a very simple epidemiological model can help understand what lies behind these decisions. There is no agenda behind this - I am not an expert and I don't know what the best approach is - but it helps to understand the rationale behind different approaches.

We know that the virus has spread widely. Data from many countries shows approximately exponential growth in the number of confirmed cases that is indicative of an epidemic. The true number of infected people is already higher than this - there is a delay (probably well over a week) between people catching the disease and it being confirmed, so our data is always lagging the 'true' number of cases.

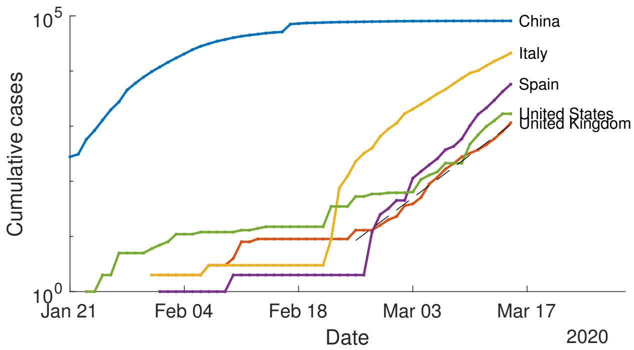

Figure 1: Confirmed cases of coronavirus for select countries (data from the WHO via https://ourworldindata.org/coronavirus-source-data). For the last two weeks, the UK cases have been doubling around every 2.8 days.

Mathematical model

The simple 'SIR' model assumes that at any time t a fraction of the population $S(t)$ is susceptible to catching the virus, while a fraction $I(t)$ are infected (and therefore infectious). These fractions evolve through time as people catch the virus, then recover and become immune (we hope - it is not yet known what degree of immunity is achieved by those who recover, but similar viruses seem to work like this).

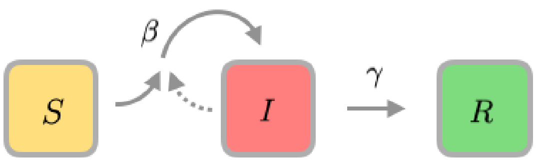

Figure 2: Schematic of the model for the 'susceptible', 'infectious', and 'removed' fractions of the population. The rate β represents how effectively the virus is transmitted, and the rate γ represents how rapidly infectious people recover.

The evolution is described by the differential equations

$$\frac{\mathrm{d}S}{\mathrm{d}t} = -\beta SI, \tag{1}$$

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \beta SI - \gamma I, \tag{2}$$

where the two parameters β and γ represent the transmission rate and the recovery rate, respectively. These encode both properties of the virus (how easily it is passed on) and people's behaviour (how often they wash their hands and how much they travel, for example). The parameters have typical values β_0 and γ_0 that represent 'normal' behaviour, but may change over time to reflect changes in behaviour.

Since this is a new strain of virus it must be presumed that we are all currently susceptible, so at time $t = 0$ (which we take to be now, 14 March), the susceptible fraction is $S_0 \approx 1$. The infected fraction I_0 can be estimated from the number of cases so far. In the UK there are 798 diagnosed cases amongst a population of around 66 million, which gives this fraction as around 1 in $100,000$. But it is likely that the true infected fraction is already much higher, so 1 in $10,000$ may be more realistic.

The models used by the WHO are a bit more complicated than this one - accounting explicitly for things like the incubation period and for different modes of transmission - but the basic behaviour is similar. This model has the advantage of simplicity, making it easy to illustrate the effect of different mitigation strategies.

Exponential growth and the effective reproductive number

Early in the epidemic, when few people have had the virus, the susceptible fraction S remains largely unaltered at its initial value S_0. Equation (2) then has solution

$$I(t) = I_0 \exp\left[(\beta S_0 - \gamma)t\right], \tag{3}$$

which shows that the infected population grows exponentially if the 'effective reproductive number',

$$R = \frac{\beta S_0}{\gamma}, \tag{4}$$

is greater than 1. This seems to be the case for coronavirus; its initial value, the 'basic reproductive number' $R_0 = \beta_0/\gamma_0$, has been estimated to be around 2.5.

As anyone who has had to pay interest on a debt will know, exponential growth compounds rapidly. If nothing were done and the normal values of β and γ were maintained, we could expect the current exponential growth to continue for some weeks, before eventually subsiding once most of us have already caught the virus and are no longer susceptible (figure 3). The peak would be in around six to seven weeks, and we could expect that more than 20% of the population may have the infection at the same time. This would clearly place unmanageable demands on the health service.

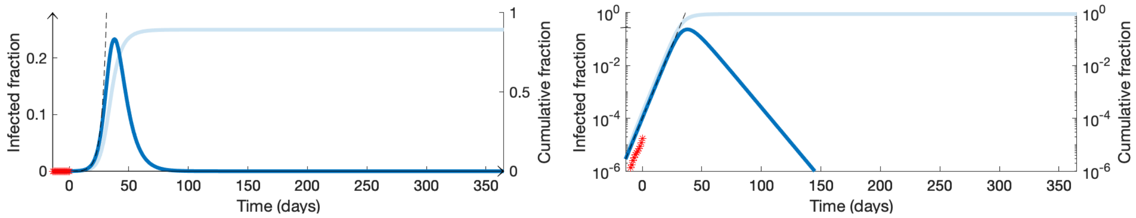

Figure 3: Modelled infected fraction of the population $I(t)$ for constant parameters $\beta_0 = 0.4228$ d^{-1}, $\gamma_0 = 0.169$ d^{-1} ($R_0 = 2.5$), and $I_0 = 10^{-4}$ at $t = 0$, shown on linear axis (left) and logarithmic axes (right; it is impossible to see what is going at the start on the linear axes). Fainter lines show the cumulative fraction of the population that has been infected. Red stars show the UK confirmed cases over the last couple of weeks, growing at the same exponential rate.

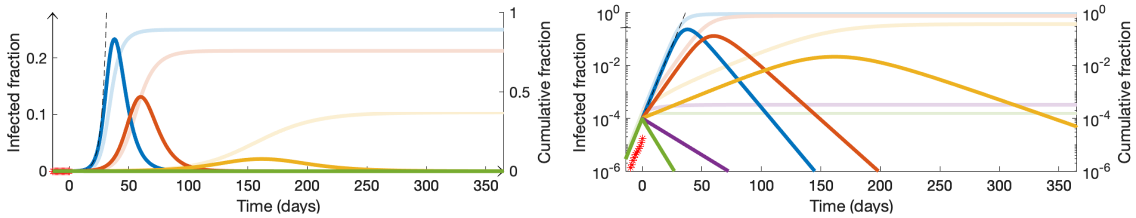

Figure 4: The effect of reducing the transmission rate β to a fraction of β_0 (1 in blue, 0.75 red, 0.5 yellow, 0.25 purple, 0 green). Reducing the transmission rate delays and broadens the peak. If $\beta < \gamma$, the infected fraction decays and the vast majority of the population remain susceptible.

Containment, Delay, and Mitigation

It is possible to reduce the spread of the infection by decreasing the transmission rate β (and therefore the effective reproductive number R). If the change is sufficiently drastic to make R less than 1, the exponential growth of the infections can be turned to exponential decay (figure 4). The more we reduce β, the lower the peak number of infections.

Decreasing the transmission rate might be achieved by a range of measures, and it seems likely that simply raising awareness will have already reduced it slightly from its original value. But we don't know by how much yet. A challenge in the current situation is knowing how much any particular action (closing schools, for example) will contribute quantitatively to a change in transmission rate, and it would appear that there is not universal agreement on this. For example, more strict hand-washing routines may already reduce it by 10%, self-isolation for those with a cough may reduce it by 20%, stopping mass gatherings by 1%, etc. (These numbers are made up for illustration - it would be interesting to know what the government actually thinks they are, and there is presumably some research on it). There is also the issue of how sustainable such actions might be over the long term, and the need to weigh up slowing the spread of the virus with maintaining some form of society and economy. Countries have taken differing approaches to this, which may also reflect cultural differences.

But the more fundamental question is what our long-term goal should be in tackling this virus. When the number of cases was small enough, the goal was to isolate infected individuals and their known contacts. The idea was that if the infected population could be kept small enough, it could be carefully managed until it is either eliminated altogether, or until a vaccine is developed. This seems to have become a less appealing strategy for two reasons. Firstly, there are now so many cases in the UK that it is not clear we could actually reduce our infected population to a manageable level (it would certainly take a long time with stringent measures to do so). Secondly, given the worldwide spread of the virus, we would then be living quite precariously, since further imports of the virus from other countries seem likely. However, it is possible this is still what we should be aiming for. Other countries like China and South Korea seem to have done this.

An alternative goal, which the UK government seems to be pursuing at the moment, is that we aim for 'herd immunity'. This occurs if we can reduce the susceptible fraction of the population to the extent that the effective reproductive number $R = \beta S_0/\gamma$ is less than 1, even with 'normal' values of β and γ (i.e. once travel and social restrictions are lifted and we go back to not washing our hands so regularly). Given the estimated value of R_0, this would likely require around 60% of the population to become immune - presumably through having caught the virus. We would automatically reach that state later this year if we did nothing (see figure 3, where more than 80% of the population have caught the virus by the end of the year), but it would happen extremely rapidly at its peak. If we take this approach, it is therefore desirable to flatten and delay the peak. This is achieved by reducing the transmission rate, but not by too much (since then not many of us would catch it).

This strategy also seems quite risky, not least because it involves an acceptance that the majority of people would catch the virus, so it would undoubtedly result in a considerable death toll. (The mortality rate is highest amongst the elderly and those with pre-existing medical conditions, so it would obviously be desirable if the 60% who need to catch the virus are, as much as possible, not those people).

It is currently not clear to me which of these goals is the best one to aim for. If we pursue the first strategy, we ought to go into complete 'lockdown' immediately, as other countries have done, in order to hit a peak number of infections as soon as possible and limit the number who are exposed to the virus. It is likely the lockdown would be needed for at least several weeks, and what happens after that is unclear, since there is a danger the epidemic could simply start again when the measures are eased (figure 5). But this may be overly pessimistic - it is possible that renewed awareness and attitudes to cleanliness, sick leave, etc. could be enough to keep the effective reproductive number R less than 1 and prevent an epidemic starting again.

If we are to pursue the second strategy, the question arises what is the best way to delay and flatten the peak. Again, there is a need to weigh up the positive aspects of any measures taken to slow the spread of the disease with the negative aspects in terms of disruption to practical living. If we assume that more strict social distancing would be impractical for more than a few weeks, then it is indeed the case that delaying those until later could help to reduce the peak number of cases (figure

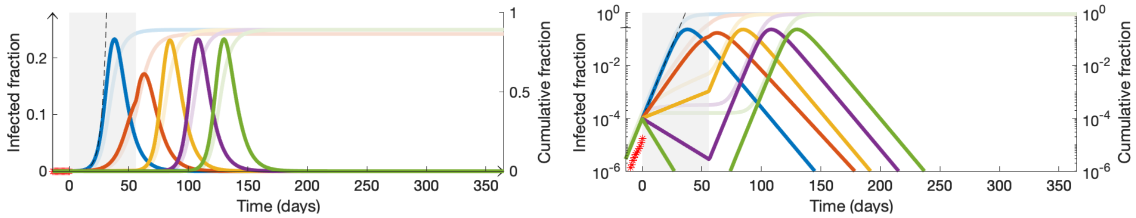

Figure 5: The effect of reducing the transmission rate β to a fraction of β_0 and holding it there for 8 weeks. The epidemic re-starts once the transmission rate returns to its normal value, and the peak number of cases is almost identical in each case.

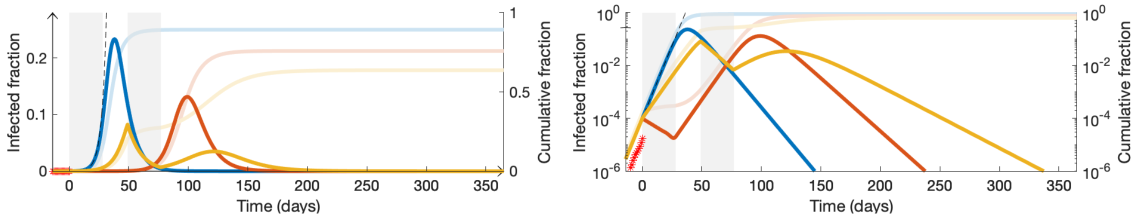

Figure 6: The effect of a permanent reduction in the transmission rate β, together with a further reduction for 4 weeks, either now (red) or in 7 weeks' time (yellow). The peak number of infections is lower when the tighter control is imposed later.

6). But that is assuming that we *need* lots of us to catch the virus, and this seems quite a bold thing to aim for when we know so little about it.

Conclusion

So what is the right course of action in terms of cancelling meetings, reducing travel, etc.? The answer to this question seems to depend on what the long-term goal is, and on how we think society will cope with different measures.

If we take the view that we do not want (or need) most of us to catch the virus, we should take the strongest measures now, in order to start reducing the numbers of infections as soon as possible. The only reason to delay is if we are aiming for a state of herd immunity and therefore want more people to catch the virus.

If we take the view that the majority of us need to catch the virus at some stage, then we should change our behaviour somewhat (in order to reduce the transmission rate and flatten the peak), but it is not necessary to limit things too much just yet.

If we are not sure - and it seems to me that it's not clear that we *need* to have lots of us catch the virus - we probably ought to be playing it safe and trying to reduce the number of infections as quickly as possible.

This is all weighed against the social, economic, and personal costs of isolating ourselves from each other. These are likely to be huge, and as yet untested.