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Modeling Pressure Extremes in Subglacial Drainage Systems

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I Introduction

An important factor controlling glacier sliding is the water pressure in the subglacial drainage system. Measurements from boreholes show that this often fluctuates in space and time, sometimes reaching the ice overburden pressure and sometimes falling to atmospheric pressure. Most current models of the drainage system implicitly assume that these extremes are not reached. We suggest a way to allow for partially filled drainage space at atmospheric pressure ('free surface flow') and for widespread uplift of the ice at overburden pressure ('flotation').

Water flows down the gradient of the hydraulic potential:

$$\phi = \rho_w g b + p_w$$

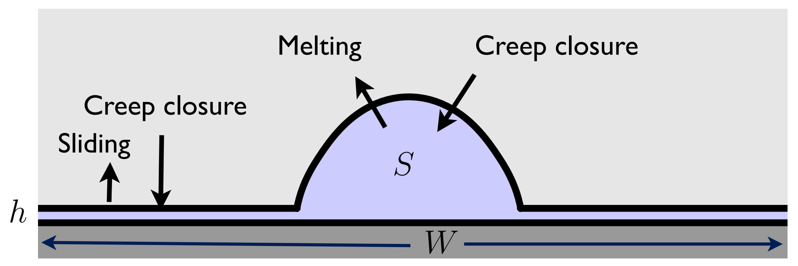
Atmospheric and overburden pressures correspond respectively to:

$$\phi_m = \rho_w g b \quad \text{and} \quad \phi_0 = \rho_w g b + \rho_i g H.$$

We assume that the potential does not pass above ϕ_0 and does not reduce below ϕ_m. Instead, modifications are made to the model when these bounds are reached.

II Model

The model distinguishes flow through channels, with cross-sectional area S, and flow through linked-cavities, described on the macro-scale as a 'porous sheet' with average depth h. This poster shows a one-dimensional flow-line model consisting of a single channel in parallel with a cavity sheet spanning the width of a model glacier. The same model can be extended to two-dimensions using a network of channels.



Variables:

- h Sheet depth
- h_w Water-filled sheet depth
- S Channel area
- S_w Water-filled channel area
- ϕ Hydraulic potential

Channel cross-section and cavity size evolve in space and time according to the processes shown in the diagram, parameterized by:

Open / closure

$$\frac{\partial S}{\partial t} = \frac{\Xi}{\rho_i L} - AS|N|^{n-1}N$$

(Melting; Creep closure)

$$\frac{\partial h}{\partial t} = \frac{u_b h_r}{l_r}\left(1 - \frac{h}{h_r}\right) - Ah|N|^{n-1}N$$

(Sliding)

$N = \phi_0 - \phi$ Effective pressure
L Latent heat
A, n Ice flow law constants
u_b Sliding speed
h_r Size of bedrock bumps
l_r Spacing of bedrock bumps

The water-filled areas may be different from S and h, so are written as S_w and h_w, satisfying conservation equations:

Mass conservation

$$\frac{\partial S_w}{\partial t} + \frac{\partial Q}{\partial x} = \frac{\Xi}{\rho_w L} + \kappa$$

(Melting; Water exchange)

$$W\frac{\partial h_w}{\partial t} + W\frac{\partial q}{\partial x} = Wm - \kappa$$

(Prescribed source)

W Glacier width

Discharge in channel and sheet are parameterized using a turbulent flow law (e. g. Darcy-Weisbach):

Water flow

$$Q = -k_c S_w^{\alpha}\left|\frac{\partial \phi}{\partial x}\right|^{\beta-2}\frac{\partial \phi}{\partial x}$$

$$q = -k h_w^{\alpha}\left|\frac{\partial \phi}{\partial x}\right|^{\beta-2}\frac{\partial \phi}{\partial x}$$

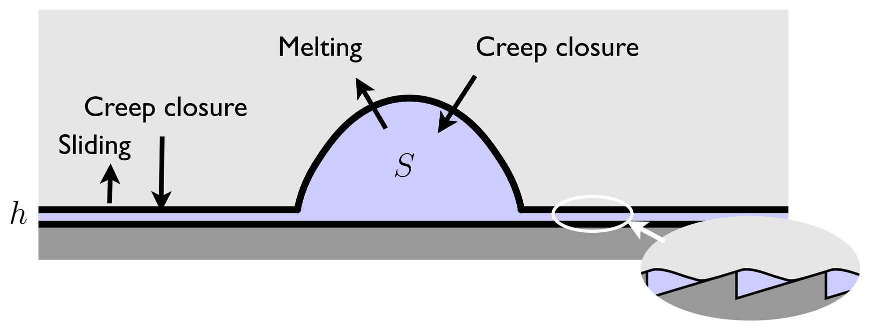
k_c, k Friction parameters
$\alpha = \frac{5}{4}$, $\beta = \frac{3}{2}$

Dissipation in the channel (heating due to water flow):

Dissipation

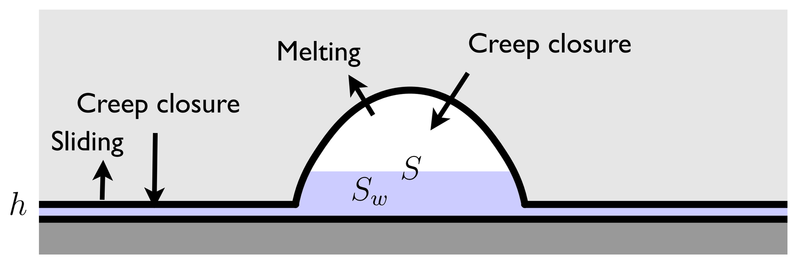
$$\Xi = k_c S_w^{\alpha}\left|\frac{\partial \phi}{\partial x}\right|^{\beta} + l_r k h_w^{\alpha}\left|\frac{\partial \phi}{\partial x}\right|^{\beta}$$

III Over- and Under-pressure



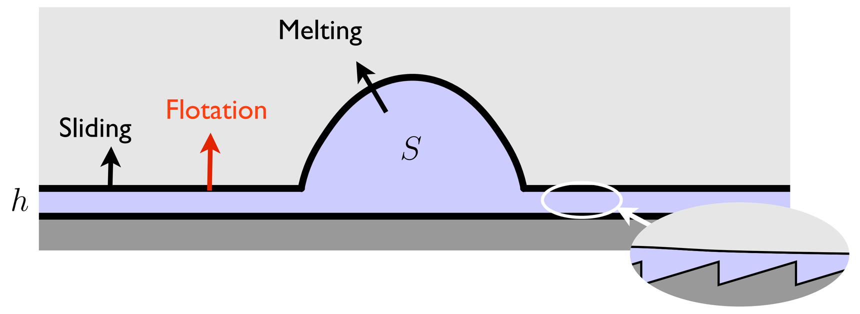
Normal pressure

If $\phi_m < \phi < \phi_0$, the drainage system is full, with $S_w = S$ and $h_w = h$.



Underpressure

At atmospheric pressure, $\phi = \phi_m$, channel and sheet may be partially full with $S_w \leq S$ and $h_w \leq h$.



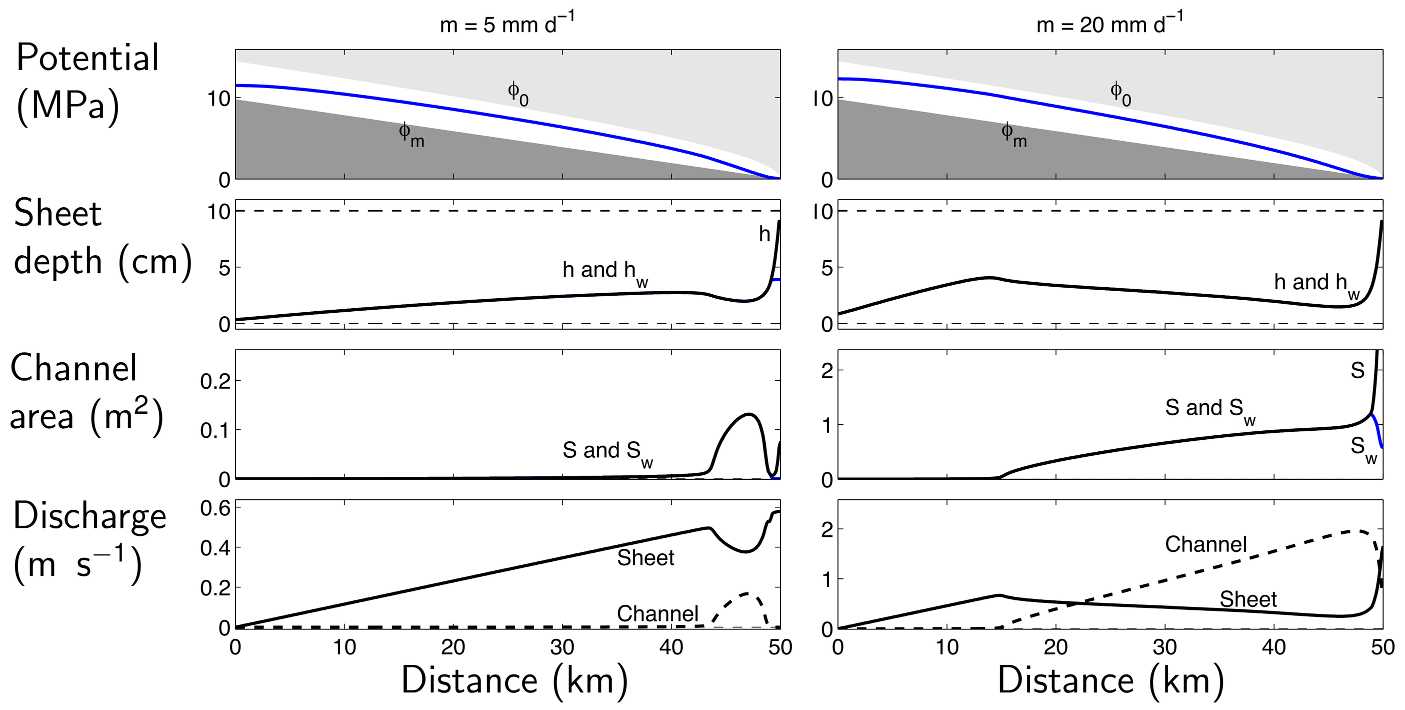
Overpressure

At overburden pressure, $\phi = \phi_0$, ice can separate from bed and we assume uplift will occur to whatever extent is required to accommodate the water at overburden pressure. The sheet evolution equation is modified to

$$\frac{\partial h}{\partial t} \geq \frac{u_b h_r}{l_r}\left(1 - \frac{h}{h_r}\right) - Ah|N|^{n-1}N$$

IV Steady-state solutions

Low discharge / High discharge

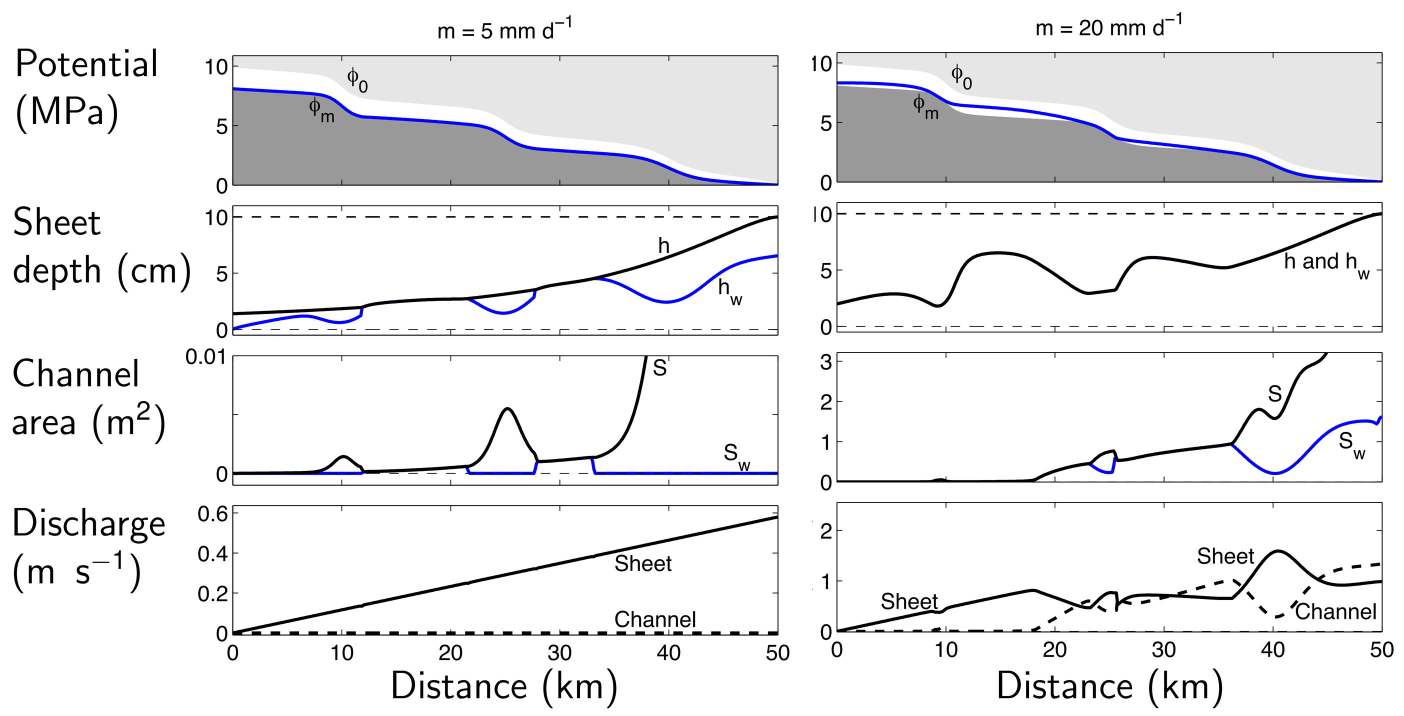


Example 1

Constant spatially-uniform input of meltwater.

For larger meltwater input, channel starts further up glacier.

Channel is only partially filled near margin.



Example 2

A wavy bed and ice surface.

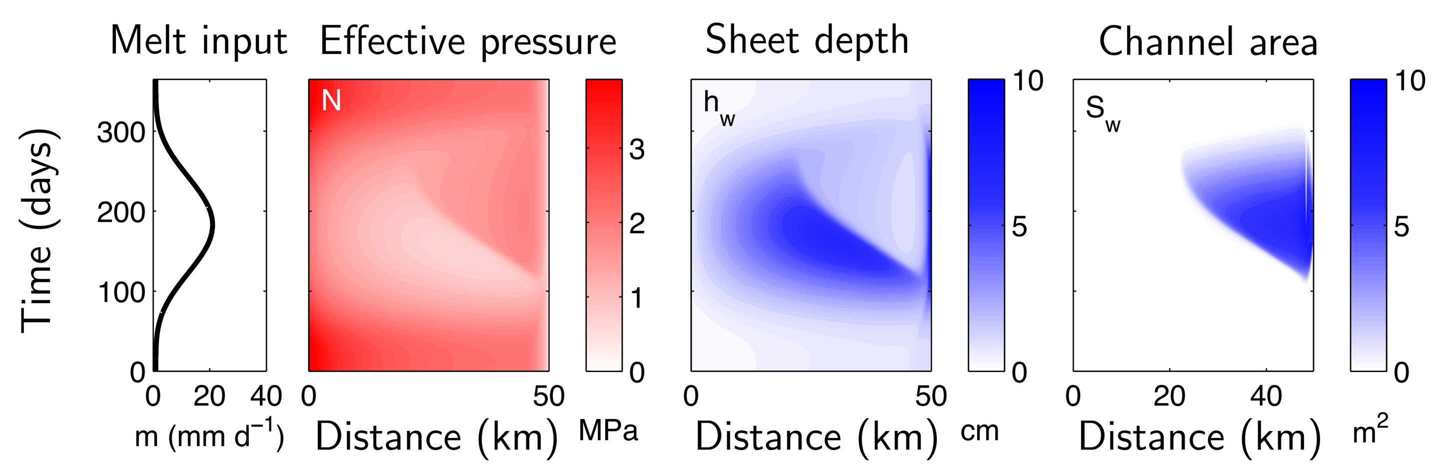
Channel is larger beneath steeper ice.

Drainage space is often only partially filled beneath steep or shallow ice.

V Variable input solutions

Idealized spatially-uniform input to glacier in Example 1.

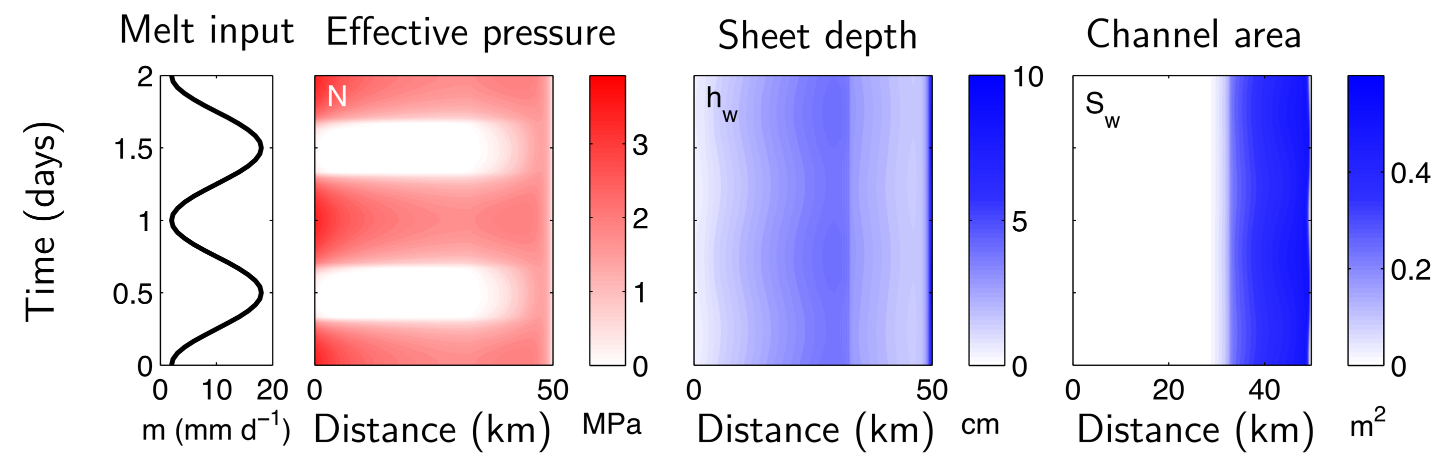
Annual cycle

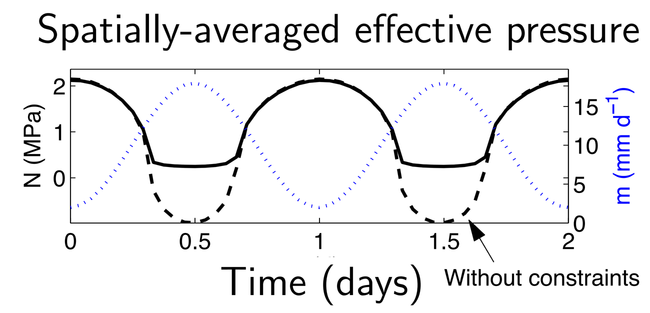


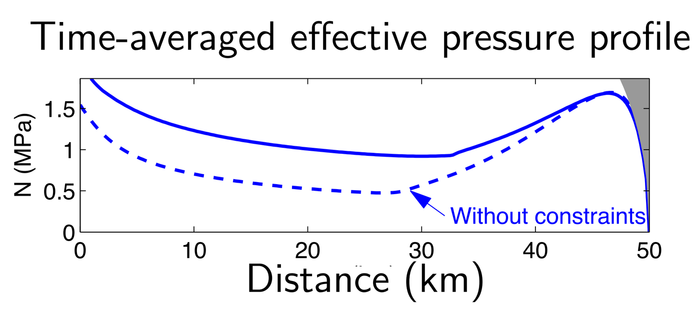
Water pressure increases (so effective pressure decreases, sliding increases) during spring.

Channels grow up glacier throughout course of summer, with resulting reduction in water pressure.

Diurnal cycle







Diurnal frequency too fast to allow significant adjustment of sheet and channel area, but causes large pressure variations.

Large input during middle of day causes water pressure to reach overburden.

Predicted average water pressure lower than from same model without constraint to overburden.

VI Summary

Varying meltwater input may be responsible for significant changes in ice speed; models of the drainage system need to account for extreme subglacial water pressures.

Bounding water pressure between atmospheric and overburden provides a method to allow for partially filled drainage space and widespread uplift of ice.

Partially filled channels at atmospheric pressure are common beneath steep or shallow ice, and at low discharge.

Pressure reaches overburden during short lived periods of high meltwater input.

Methods

The numerical procedure evolves S, S_w, h and h_w explicitly in time. At each timestep, ϕ is calculated from the elliptic problem that is obtained by combining all of the equations over the saturated regions where $S_w = S$ and $h_w = h$ (elsewhere, at atmospheric pressure, $\phi = \phi_m$ is known). The bounds ϕ_m and ϕ_0 provide constraints on the solution for ϕ; a variational approach is used to account for these.

For more details, see the following papers to be published in *Journal of Fluid Mechanics*:

Schoof, C., Hewitt, I. J. and Werder, M. A. Flotation and free surface flow in a model for subglacial drainage. Part I: Linked cavities.

Hewitt, I. J., Schoof, C. and Werder, M. A. Flotation and free surface flow in a model for subglacial drainage. Part II: Channel flow.