Modelling melting rates in upwelling mantle

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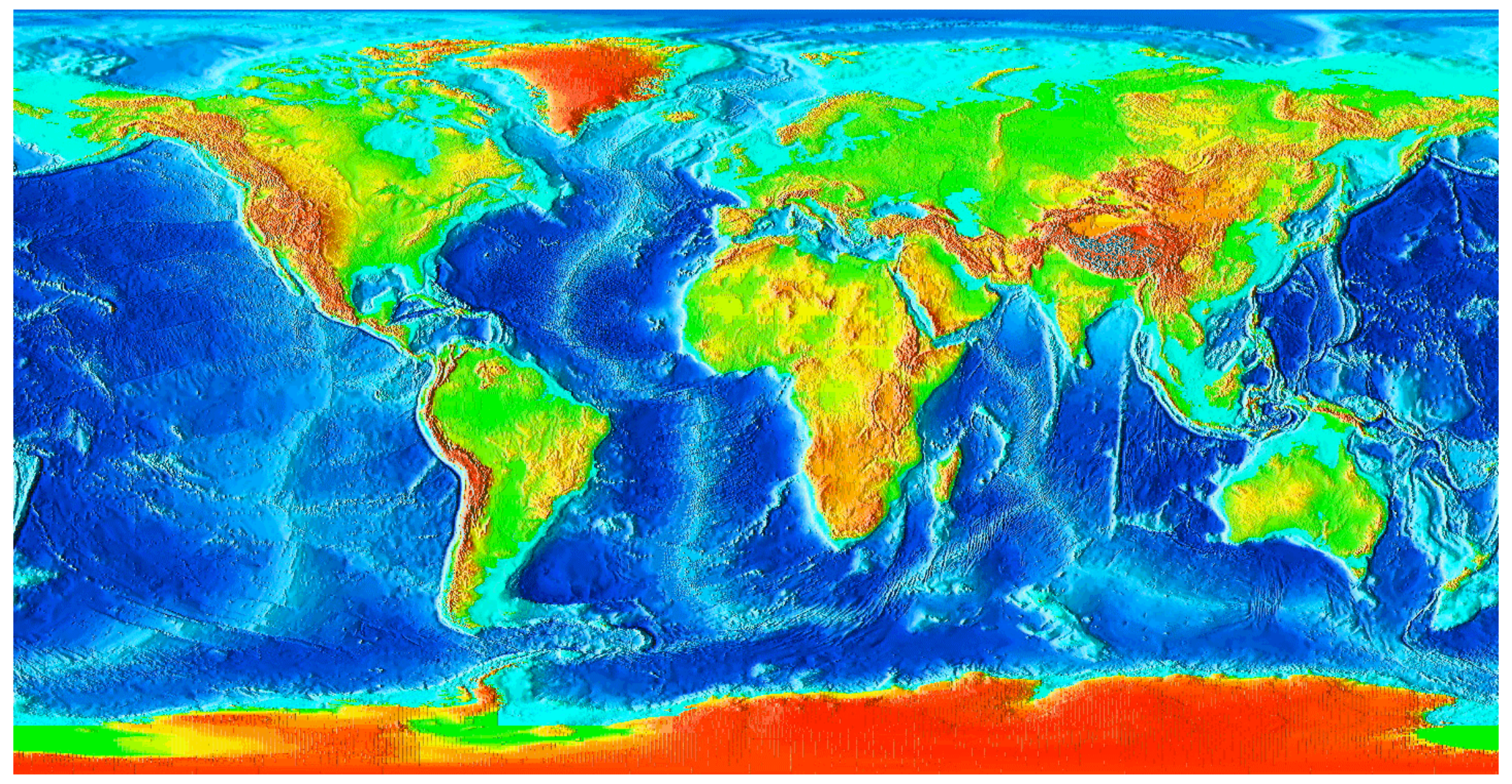
1. Introduction

Upwelling mantle rock beneath mid-ocean ridges undergoes partial melting. The magma that is produced is buoyant and rises through the residual porous matrix towards the surface. Models of this process often ignore melting or assume a prescribed melting rate.

This work aims to determine the melting rate consistently from simple physical models. Under assumptions of thermodynamic equilibrium and using simple illustrative phase constraints, melting is found to be proportional to the average upwelling velocity of matrix and melt.

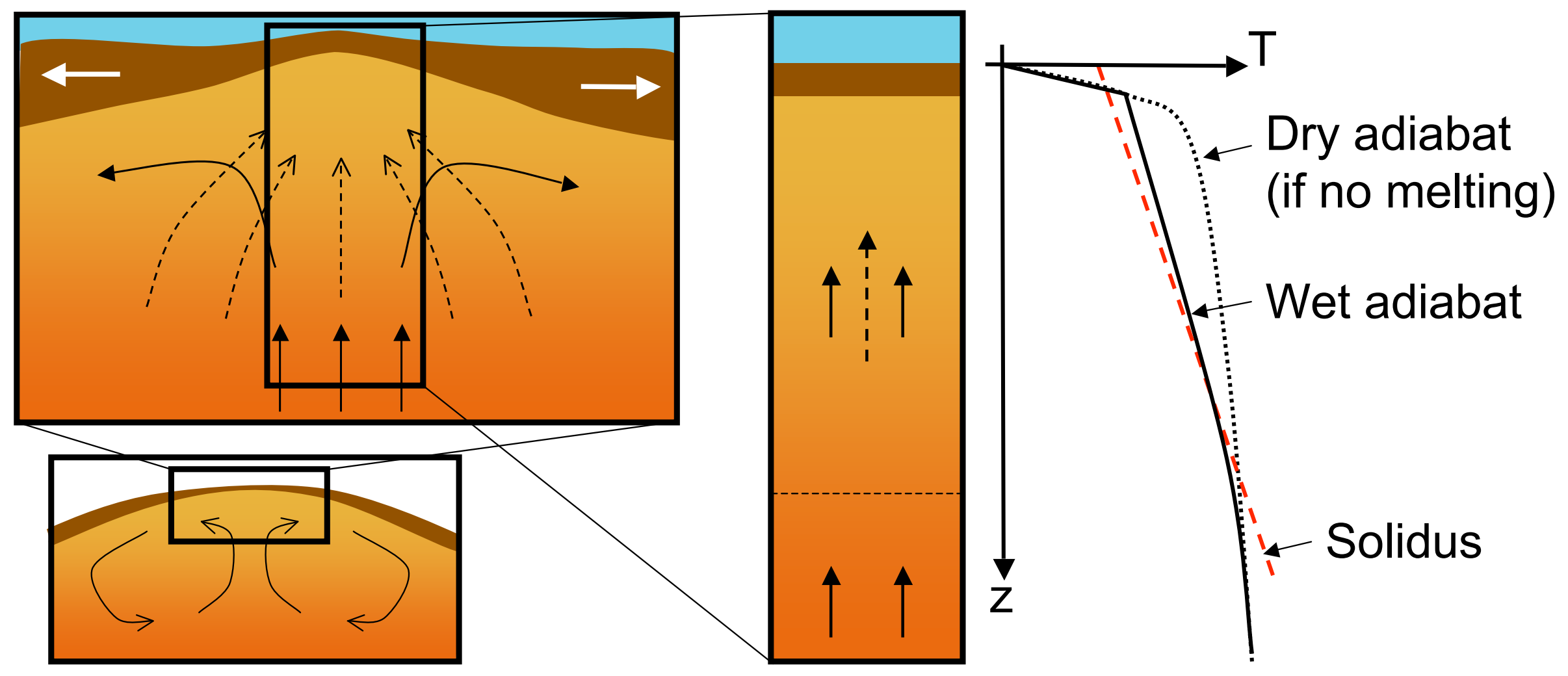
Feedbacks on the melting or dissolution rate have previously been suggested to cause a reactive instability that can result in localization of melt into high porosity channels. The new calculation of melting rate made here suggests this is indeed a possibility, although uniform porous flow may be linearly stable.



Topography of the Earth's crust, showing mid-ocean ridges in light blue.

2. Why melting?

Melting occurs because of the pressure dependence of the solidus - as the rock decompresses adiabatically its temperature intersects the solidus and it must partially melt. Since some components of the rock are more fusible than others, melting alters the composition and therefore the solidus.



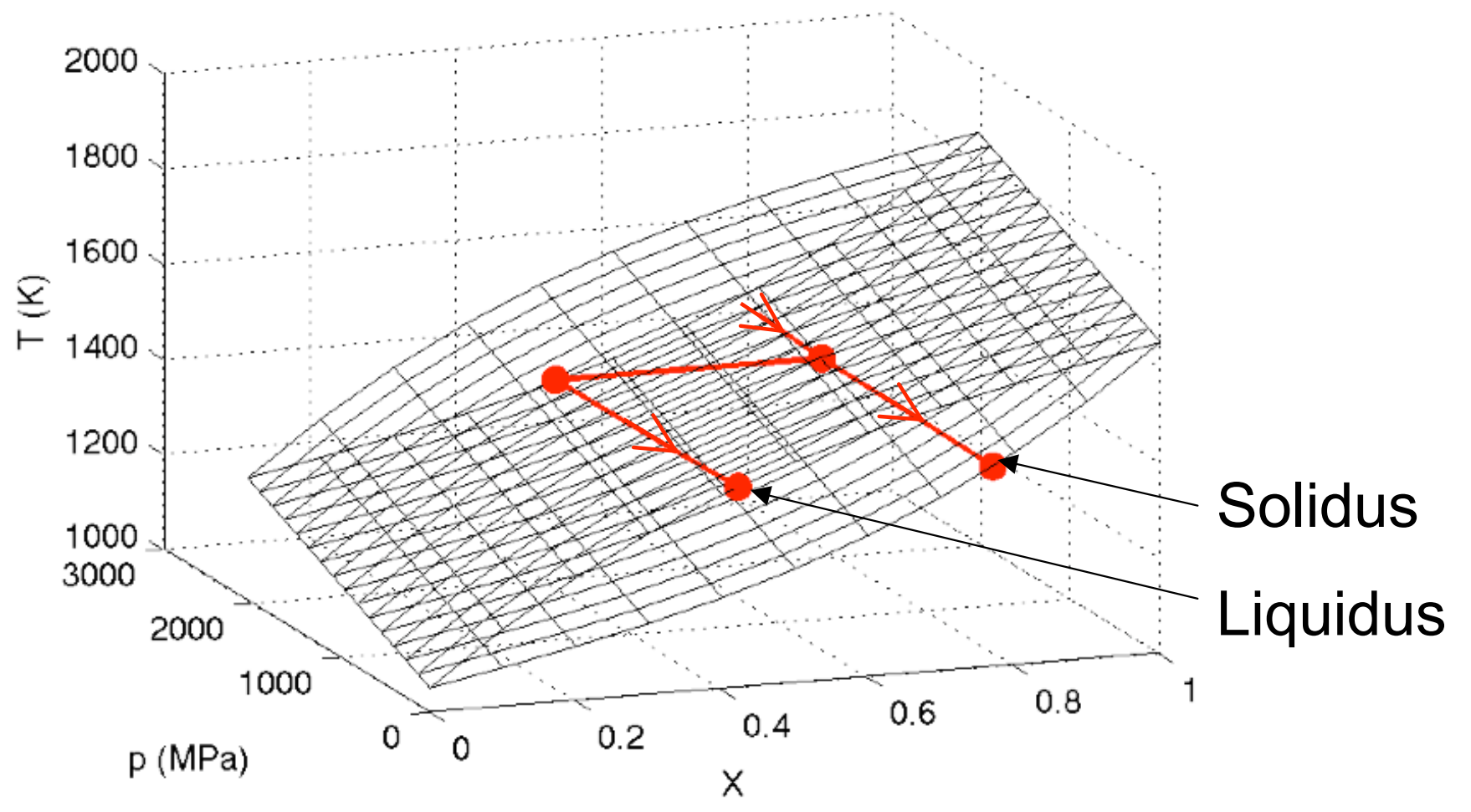
An upwelling columnn beneath a mid ocean ridge. Solid arrows show solid (matrix) motion, dashed arrows show melt motion.

3. Thermodynamic equilibrium

Local thermodynamic equilibrium is assumed to be maintained. For a rock comprising two chemical components, A and B, this requires two phase constraints between temperature T, pressure p, and composition X:

Solidus $$T = T_{S0} + \gamma p + \lambda f_S(X_s) \quad (1)$$

Liquidus $$T = T_{L0} + \gamma p + \lambda f_L(X_l) \quad (2)$$



4. Model

The model requires conservation of momentum (4,5), total mass (6,7), individual components (8,9) and energy (10). Compaction of the matrix is related to the effective pressure (3). A Boussinesq approximation is used.

T	Temperature
X_s	Concentration of A in solid
X_l	Concentration of A in melt
ϕ	Melt fraction
$\mathbf{u}$	Melt velocity
$\mathbf{V}$	Matrix velocity
N	Effective pressure

$$N \equiv p_s - p_l = -\frac{\eta_s}{\phi}\nabla\cdot\mathbf{V} \quad (3)$$

$$\phi(\mathbf{u}-\mathbf{V}) = \frac{k_0\phi^n}{\eta_l}(\Delta\rho g\mathbf{k} + \nabla N) \quad (4)$$

$$0 = -\nabla p_s - \rho g\mathbf{k} \quad (5)$$

$$\frac{\partial\phi}{\partial t} + \rho\nabla\cdot[\phi\mathbf{u}] = \frac{m}{\rho} \quad (6)$$

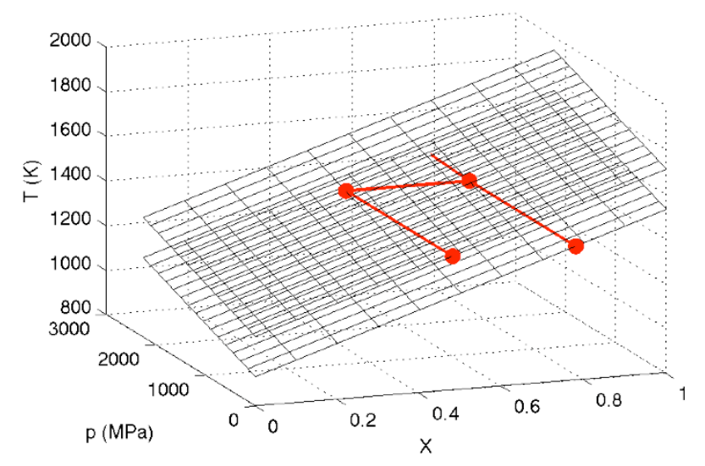
$$\nabla\cdot[\mathbf{V} + \phi(\mathbf{u}-\mathbf{V})] = 0 \quad (7)$$

$$\frac{\partial}{\partial t}[\phi X_l] + \nabla\cdot[\phi\mathbf{u}X_l] = \frac{m^1}{\rho} \quad (8)$$

$$\frac{\partial}{\partial t}[X_s - \phi(X_s - X_l)] + \nabla\cdot[(1-\phi)\mathbf{V}X_s + \phi\mathbf{u}X_l] = 0 \quad (9)$$

$$mL + \rho c\frac{\partial T}{\partial t} + \rho[\mathbf{V} + \phi(\mathbf{u}-\mathbf{V})]\cdot[c\nabla T + \alpha gT\mathbf{k}] = 0 \quad (10)$$

Imposing phase constraints (1,2) determines the overall mass transfer rate (m) and the mass transfer rate of each component.

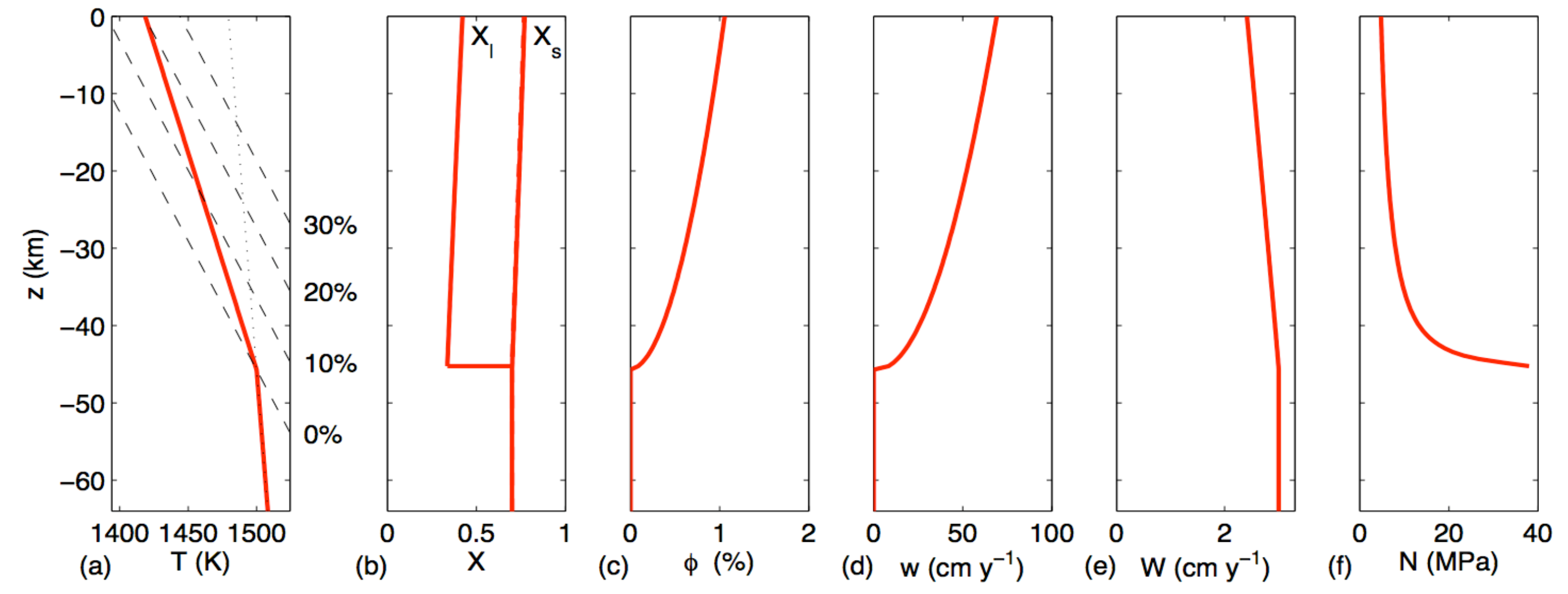


eg. for a linear solidus and liquidus

$$T = T_{S0} + \gamma p + \lambda X_s \quad (11)$$

$$X_s = X_l + \Delta X \quad (12)$$

5. One-dimensional steady state



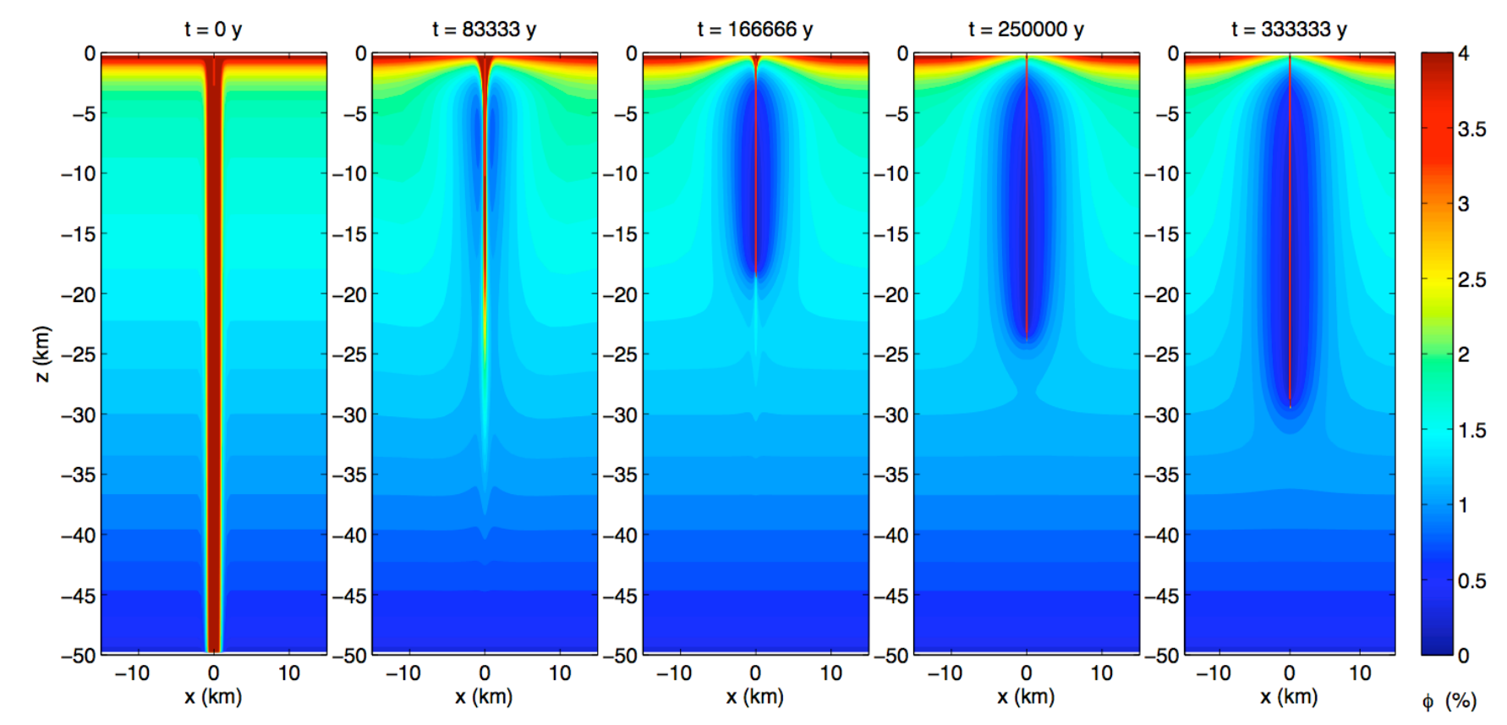
Path followed by one-dimensional upwelling rock. Starts below solidus cooling adiabatically, then intersects solidus and starts melting, with liquid and solid composition following liquidus and solidus respectively (see also the phase diagram above, to which this corresponds).

6. Melting rate

Taking an idealized linear phase constraint (11,12), the conservation laws (6), (7), (9) and (10) combine to give an explicit expression for the melting rate:

$$m = \rho\underbrace{\big[(1-\phi)\mathbf{V} + \phi\mathbf{u}\big]}_{\text{Average upwelling velocity}}\cdot\mathbf{k}g\frac{\gamma\rho c - \alpha T}{L + \lambda c\Delta X}$$

If melt velocity dominates, this can lead to runaway melting and the growth of a channel. The one-dimensional profile above is, however, linearly stable.

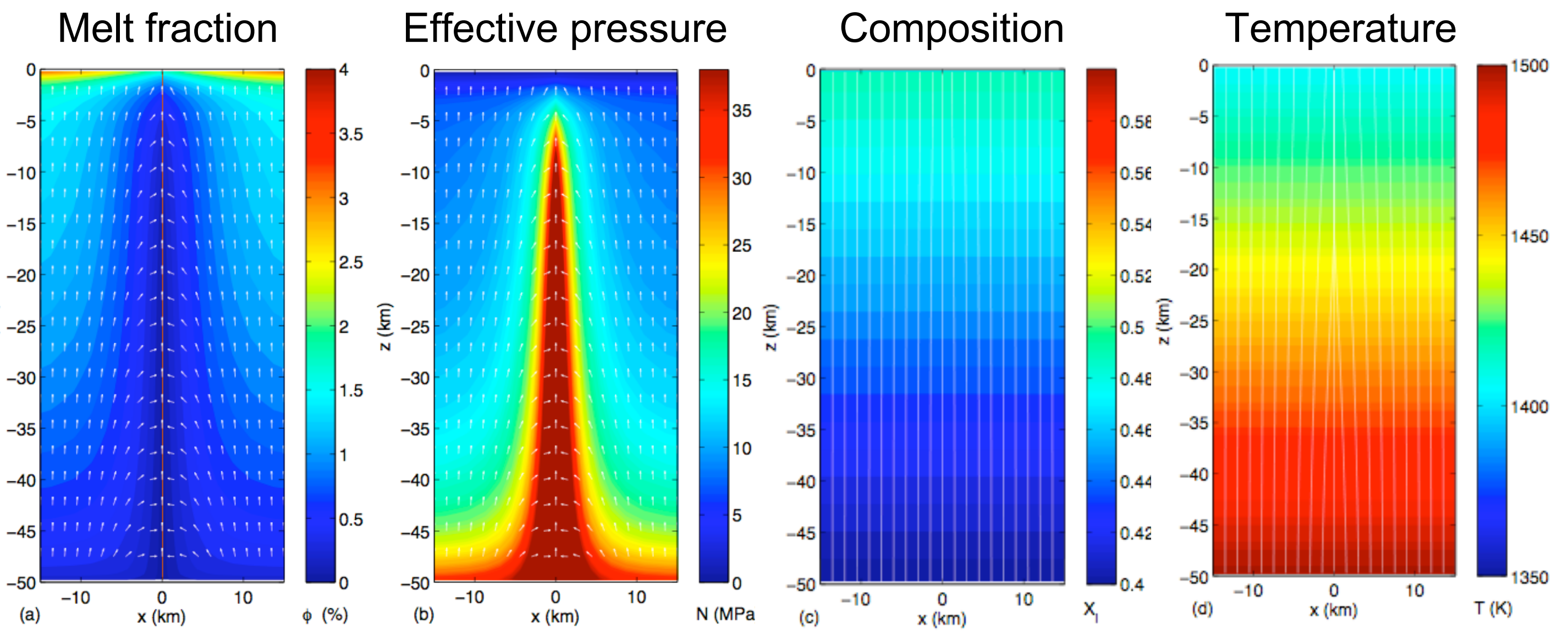


A finite perturbation to the porosity causes melt to localize and the enhanced melting leads to the growth of an open channel.

7. A two-dimensional steady state

If an open channel develops the pressure within it is approximately magmastatic and melt flows into it.

Melting rate is much larger in the channel, but temperature and composition follow the same one-dimensional profiles as above.



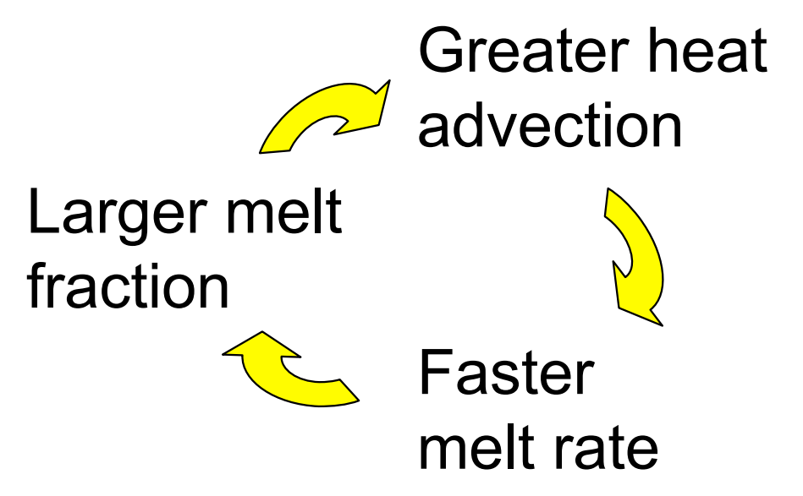
8. Conclusions

Melting rate must be determined using conservation laws.

With simple phase constraints, melting rate is proportional to *average* upwelling rate.

Uniform one-dimensional solutions appear to be linearly stable.

Finite perturbations can, however, lead to locally enhanced melting and the formation of channels.



Greater heat advection

Larger melt fraction

Faster melt rate

References

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