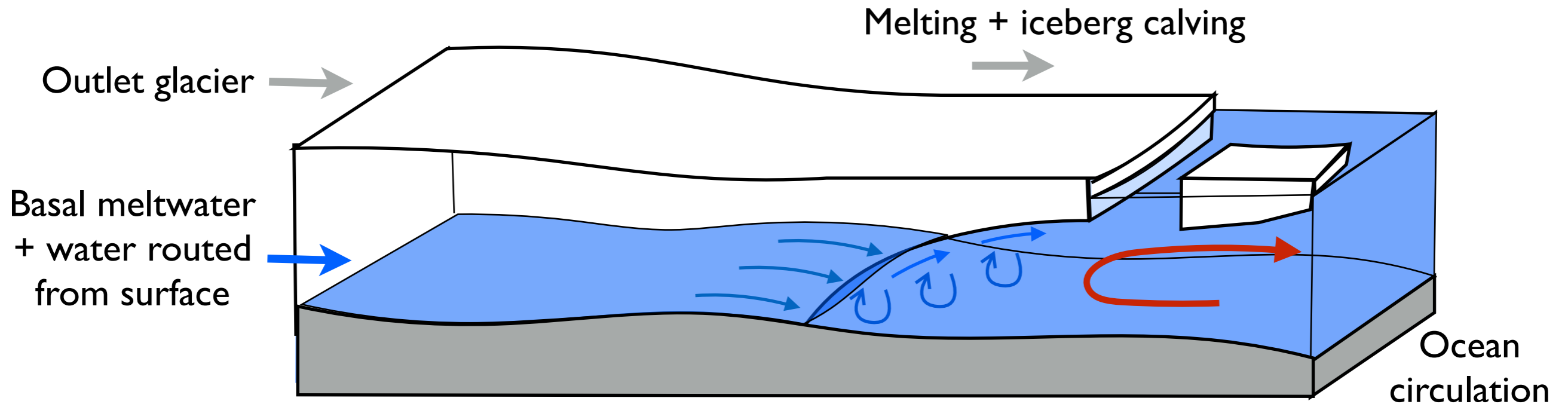
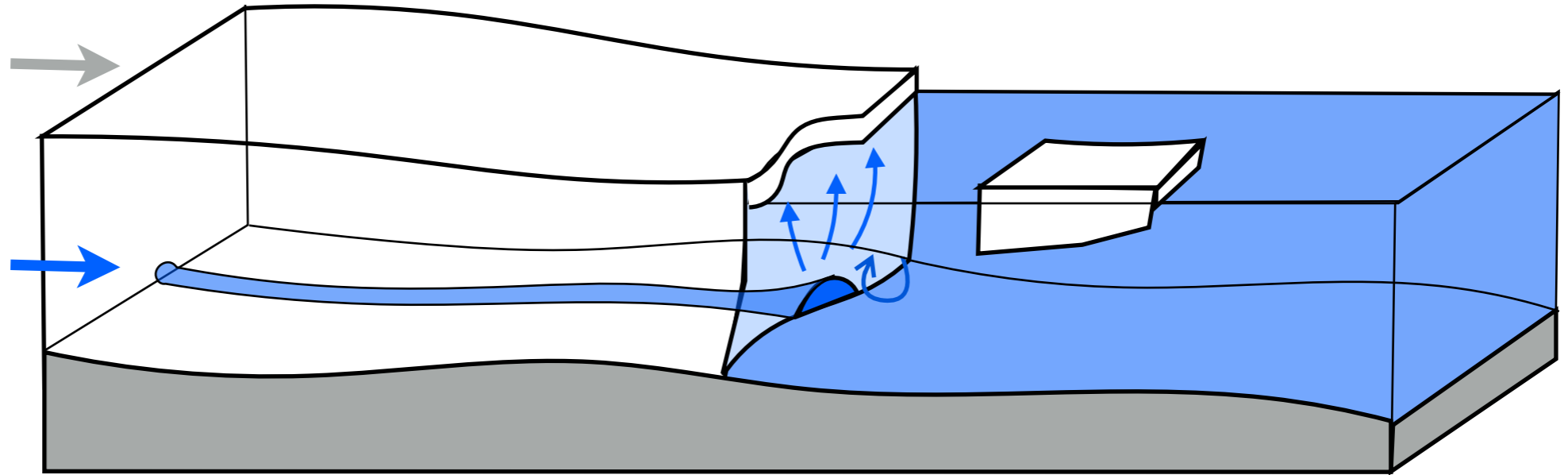


# Modelling subglacial drainage and its role in ice-ocean interaction

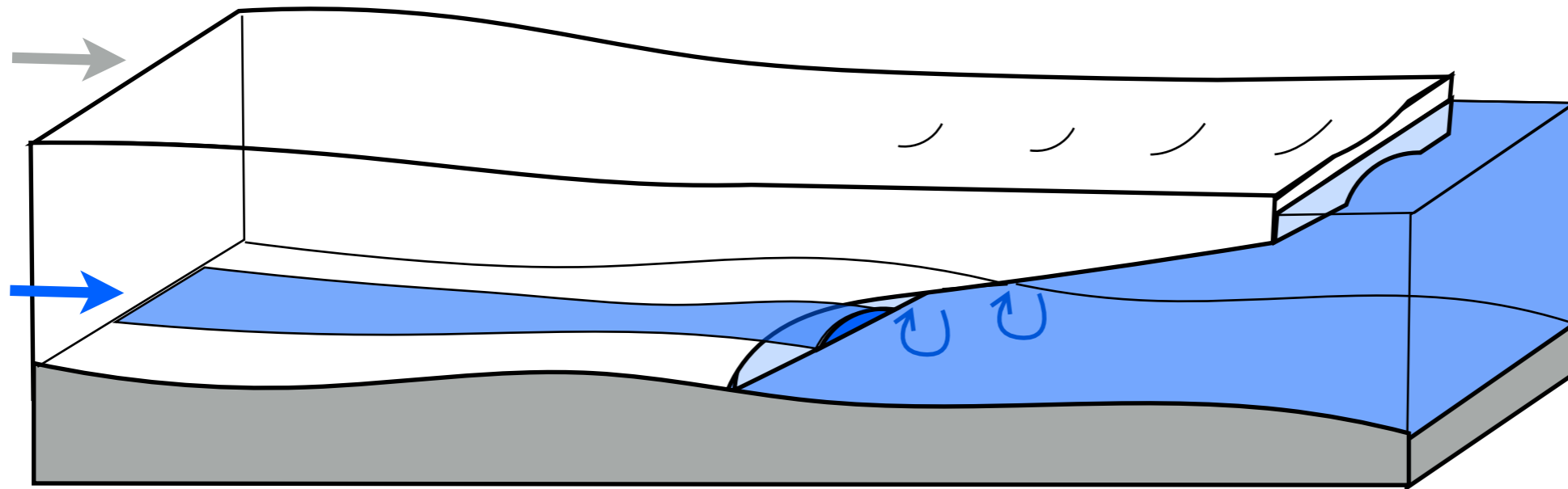
Ian Hewitt (University of Oxford), Michael Dallaston (Imperial College London), Andrew Wells (University of Oxford)



I. What do models tell us about how subglacial discharge is delivered at grounding lines?

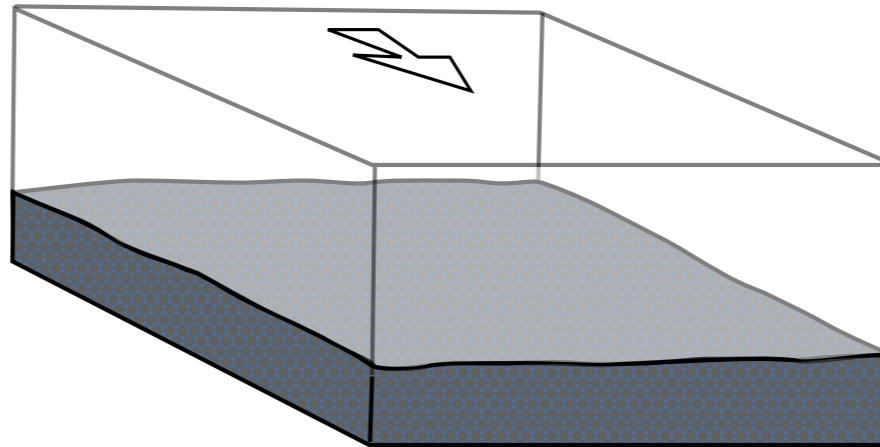


II. How does the spatial distribution of subglacial discharge affect the shape of ice shelves?

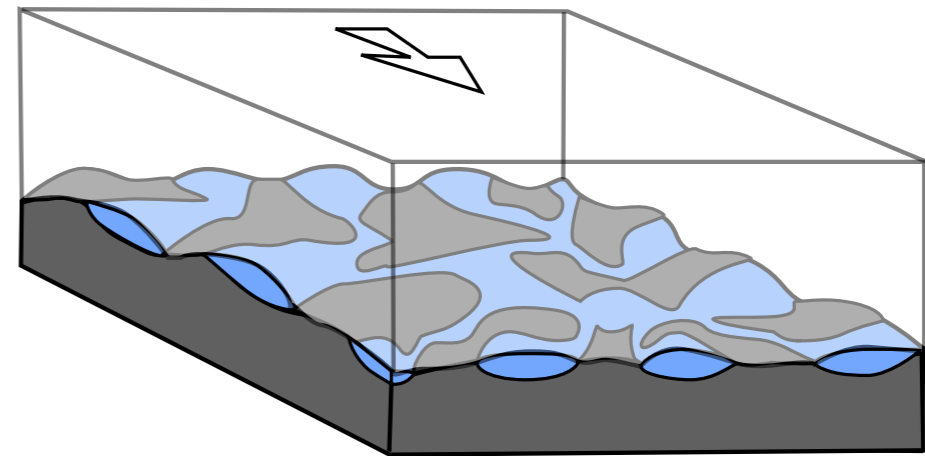


# Subglacial drainage

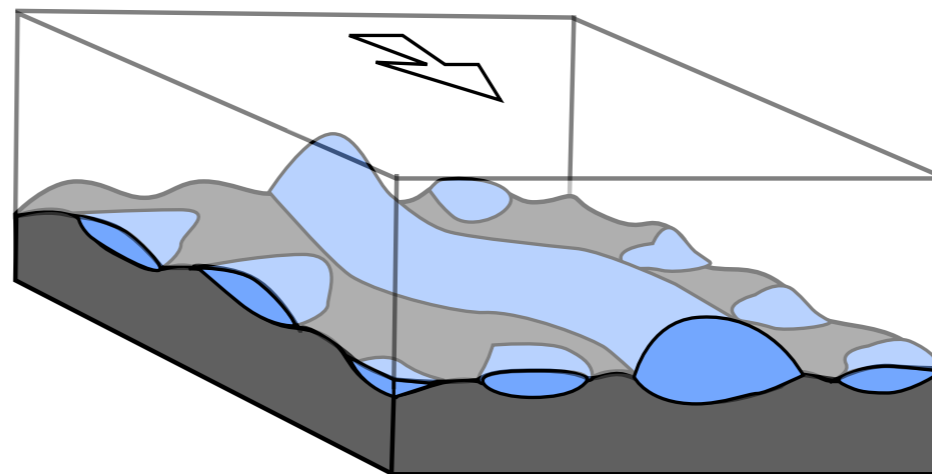
Permeable sediments



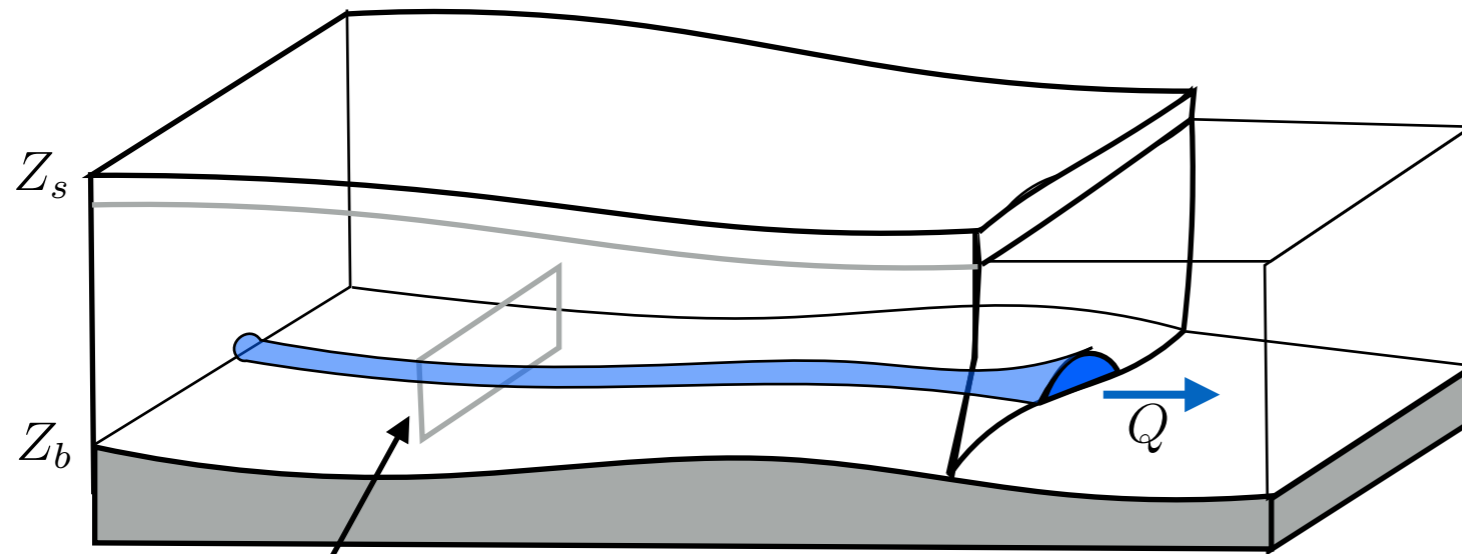
'Distributed' systems



'Channel' systems



# Channel dynamics

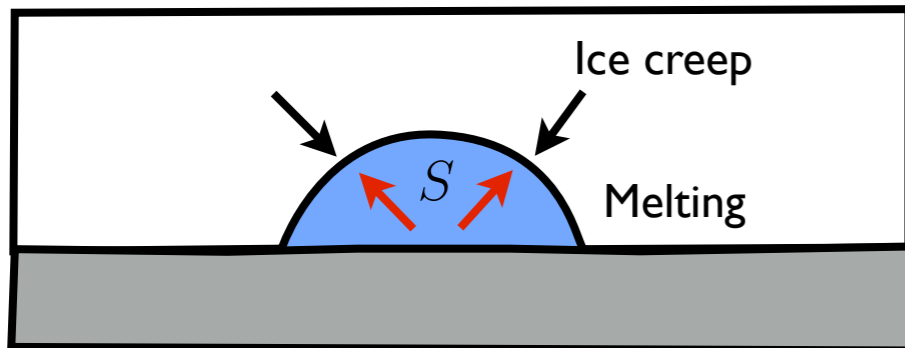


Hydraulic potential

$$\phi = \rho_w g Z_b + p_w$$

Discharge (turbulent flow)

$$Q = -K_c S^{4/3} \left| \frac{\partial \phi}{\partial s} \right|^{-1/2} \frac{\partial \phi}{\partial s}$$



Cross-sectional area

$$\frac{\partial S}{\partial t} = \frac{\rho_w}{\rho_i} M - \frac{2A}{n^n} S |N|^{n-1} N$$

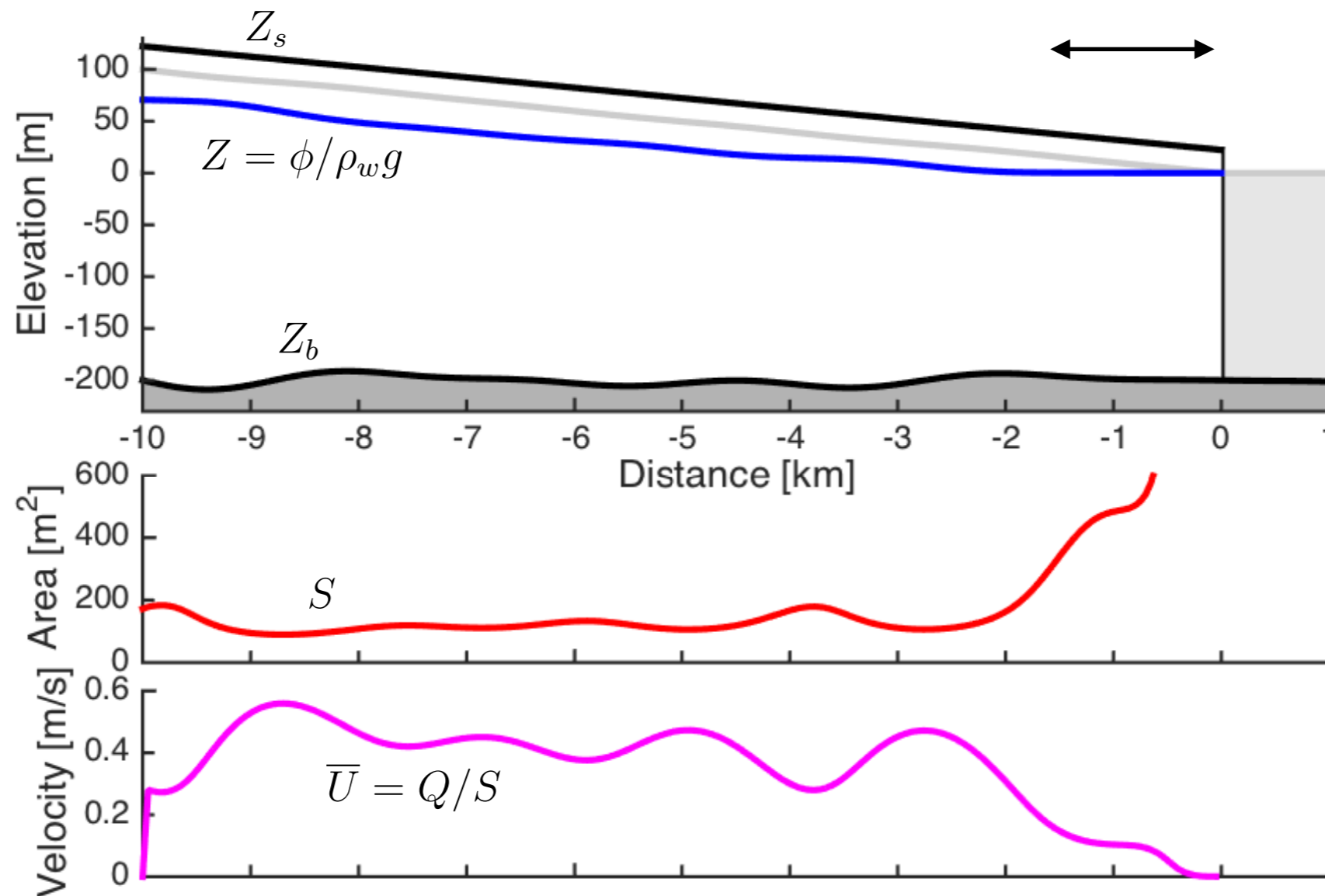
$$N = p_i - p_w$$

- Most of the potential energy dissipated by turbulence is converted to latent heat

$$M = -\frac{1 - \rho_w c \gamma}{\rho_w L} Q \frac{\partial \phi}{\partial s} - \frac{\rho_w g c \gamma}{L} Q \frac{\partial Z_b}{\partial s}$$

# Channel dynamics

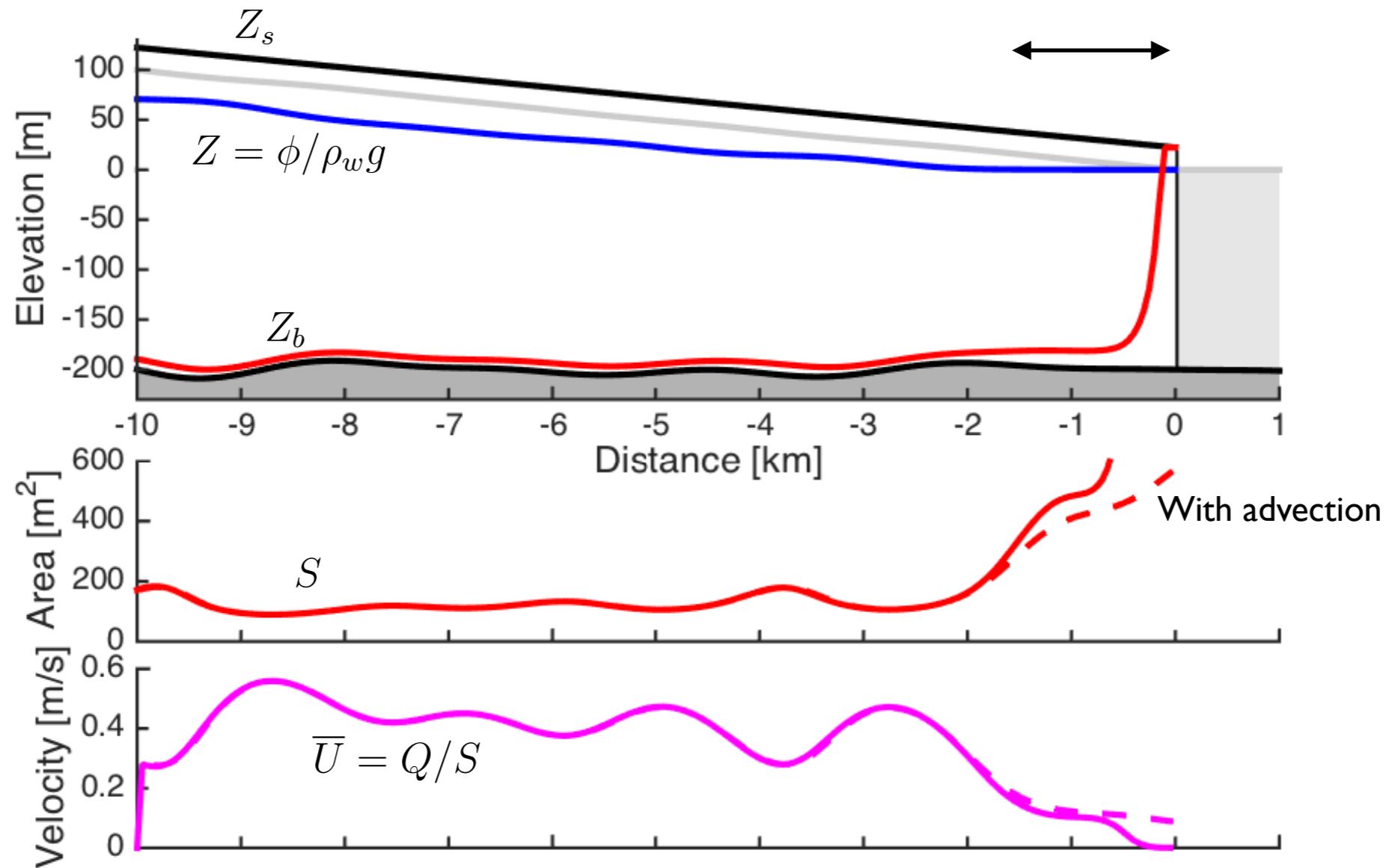
Steady-state channel with constant discharge



- Melting rate and creep closure rate are reduced near grounding line.
- Results in trumpet-like shape of channels, and relatively low water speed at outlet.

# Channel dynamics

Steady-state channel with constant discharge



- However, advection of channels with the ice prevents them becoming too large

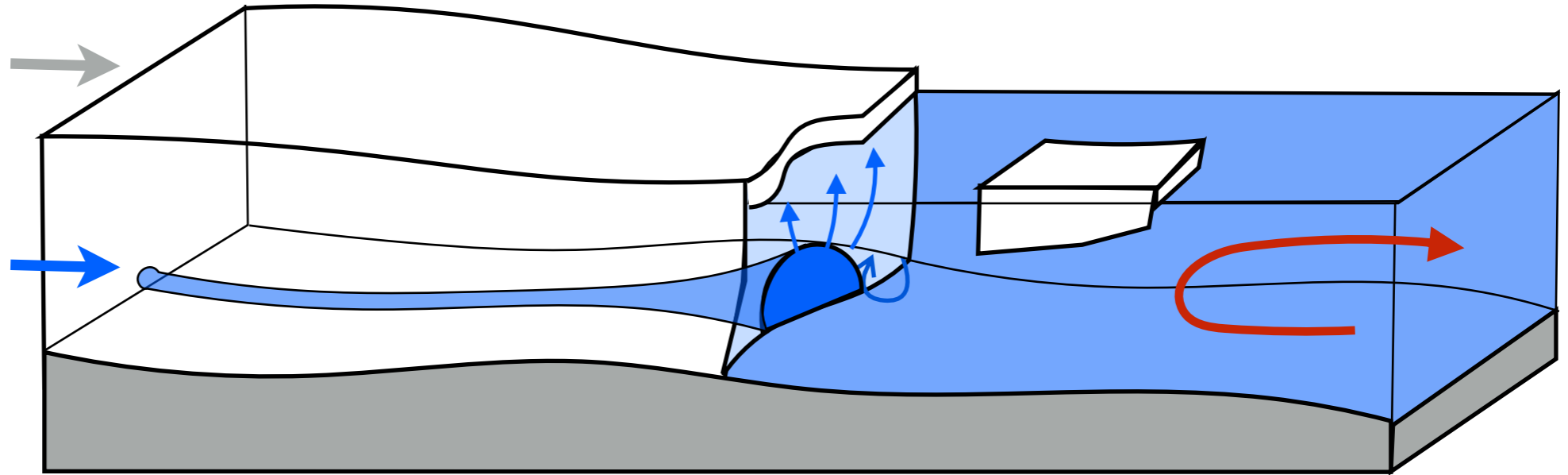
$$\frac{\partial S}{\partial t} + \mathbf{u}_b \cdot \nabla S = \frac{\rho_w}{\rho_i} M - \frac{2A}{n^n} S N^n$$

- Further analysis indicates outflow water speed  $\bar{U} \approx C Q^{2/11} u_b^{9/44} (\sin \theta)^{9/44}$   $C \approx 3 \text{ m}^{1/4} \text{ s}^{-27/44}$

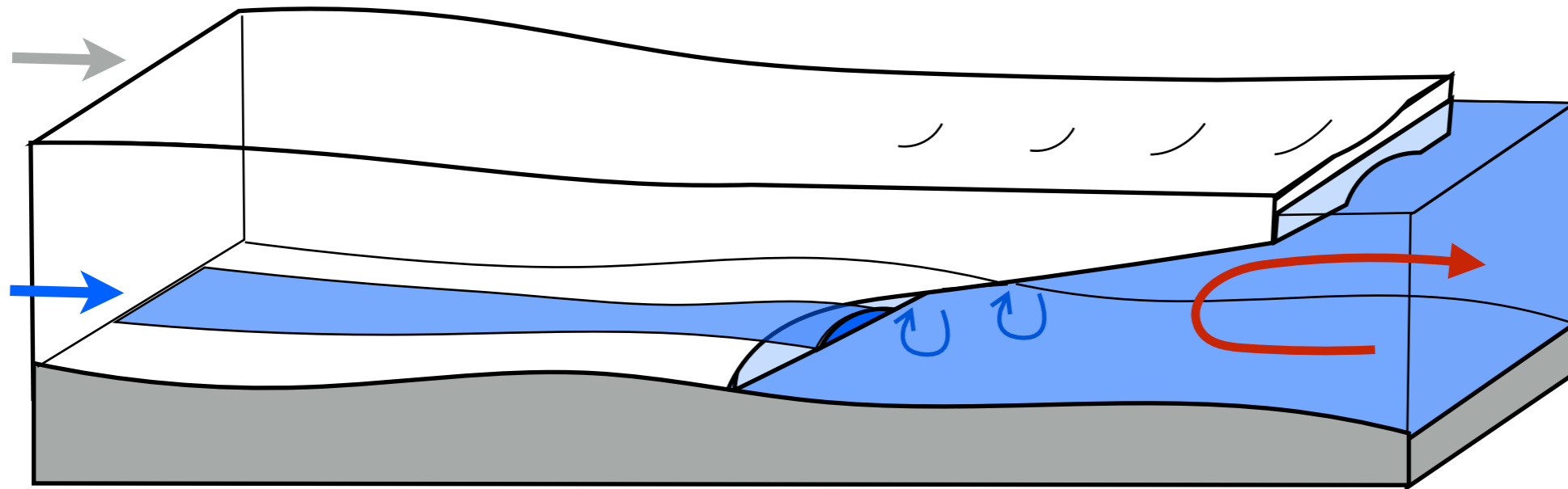
# General comments

- Routing of subglacial water controlled primarily by topography (probably largest factor in determining portal locations).
- Expect large seasonal signal of subglacial discharge when surface meltwater present.
- Subglacial drainage system exhibits instabilities that likely lead to episodic discharge (in addition to weather-driven episodes).

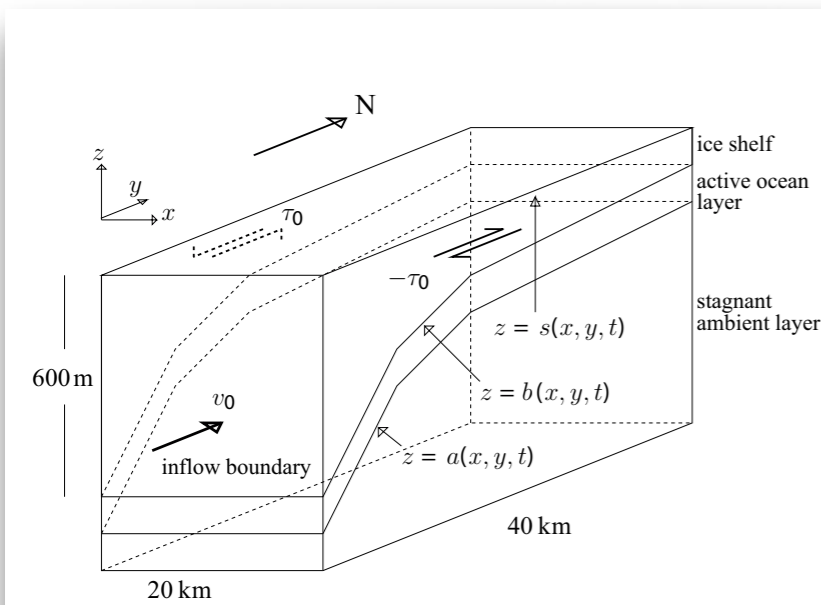
I. What do models tell us about how subglacial discharge is delivered at grounding lines?



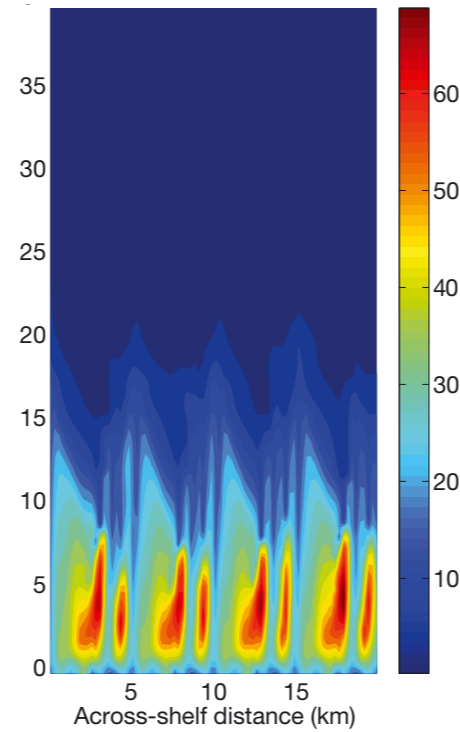
II. How does the spatial distribution of subglacial discharge affect the shape of ice shelves?



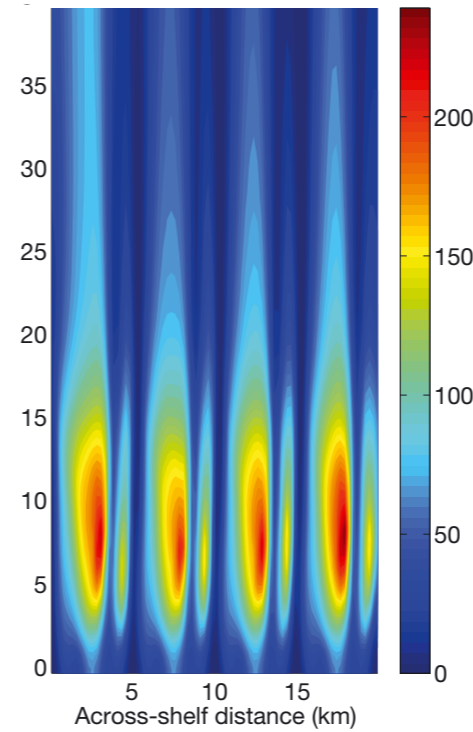
# Previous modelling results



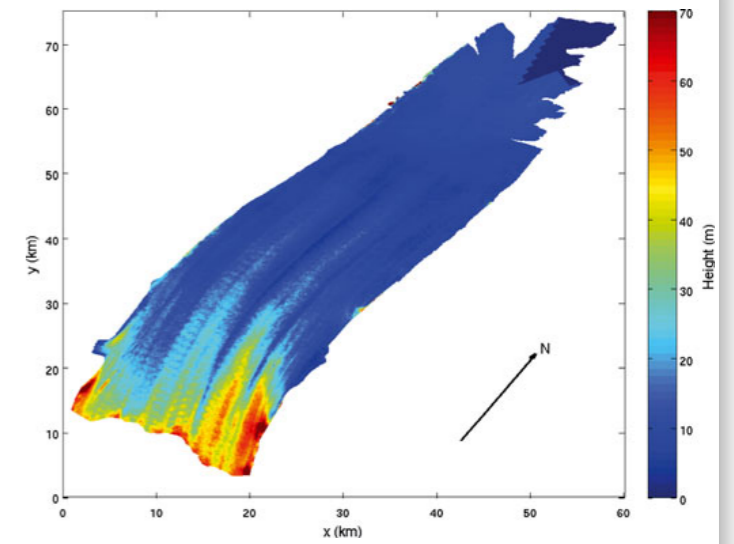
Melt rate



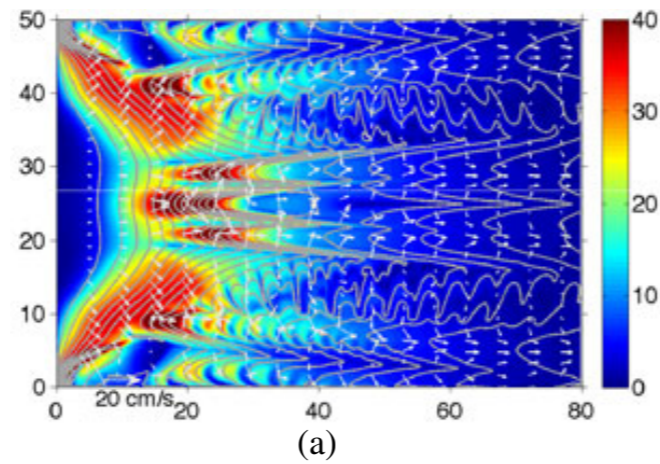
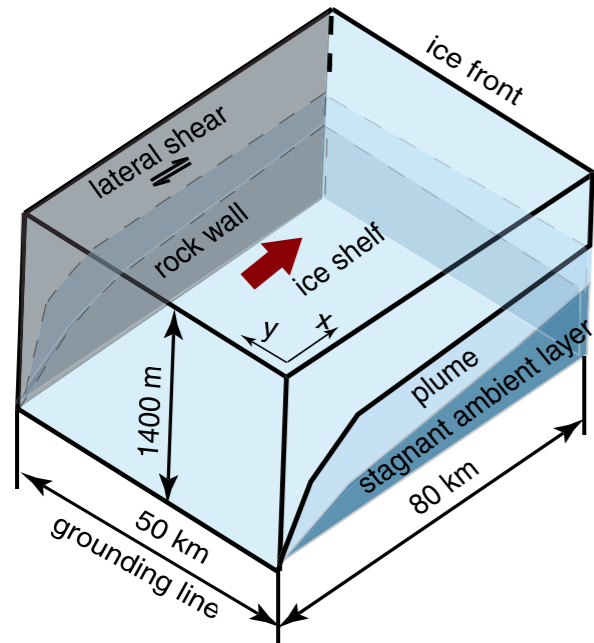
Channel depth



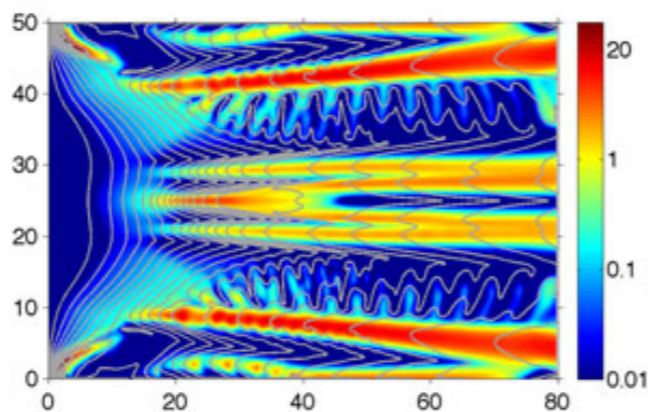
Petermann ice shelf



Gladish et al 2012



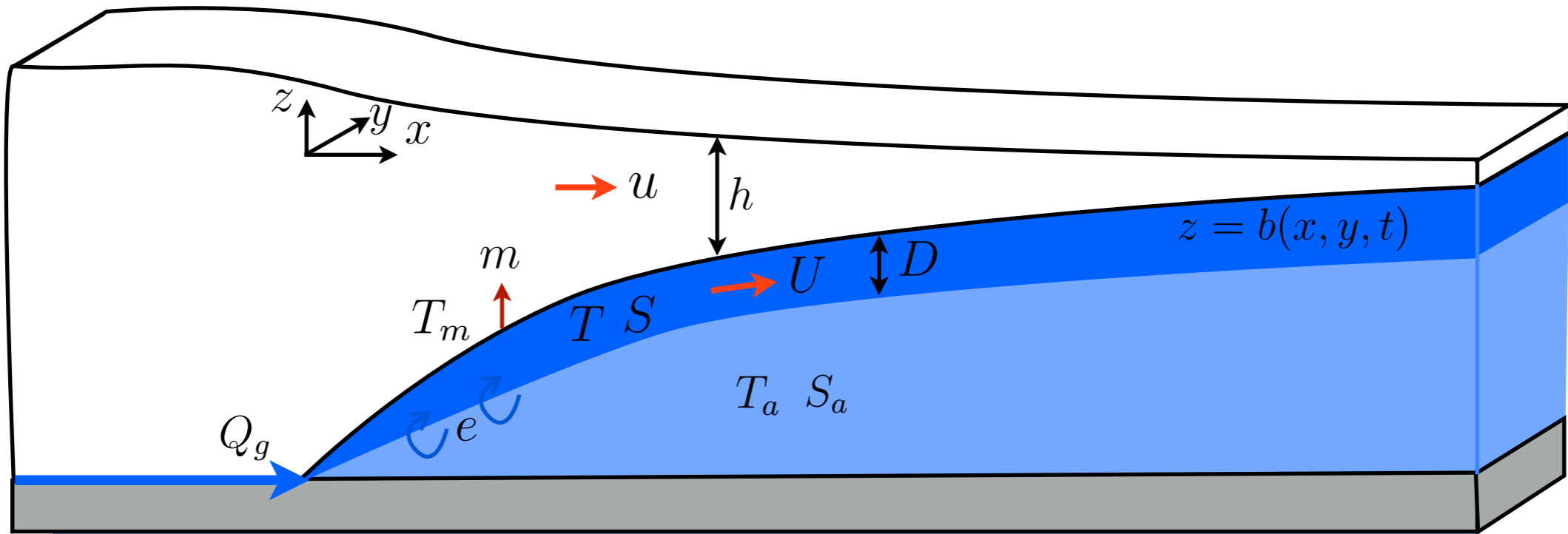
Melt rate



Plume thickness

Sergienko 2013

# A simplified model - ice



Depth-integrated model for ice shelf (standard)

Parameterised interface melting

$$\frac{\partial h}{\partial t} + \nabla \cdot (h\mathbf{u}) = -(\rho_o/\rho_i)m,$$

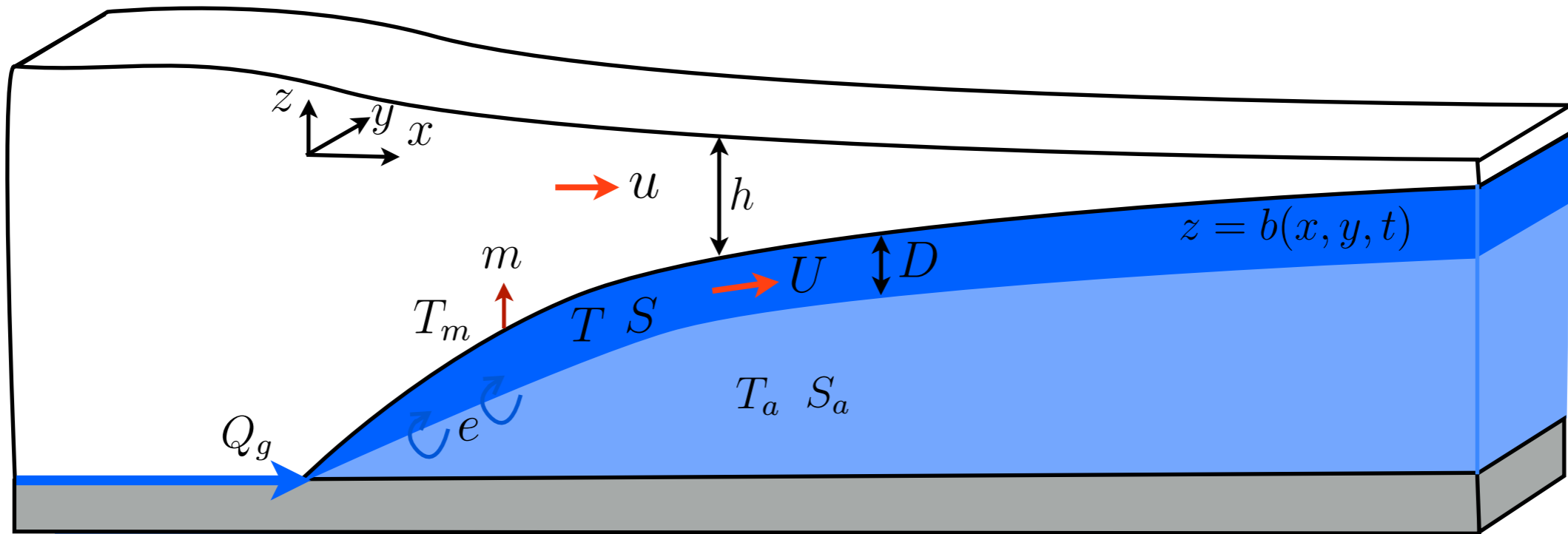
$$mL = c\gamma_T|\mathbf{U}|(T - T_m).$$

$$\frac{\partial}{\partial x} \left[ 2\eta h \left( 2\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right] + \frac{\partial}{\partial y} \left[ \eta h \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] - (1 - \rho_i/\rho_o) \rho_i g h \frac{\partial h}{\partial x} = 0,$$

$$\frac{\partial}{\partial x} \left[ \eta h \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] + \frac{\partial}{\partial y} \left[ 2\eta h \left( \frac{\partial u}{\partial x} + 2\frac{\partial v}{\partial y} \right) \right] - (1 - \rho_i/\rho_o) \rho_i g h \frac{\partial h}{\partial y} = 0,$$

+ prescribed ice depth and speed over grounding line

# A simplified model - plume



## Simplified plume model (conservation laws)

Parameterised entrainment  $e = E_0 |U| |\nabla b|$ ,

$$\nabla \cdot (DU) = e + m,$$

Along slope buoyancy due to salinity

(coupling to ice dynamics)

$$\nabla \cdot (DUU) = Dg\beta_S S_\Delta \left( \frac{\partial b}{\partial x} - \frac{\partial D}{\partial x} \right) + \nabla \cdot (\kappa D \nabla U) - C_d |U| U$$

$$\nabla \cdot (DUV) = Dg\beta_S S_\Delta \left( \frac{\partial b}{\partial y} - \frac{\partial D}{\partial y} \right) + \nabla \cdot (\kappa D \nabla V) - C_d |U| V$$

Turbulent eddy viscosity

(smooths small-scale velocity differences)

$$\nabla \cdot (DUS) = eS_a + \nabla \cdot (\kappa D \nabla S) + mS_i,$$

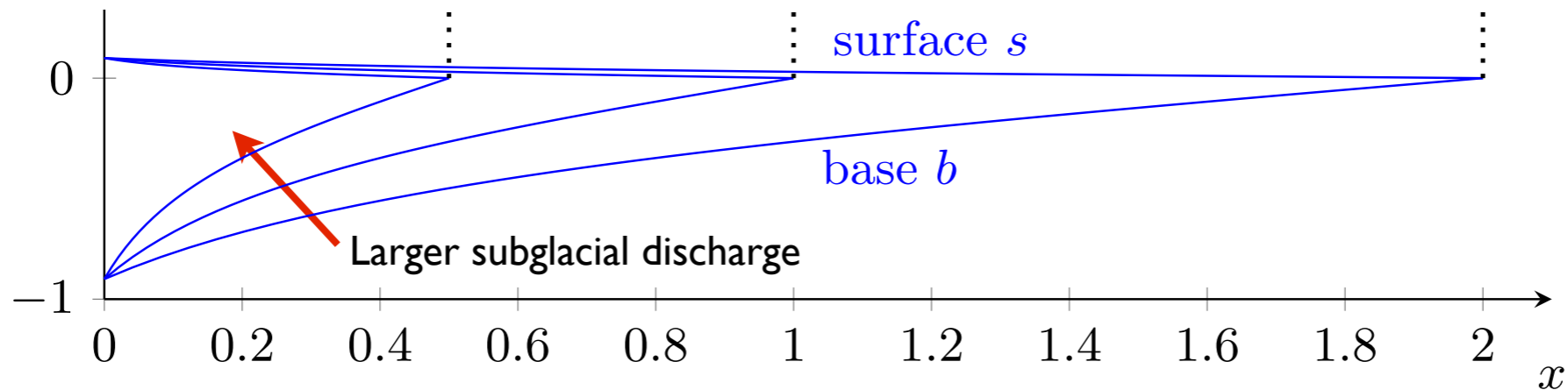
$$\nabla \cdot (DUT) = eT_a + \nabla \cdot (\kappa D \nabla T) + mT_m - \frac{mL}{c}.$$

Heat exchange with ice

+ prescribed subglacial discharge at grounding line

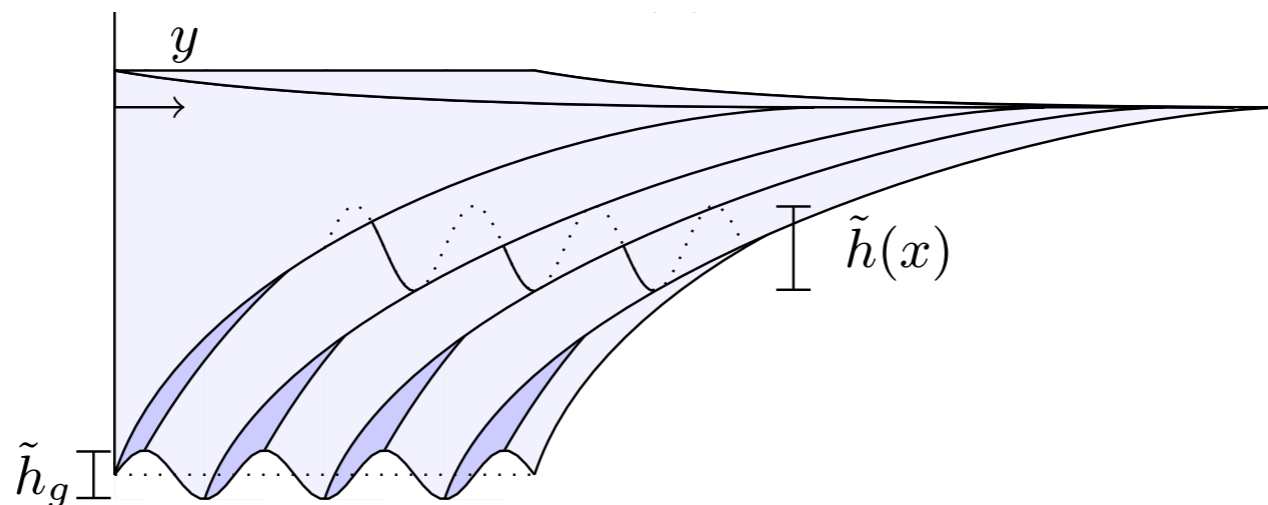
# Reduced model results

One-dimensional steady-state ice-shelf shape (melting rate approximately uniform)



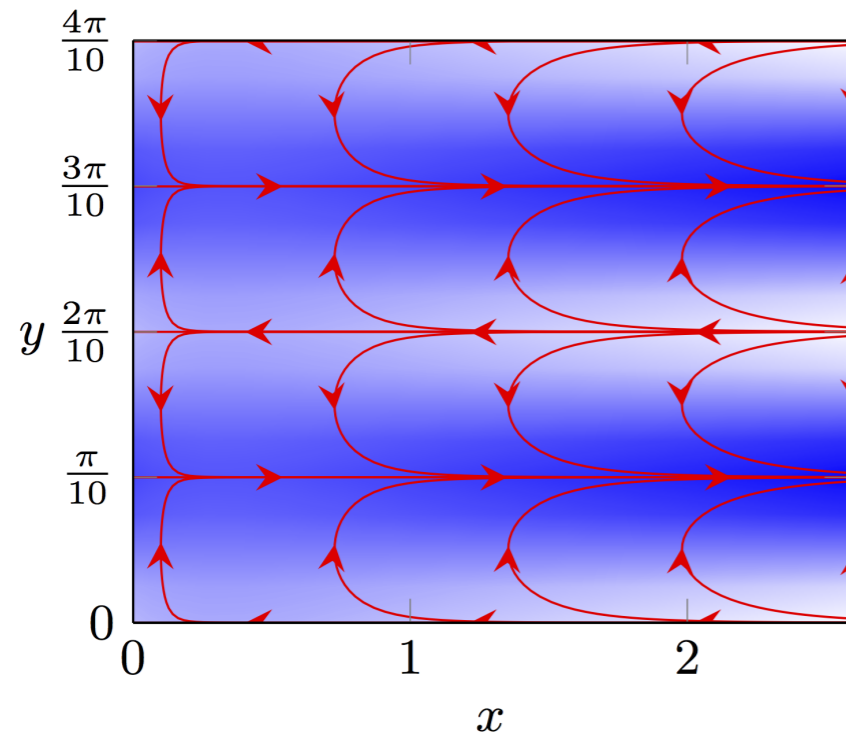
## Linear stability analysis

- Small transverse perturbation of arbitrary wavenumber  
(due to variable grounding-line ice depth or variable subglacial discharge)

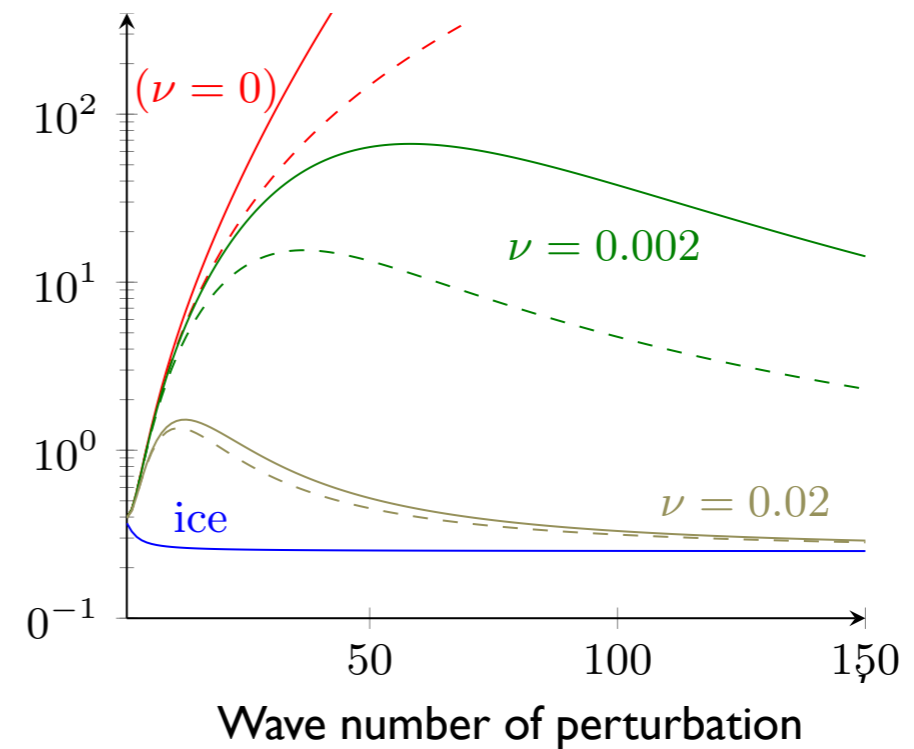


# Reduced model results

Ice depth perturbation



Amplitude of perturbation



- Perturbations at grounding line grow downstream, driven by transverse flow into channels and enhanced buoyancy-driven acceleration.
- Transverse ice flow is relatively ineffective at smoothing out channels.
- Stabilisation of small wavelengths is due to turbulent mixing in the plume layer.

# Summary

A satellite image showing a glacier system. The glacier is a mix of white and light brown, with a network of dark blue and black channels (subglacial drainage channels) visible. The channels are more prominent near the grounding line, where the glacier meets the ocean. The ocean is a deep blue on the left side of the image.

Subglacial drainage channels are likely to trumpet out near grounding line.

Expect a smooth transition from subglacial melting (potential energy) to frontal melting (ocean heat).

Uneven spatial distribution of discharge and/or basal topography at grounding line can cause channelisation due to enhanced melting of the shelf.

Primary factors in channelisation are flow-focussing and buoyancy-driven acceleration, counteracted by turbulent mixing.