Models for Polythermal Ice Sheets and Glaciers
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Motivation
- Temperate ice (at the pressure melting point, containing small amounts of liquid water) is widespread in mountain glaciers, and near the base of ice sheets.
- The water content of temperate ice reduces its effective viscosity, with potentially important dynamical consequences.
- Models of polythermal ice need to account for the evolution and transport of water content as well as sensible heat.

Take home points
- We derive three different models, incorporating different descriptions of water transport, including gravity-driven drainage.
- We compare predictions of porosity and cold-temperate boundary locations.
- All the models are simple enough to incorporate in existing ice-sheet models with little modification.

Models
Stokes flow
\[ \nabla \cdot u = 0 \quad \nabla \cdot \tau - \nabla p + \rho g = 0 \]
\[ \tau_{ij} = A^{-1/n} \varepsilon^{1/n} \delta_{ij} \]

Energy
\[ \rho_e \left( \frac{\partial T}{\partial t} + u \cdot \nabla T \right) = \nabla \cdot (k \nabla T) + \tau_{ij} \varepsilon_{ij} \quad \phi = 0, \quad T \leq T_m \]
\[ \rho_e L \left( \frac{\partial \phi}{\partial t} + u \cdot \nabla \phi \right) + \rho_e L \nabla \cdot j = \tau_{ij} \varepsilon_{ij}, \quad T = T_m, \quad \phi > 0 \]

Darcy’s law
\[ j = \frac{k_{uw} \phi^2}{n_e} (\rho_e k - \nabla p_e) \]

1) Compaction model
\[ p_w = p - p_e \]
\[ \nabla p \approx \rho g \]
\[ j = \frac{k_{uw} \phi^2}{n_e} (\rho(u - v) + \nabla p_e) \quad p_e = \frac{1}{\phi} \nabla \cdot j \]

2) Modified enthalpy gradient model
\[ p_w = p - p_e(\phi) \]
\[ j = \frac{k_{uw} \phi^2}{n_e} (\rho(u - v) - \nabla \phi) \quad \nu = -\frac{k_{uw} \phi^2}{n_e} \frac{d p_e}{d \phi} \]

3) Standard enthalpy gradient model
\[ j = -\nu \nabla \phi \]

Slab glacier
Vertical advection downwards / upwards

Ice cap
Modified enthalpy gradient model

Boundary conditions
- Finite volume method naturally incorporates conditions at cold-temperate boundaries
\[ \rho_e L [\phi (u - v)^+] \cdot n = -k \nabla T^- \cdot n - \rho_e L j \cdot n \quad j \cdot n = 0 \quad [ \phi = 0 \quad \text{enthalpy gradient models}] \]


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