Models for Polythermal Ice Sheets and Glaciers

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Motivation

- Temperate ice (at the pressure melting point, containing small amounts of liquid water) is widespread in mountain glaciers, and near the base of ice sheets.
- The water content of temperate ice reduces its effective viscosity, with potentially important dynamical consequences.
- Models of polythermal ice need to account for the evolution and transport of water content as well as sensible heat.

Models

Stokes flow

$$\boxed{\nabla\cdot\mathbf{u}=0 \qquad \nabla\cdot\boldsymbol{\tau}-\nabla p+\rho\mathbf{g}=0} \qquad \tau_{ij}=A^{-1/n}\dot{\varepsilon}^{1/n-1}\dot{\varepsilon}_{ij}$$

Energy

$$\rho c\left(\frac{\partial T}{\partial t}+\mathbf{u}\cdot\nabla T\right)=\nabla\cdot(k\nabla T)+\tau_{ij}\dot{\varepsilon}_{ij}, \qquad \phi=0, \qquad T\le T_m$$

$$\rho_w L\left(\frac{\partial \phi}{\partial t}+\mathbf{u}\cdot\nabla \phi\right)+\rho_w L\nabla\cdot\mathbf{j}=\tau_{ij}\dot{\varepsilon}_{ij}, \qquad T=T_m, \qquad \phi>0$$

Darcy's law

$$\mathbf{j}=\frac{k_0\phi^2}{\eta_w}\left(\rho_w\mathbf{g}-\nabla p_w\right)$$

1) Compaction model $\quad p_w=p-p_e \qquad \nabla p\approx\rho\mathbf{g}$

$$\boxed{\mathbf{j}=\frac{k_0\phi^2}{\eta_w}\left((\rho_w-\rho)\mathbf{g}+\nabla p_e\right) \qquad p_e=\frac{\eta}{\phi}\nabla\cdot\mathbf{j}}$$

2) Modified enthalpy gradient model $\quad p_w=p-p_s(\phi)$

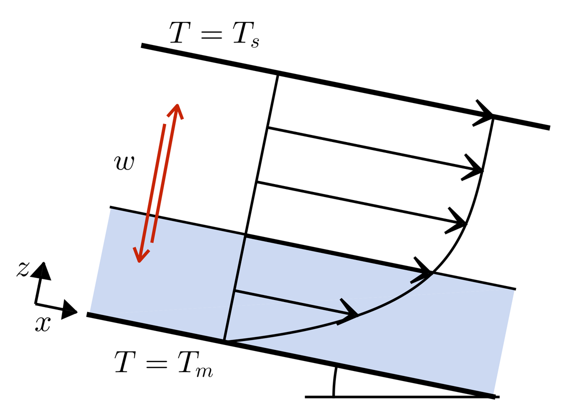
$$\boxed{\mathbf{j}=\frac{k_0\phi^2}{\eta_w}(\rho_w-\rho)\mathbf{g}-\nu\nabla\phi} \qquad \nu=-\frac{k_0\phi^2}{\eta_w}\frac{\mathrm{d}p_s}{\mathrm{d}\phi}$$

3) Standard enthalpy gradient model $\quad \boxed{\mathbf{j}=-\nu\nabla\phi}$

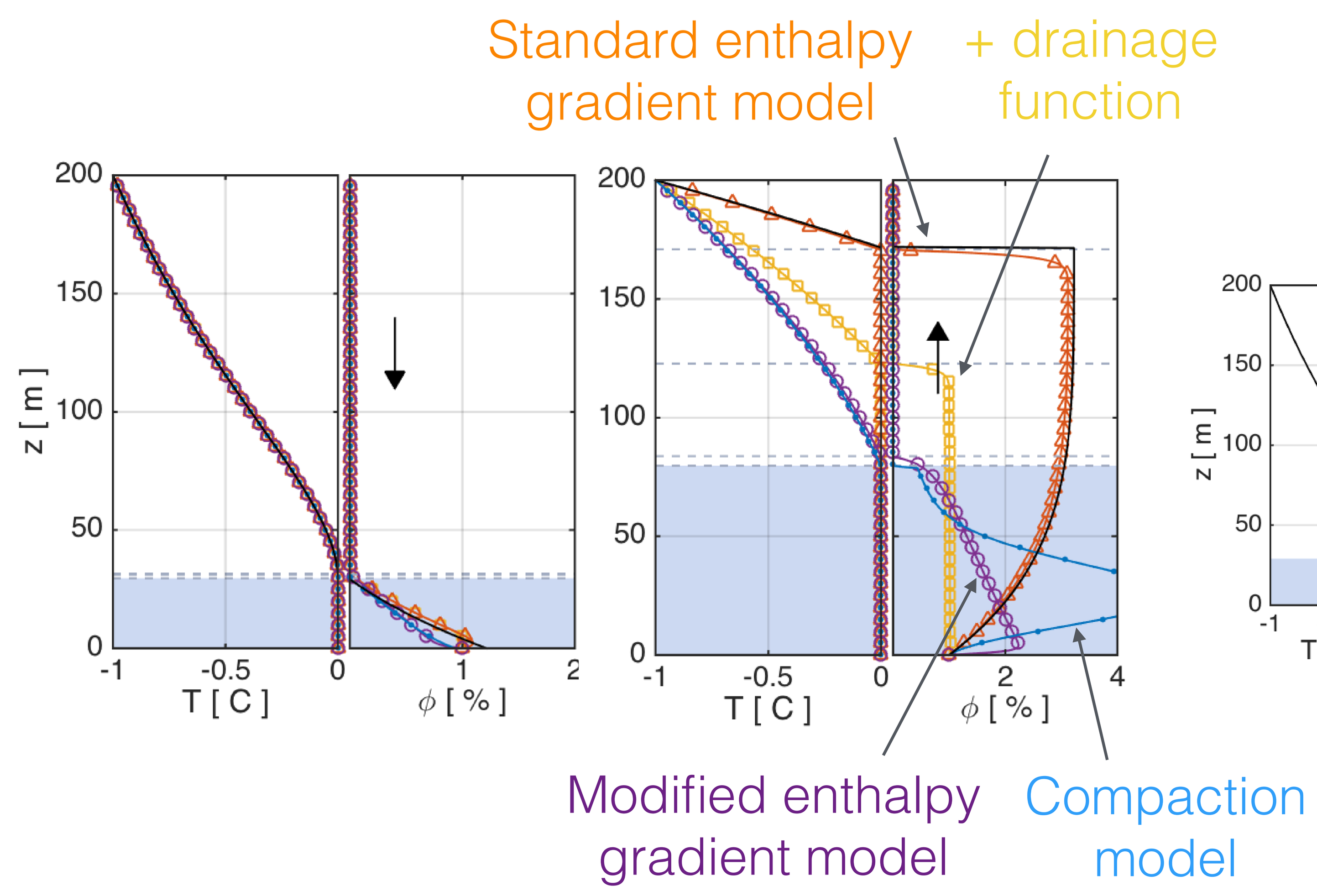
Take home points

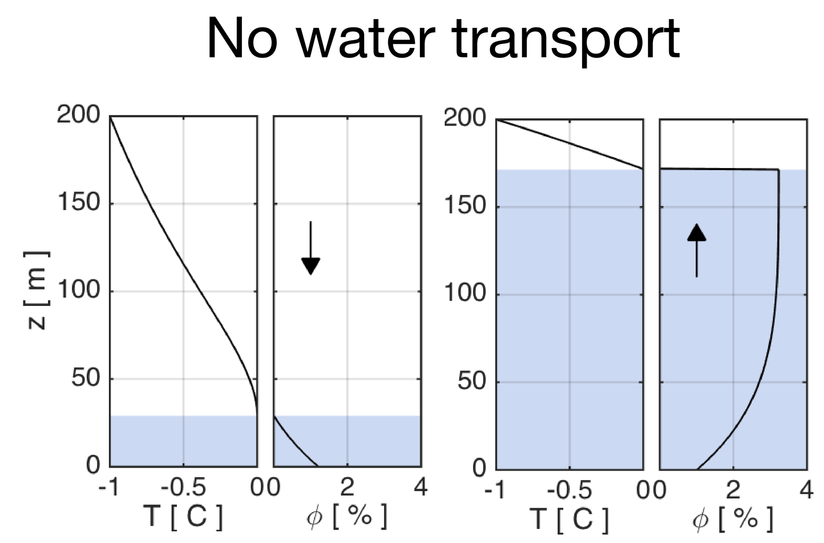
- We derive three different models, incorporating different descriptions of water transport, including gravity-driven drainage.
- We compare predictions of porosity and cold-temperate boundary locations.
- All the models are simple enough to incorporate in existing ice-sheet models with little modification.

Slab glacier

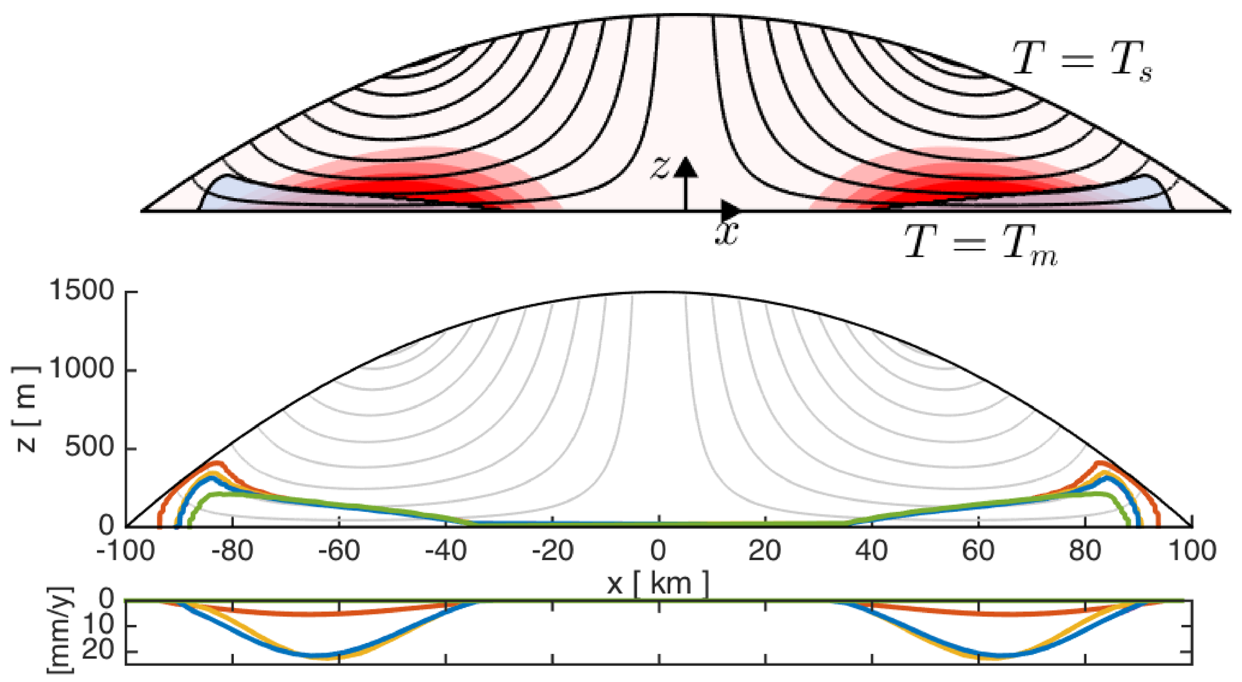


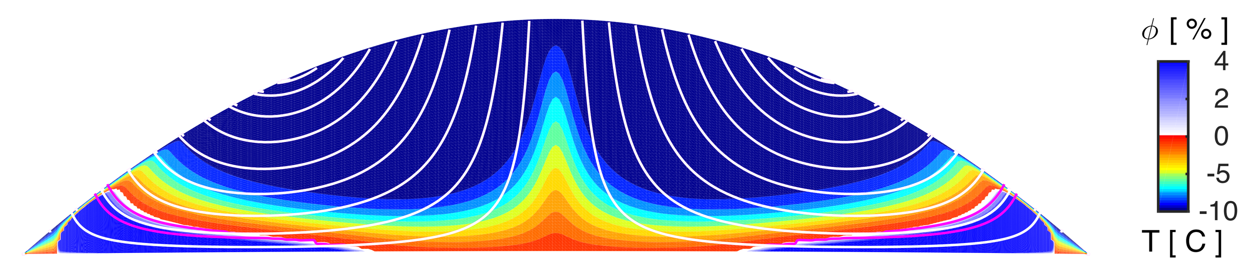
Vertical advection downwards / upwards



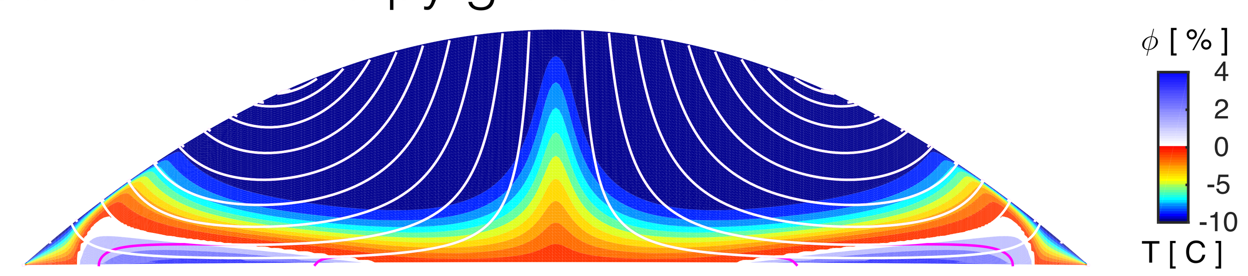


Ice cap





Standard enthalpy gradient model



Compaction model

Boundary conditions

- Finite volume method naturally incorporates conditions at cold-temperate boundaries $\quad \rho_w L[\phi(\mathbf{u}-\mathbf{v})]^+\cdot\mathbf{n}=-k\nabla T^-\cdot\mathbf{n}-\rho_w L\,\mathbf{j}\cdot\mathbf{n} \qquad \mathbf{j}\cdot\mathbf{n}=0\ \ [\,\phi=0$ enthalpy gradient models]

References: Schoof, C. & Hewitt, I.J. A model for polythermal ice incorporating gravity-driven moisture transport. *J. Fluid Mech. to appear.* Hewitt, I.J. & Schoof, C. Models for polythermal ice sheets and glaciers. *submitted.*
This work draws heavily upon the works of Nye & Frank (1973), Fowler & Larson (1978), Hutter (1982), Fowler (1985), Greve (1997), Aschwanden et al. (2012), and Kleiner et al, (2015).

Acknowledgements: IJH is supported by a Marie Curie FP7 Career Integration Grant PCIG13-GA-2013-618007 ImPISM. CGS acknowledges the support of NSERC grants 357193-13 and 446042-13 and a Killam Faculty Research Fellowship at UBC.