Subglacial conduits in sediments

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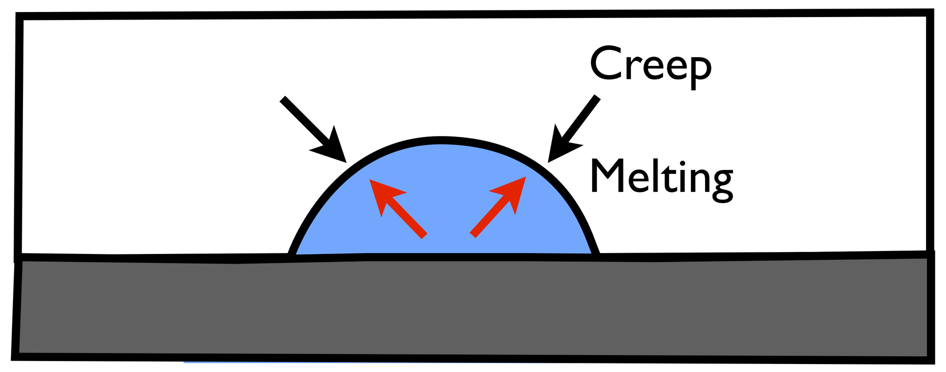
Motivation

- Hard-bedded Röthlisberger channels have become a **canonical model** for subglacial drainage.
- A (relatively) simple set of equations describe their dynamics.
- Model is successful at explaining many jökulhlaups and aspects of drainage system evolution.
- It would be helpful to have **an analogous model** for sediment-floored channels.

Röthlisberger channel



$$Q = K_c S^\alpha \Psi^{1/2}$$

$$\Psi = \Psi_0 + \frac{\partial N}{\partial x}$$

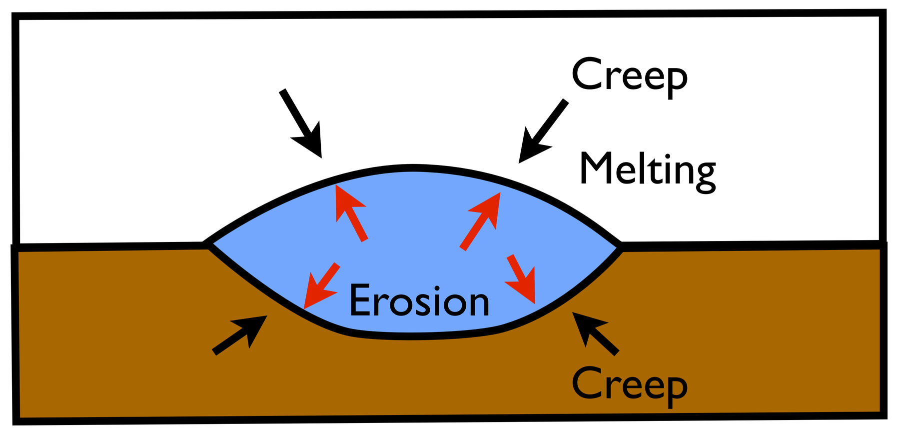
$$\frac{\partial S}{\partial t} + \frac{\partial Q}{\partial x} = \frac{\Psi Q}{\rho_w L} + M$$

$$\frac{\partial S}{\partial t} = \frac{\Psi Q}{\rho_i L} - \tilde{A} S |N|^{n-1} N$$

- **Controlling physics**: turbulent heating+melting, creep closure of ice walls

$$N \propto \Psi^{11/24} Q^{1/12}$$

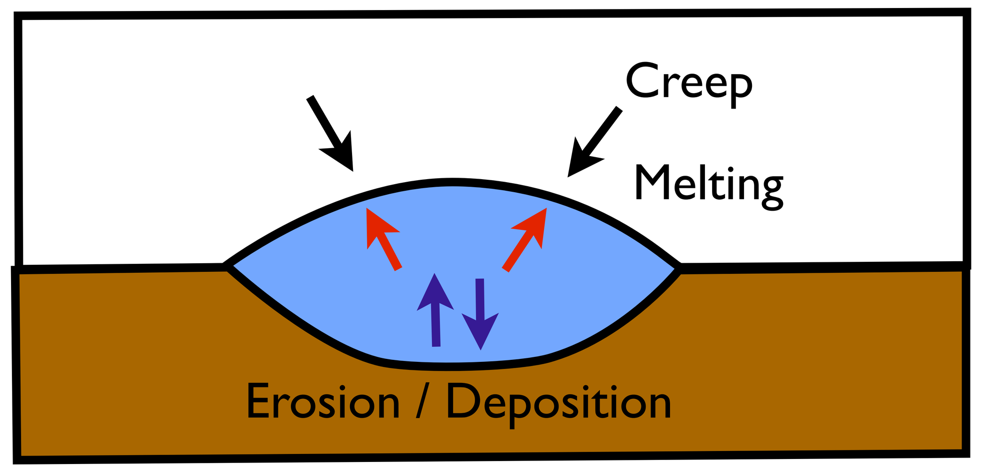
Canals

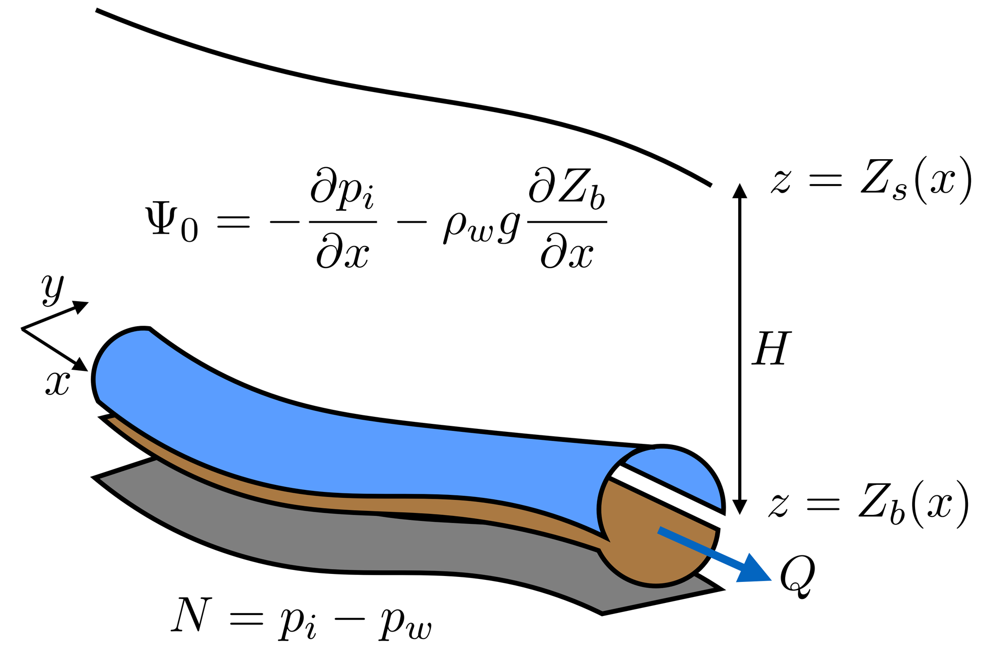


$$N \propto \Psi^{2/3} Q^{-1/3}$$

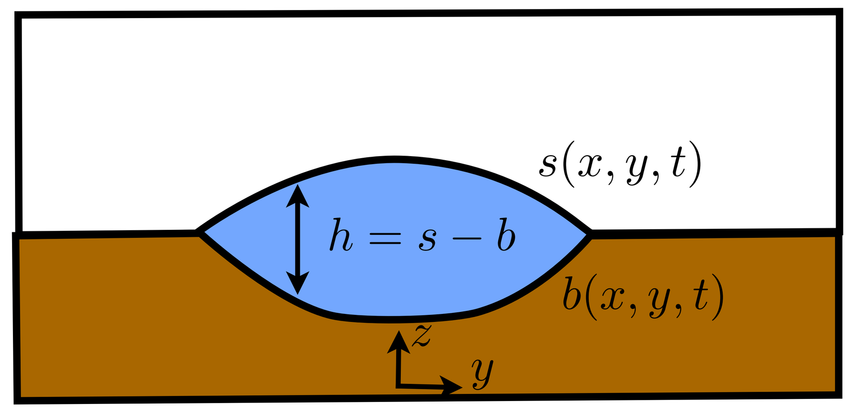
- Walder & Fowler (1994) described a **steady-state** theory for wide-aspect-ratio channels, often referred to as canals.
- Predict **inverse relationship** between discharge and effective pressure.
- Extensions to dynamic evolution previously explored by Fowler & Ng (1996), and Ng (2000).

Channels in Sediments

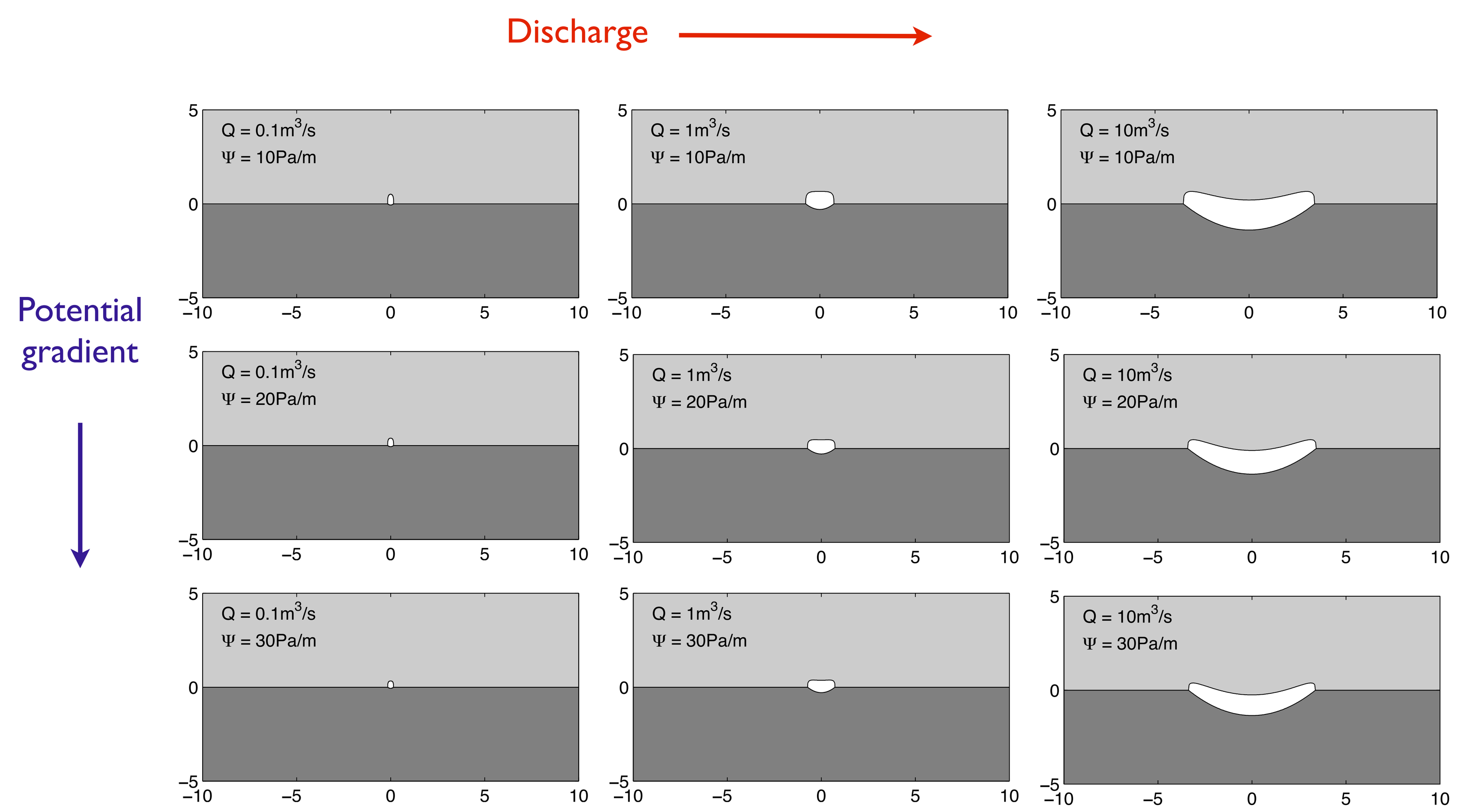




- **Controlling physics**: turbulent heating+melting, creep closure of ice walls, sediment erosion +deposition, (creep closure of sediment walls)
- Lumped model derived from analysis and approximation of a more detailed wide-channel theory.



Steady-state cross sections



$$N \propto \Psi^{1/3} Q^{-1/9}$$

$$Q = K_1 \Psi^{3/8} S^{9/8}$$

$$\ell = K_2 \Psi^{1/4} S^{3/4}$$

$$S_s = K_3 \Psi^{3/2} S^{3/2}$$

$$Q_s = K_4 \Psi^{15/8} S^{13/8}$$

$$\Psi = \Psi_0 + \frac{\partial N}{\partial x}$$

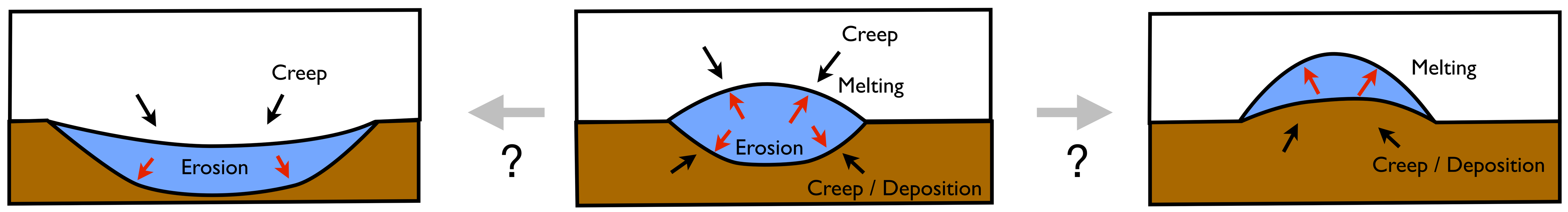
$$\frac{\partial S}{\partial t} + \frac{\partial Q}{\partial x} = \frac{\Psi Q}{\rho_w L} + M$$

$$\frac{\partial S}{\partial t} = \frac{\Psi Q}{\rho_i L} - \mathcal{C} \tilde{A}_i \ell^2 N^n + \frac{\overline{E}}{\rho_s}$$

$$\frac{\partial S_s}{\partial t} + \frac{\partial Q_s}{\partial x} = \overline{E} + M_s$$

- Dynamic channel evolution according to this model has yet to be explored.

Implications for bed erosion



References: Fowler, A.C. & Ng, F.S.L.1996 The role of sediment transport in the mechanics of jökulhlaups, Ann. Glaciol. 22. Kovacs, A. & Parker, G. 1994 A new vectorial bedload formulation and its application to the time evolution of straight river channels. J. Fluid. Mech. 267. Ng, F.S.L. 2000 Canals under sediment-based ice sheets. Ann. Glaciol. 30. Nye, J.F. 1976 Water flow in glaciers: jökulhlaups, tunnels and veins. J. Glaciol. 17. Parker, G. 1978 Self-formed rivers with equilibrium banks and mobile bed. Part 1. The sand-silt river. J. Fluid Mech. 89. Röthlisberger, H. 1972 Water pressure in intra- and subglacial channels. J. Glaciol. 11. Walder, J.S. & Fowler, A.C. 1994 Channelized subglacial drainage over a deformable bed. J. Glaciol. 140.