A continuum model for snow and firn on the surface of ice sheets

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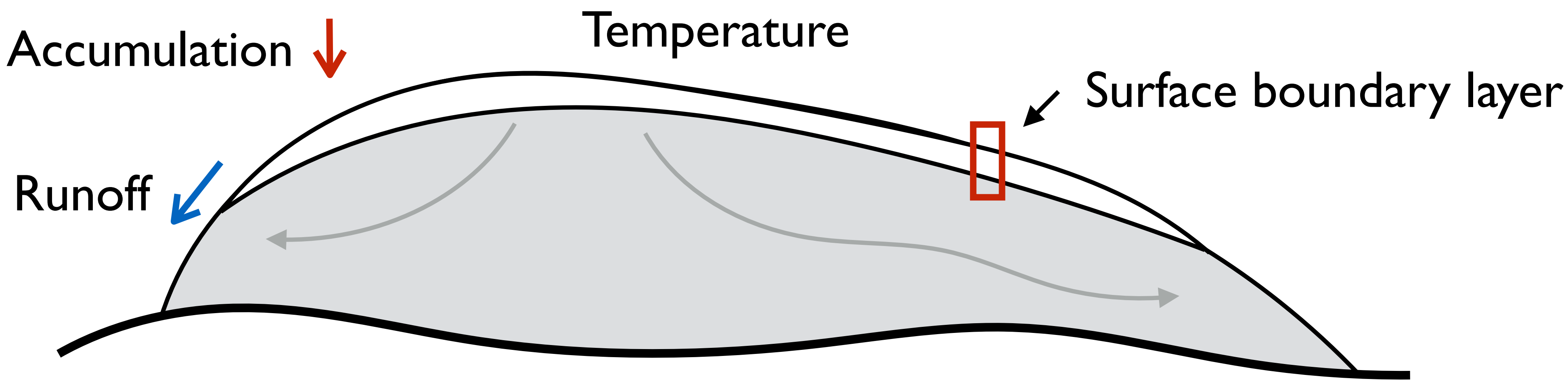
Motivation - boundary conditions for ice-sheet modelling

- Large-scale (long time-scale) evolution of an ice-sheet model requires surface boundary conditions for **mass flux** and **temperature**.

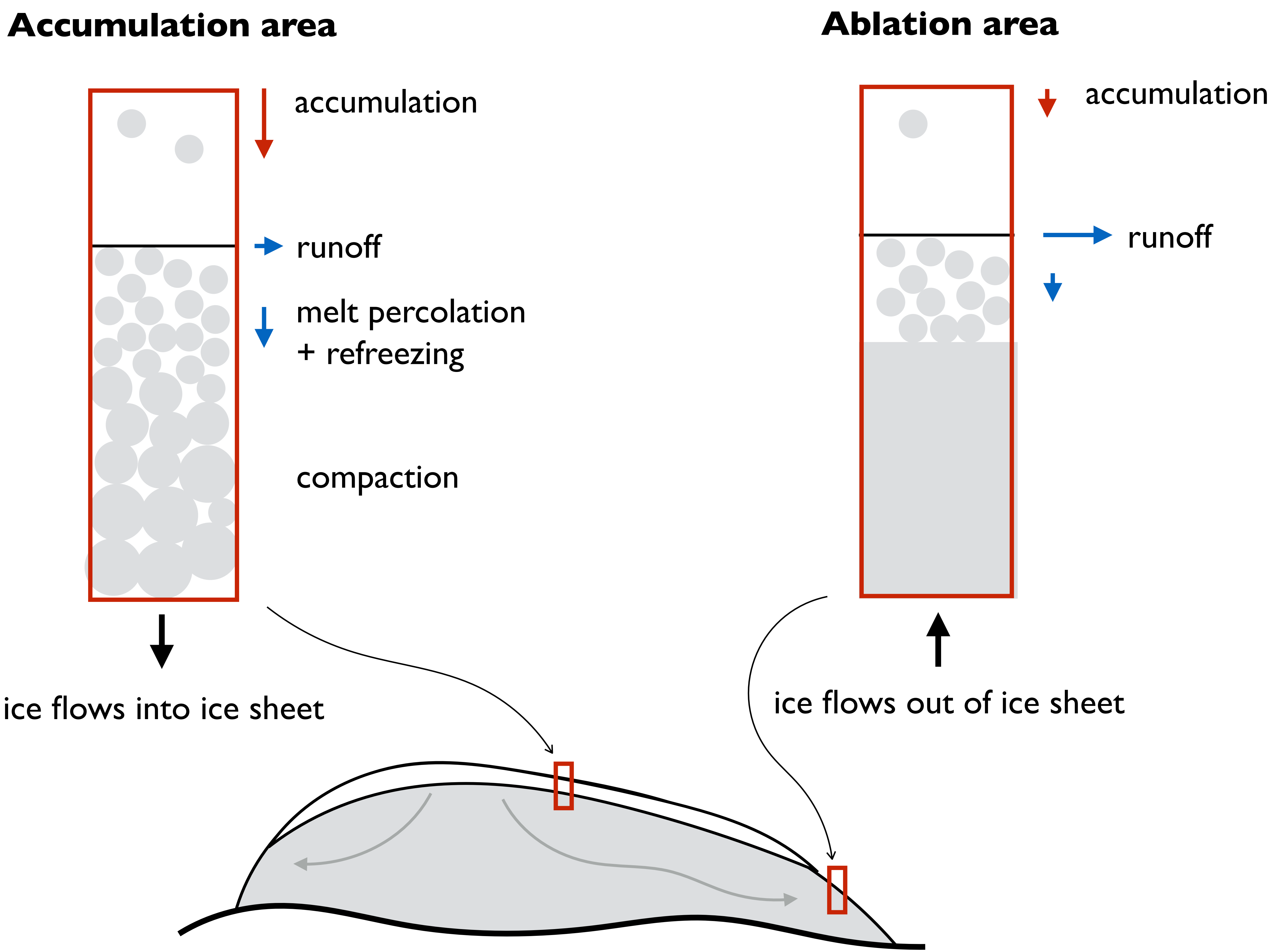


- The snow/firn layer on the surface transmits the ***actual*** surface conditions to ***effective*** surface conditions that the ice-sheet model sees.

- These effective conditions *differ* from averages of the actual surface quantities due to **melt percolation and refreezing**.

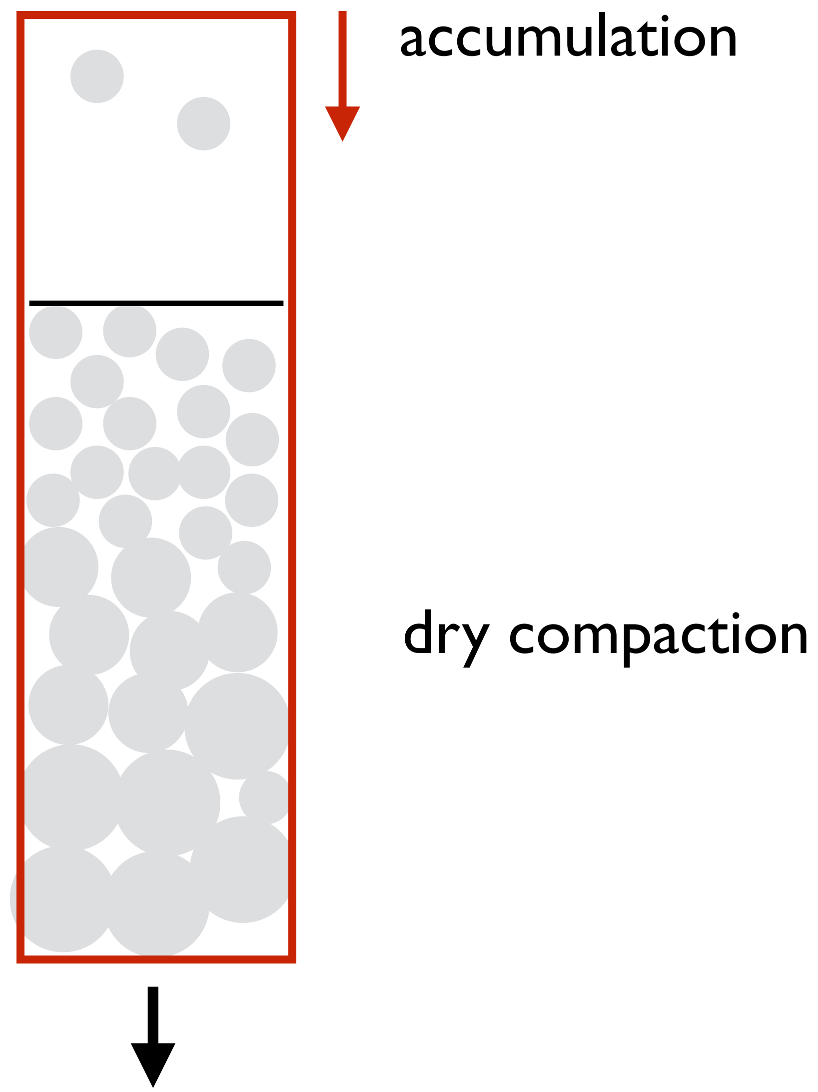
- We develop a continuum model that allows us to calculate the appropriate conditions for given surface forcing. (cf. IMAU-FDM, SNOWPACK, etc)

The near-surface boundary layer



Two easy cases

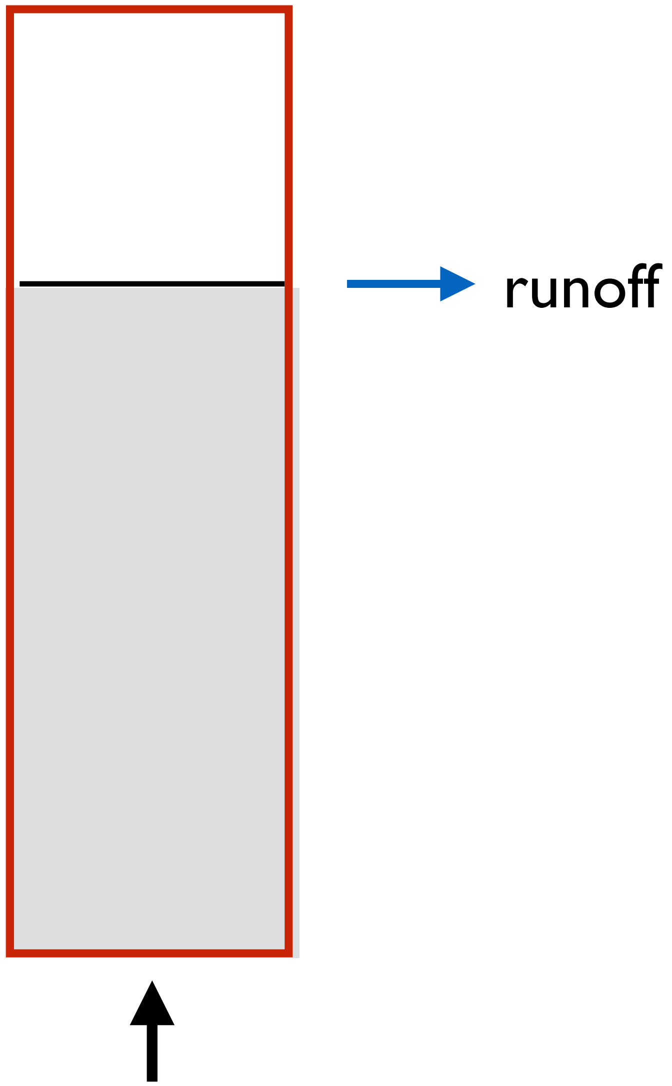
No melting



mass flux = accumulation

temp. = mean surface temp.

No accumulation



mass flux = surface melt rate

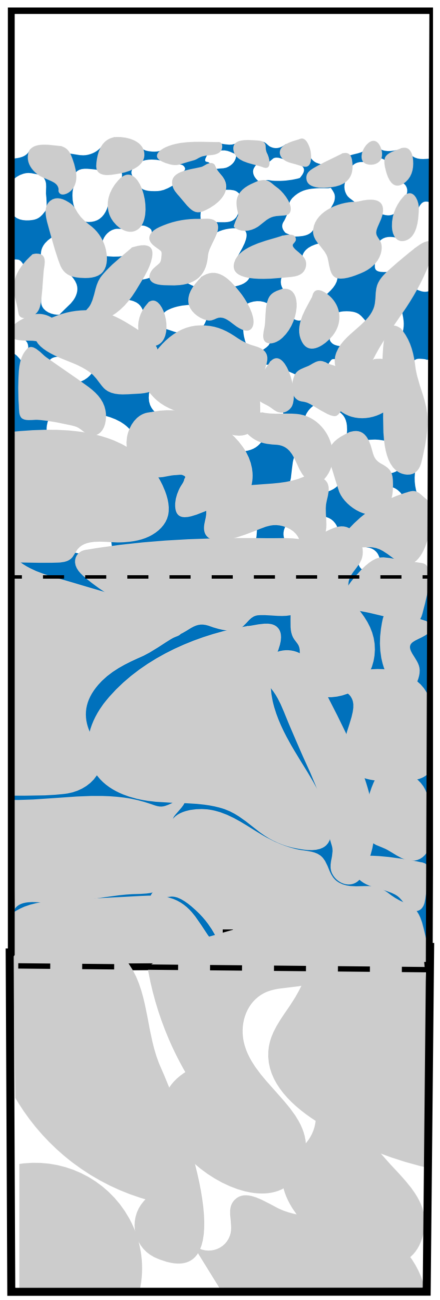
temp. = mean ‘capped’ surface temp.

A continuum model

T Temperature

ϕ Porosity

S Saturation



Conservation equations

$$\frac{\partial}{\partial t}[(1-\phi)\rho_i] + \nabla \cdot [(1-\phi)\rho_i \mathbf{u}_i] = -m$$

$$\frac{\partial}{\partial t}[\phi S \rho_w] + \nabla \cdot [\phi S \rho_w \mathbf{u}_i + \rho_w \mathbf{q}_w] = m$$

← refreezing

Darcy flux

$$\mathbf{q}_w = \frac{k(\phi) k_f(S)}{\eta_w}(\rho \mathbf{g} - \nabla p_w)$$

$$S = 1 \quad \text{or} \quad p_w = -p_c(S)$$

← capillary pressure

Compaction / refreezing

$$\frac{\partial \phi}{\partial t} + \mathbf{u}_i \cdot \nabla \phi = \frac{m}{\rho_i} - \mathcal{C}$$

← compaction rate

$$\mathcal{C} = \frac{p}{\eta_f}\phi \quad \text{or} \quad \mathcal{C} = c_{0,1}(T, a)\,\phi$$

Energy equation

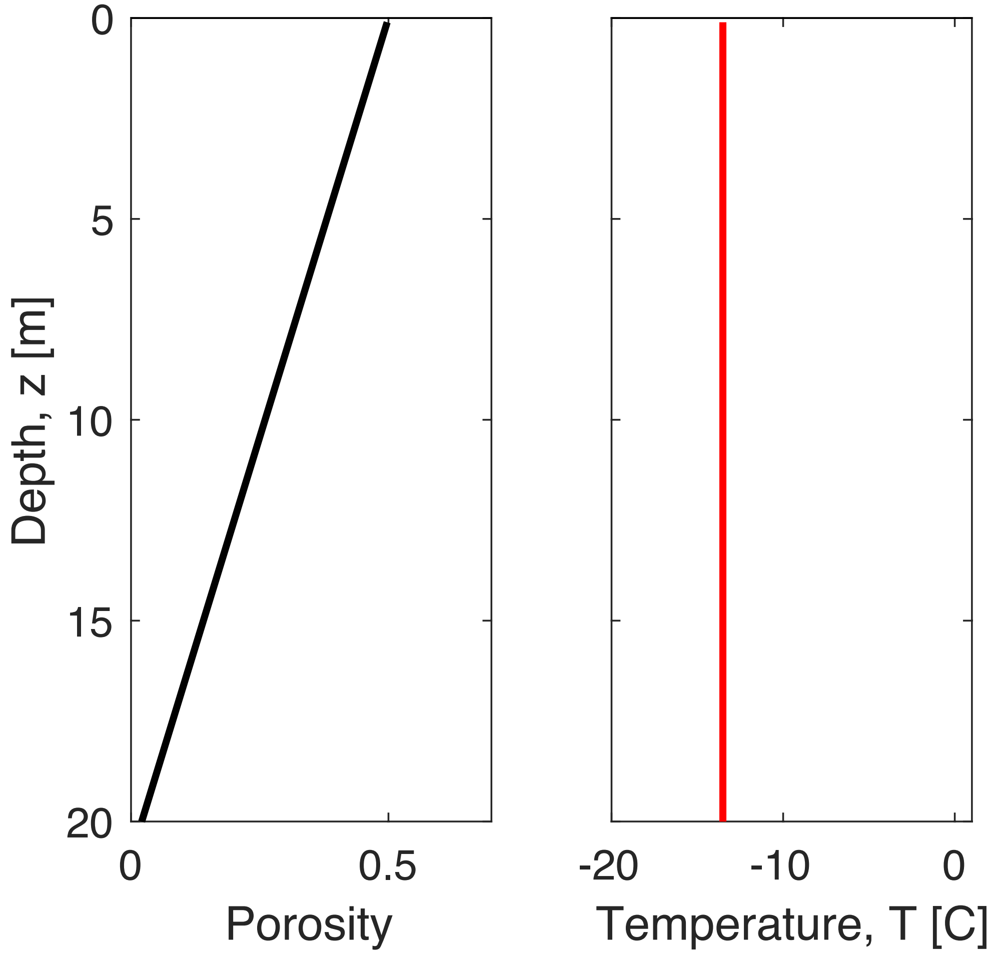
$$\frac{\partial}{\partial t}[(1-\phi)\rho_i c_i T] + \nabla \cdot [(1-\phi)\rho_i c_i T \mathbf{u}_i] = \nabla \cdot [(1-\phi) k_i \nabla T] - mL$$

+ mass- & energy-conserving jump conditions

Meyer & Hewitt 2017 The Cryosphere   see also earlier work by Colbeck, Gray, Morland, and others

Example - surface melting of a cold stratified snow column

'Movie'



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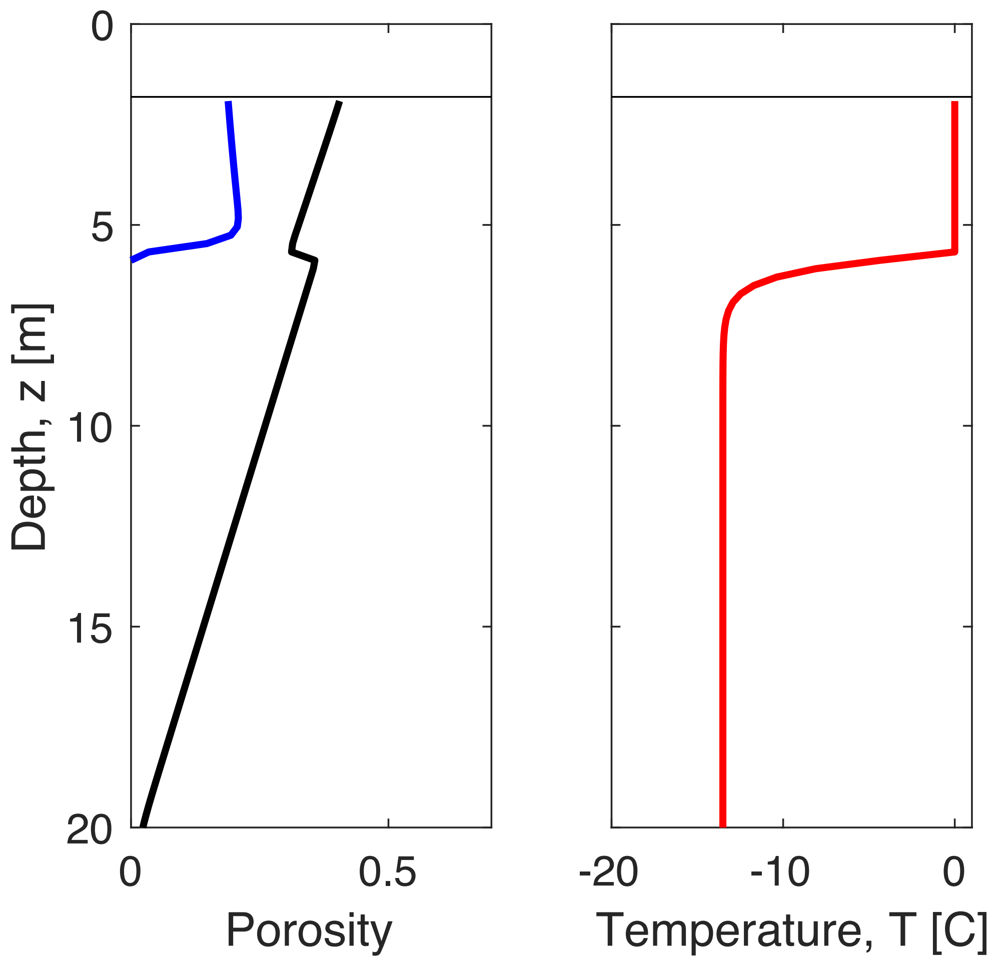

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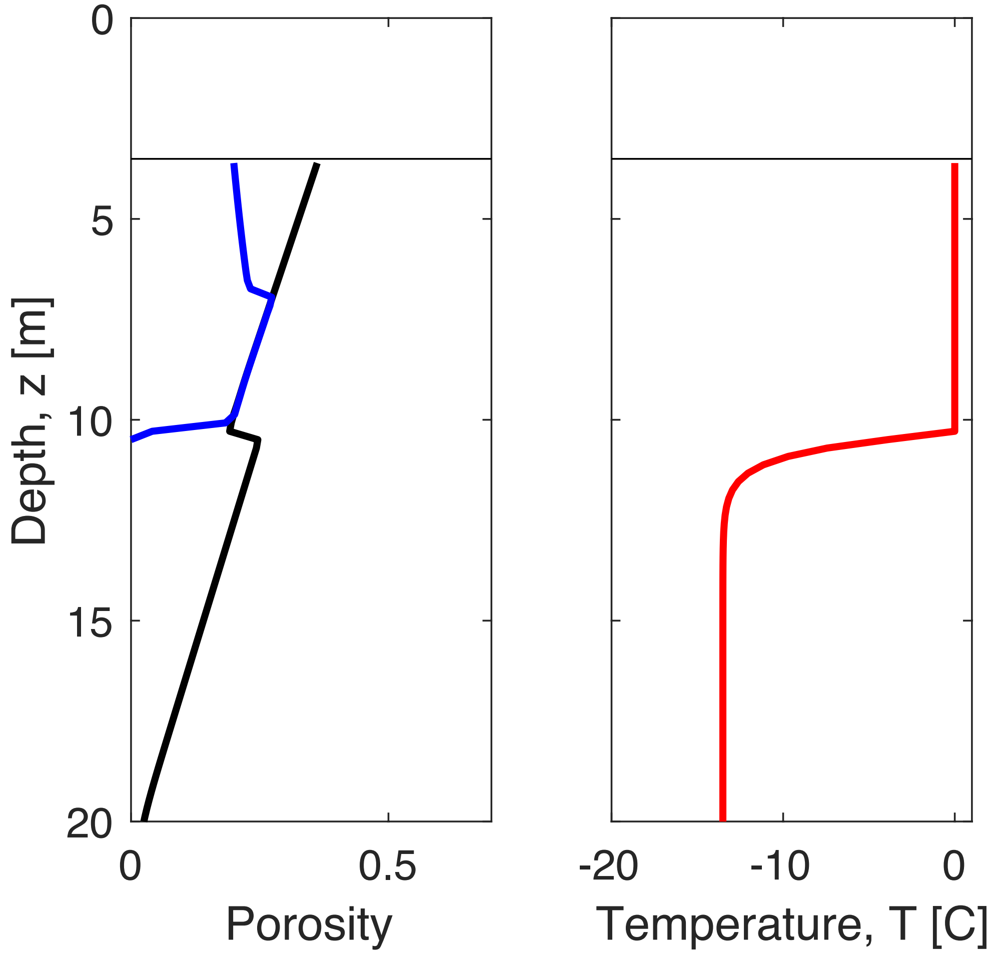

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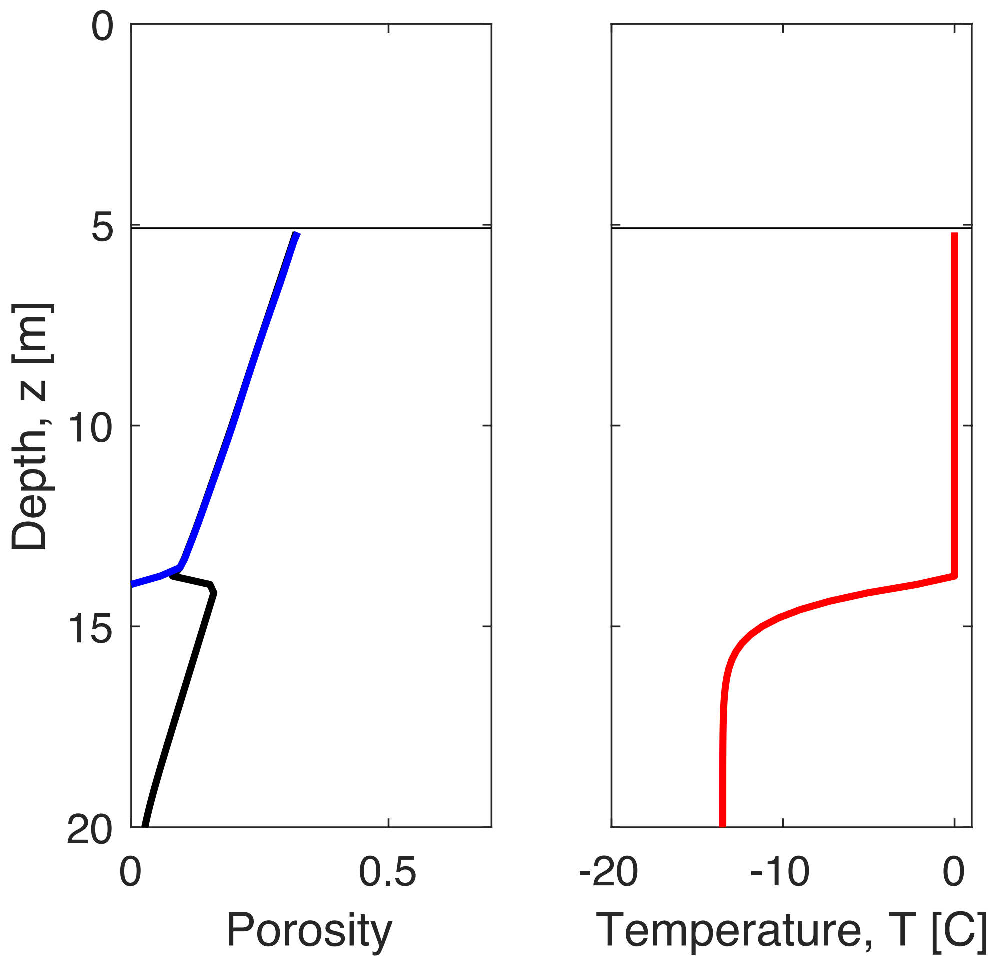

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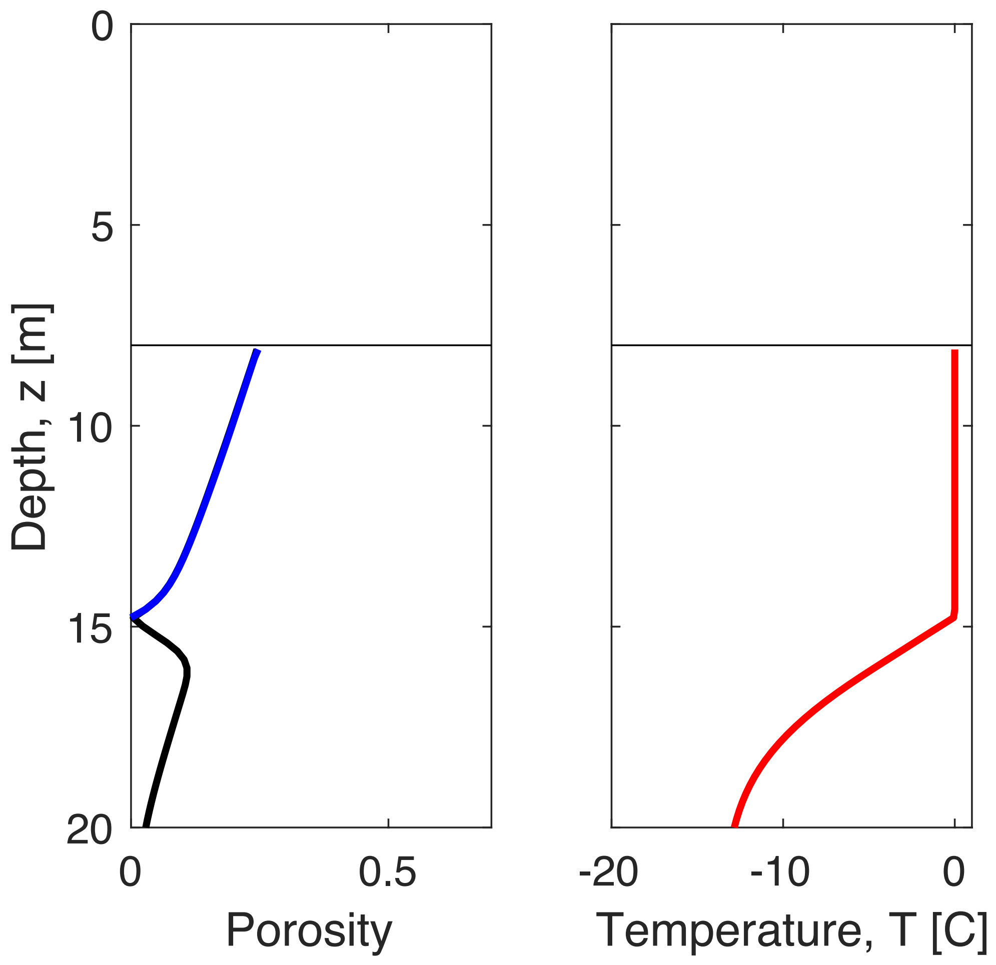

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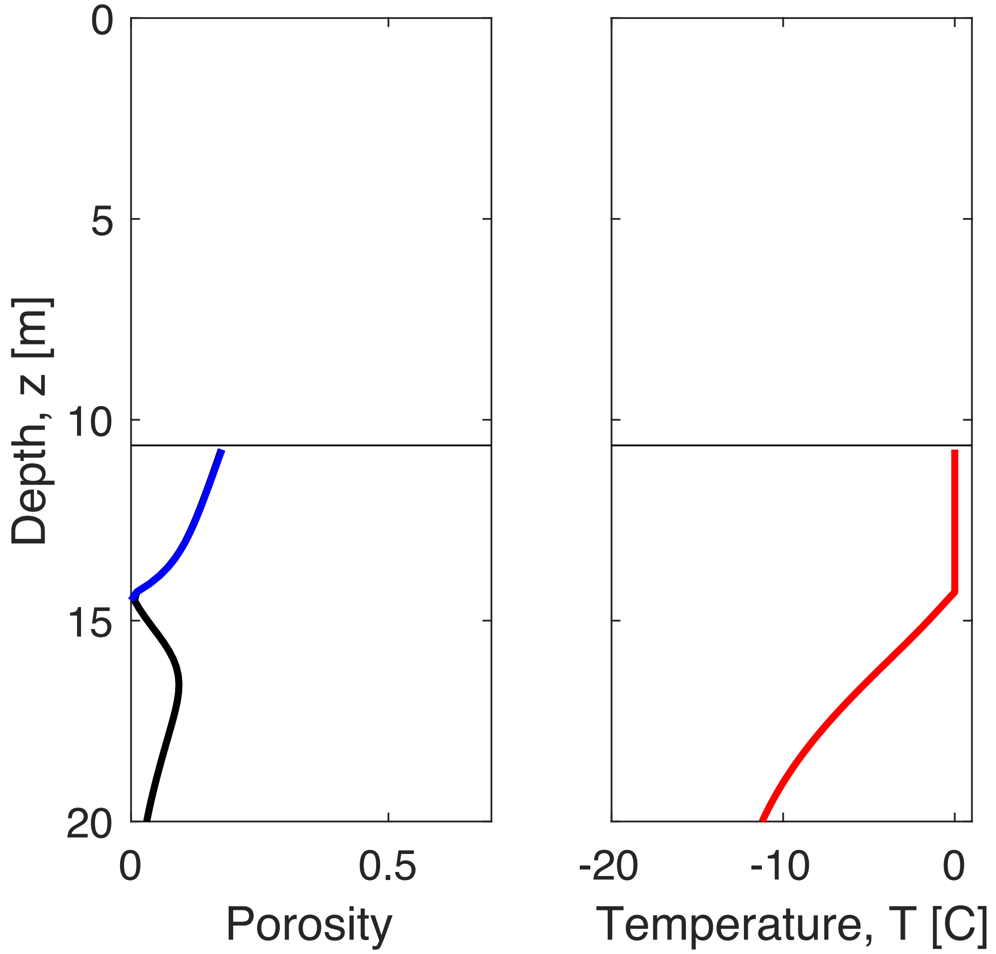

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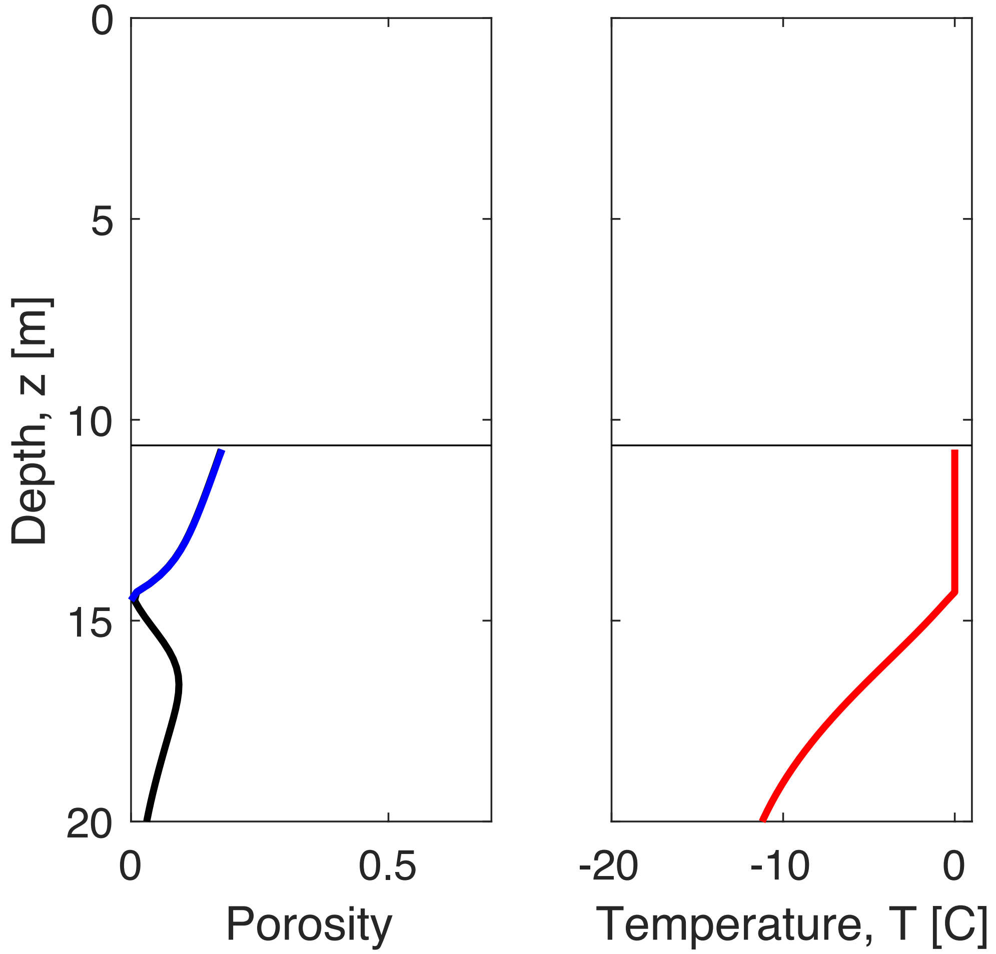
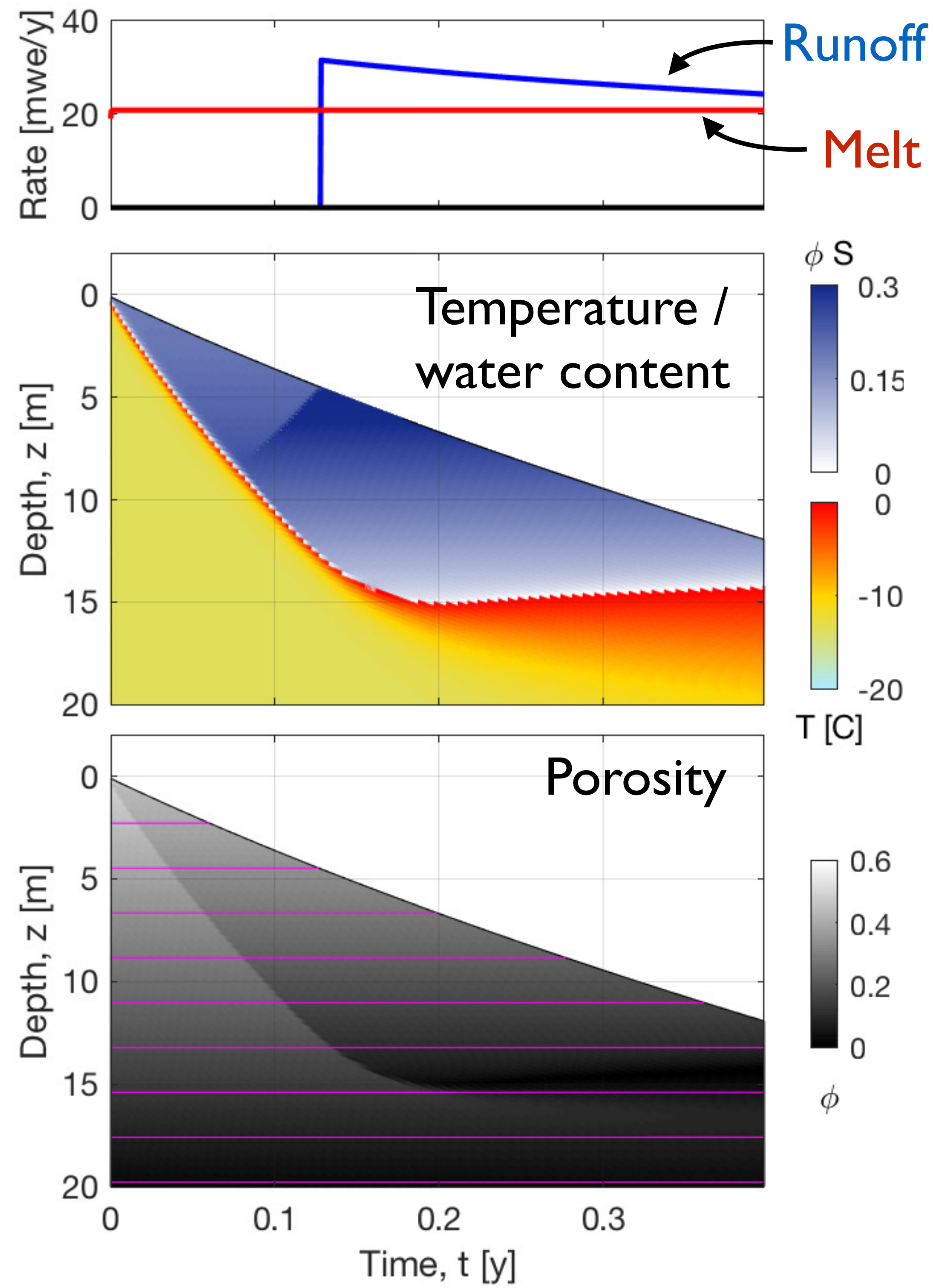
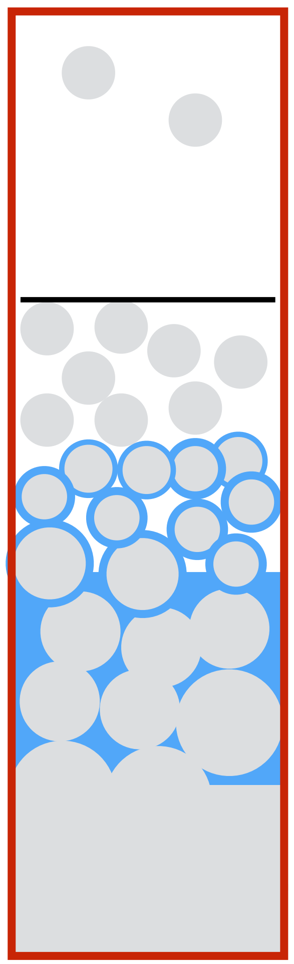


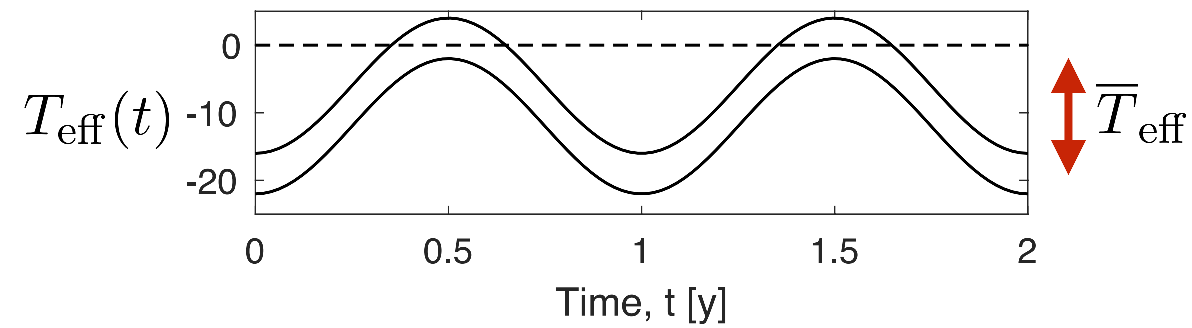


Periodic forcing experiments



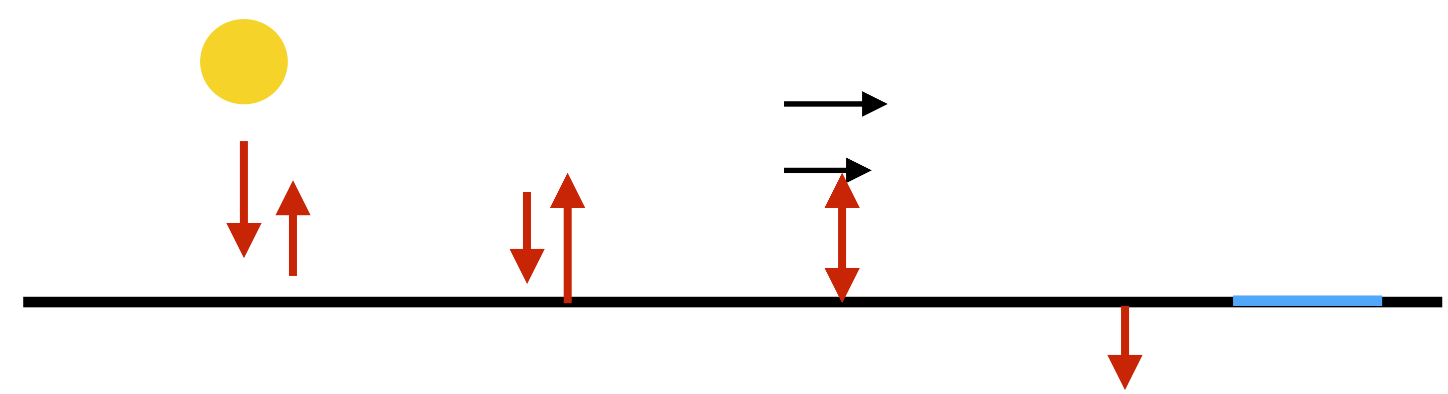
prescribed **accumulation** (constant)

prescribed **forcing temperature**



in linearised surface energy balance

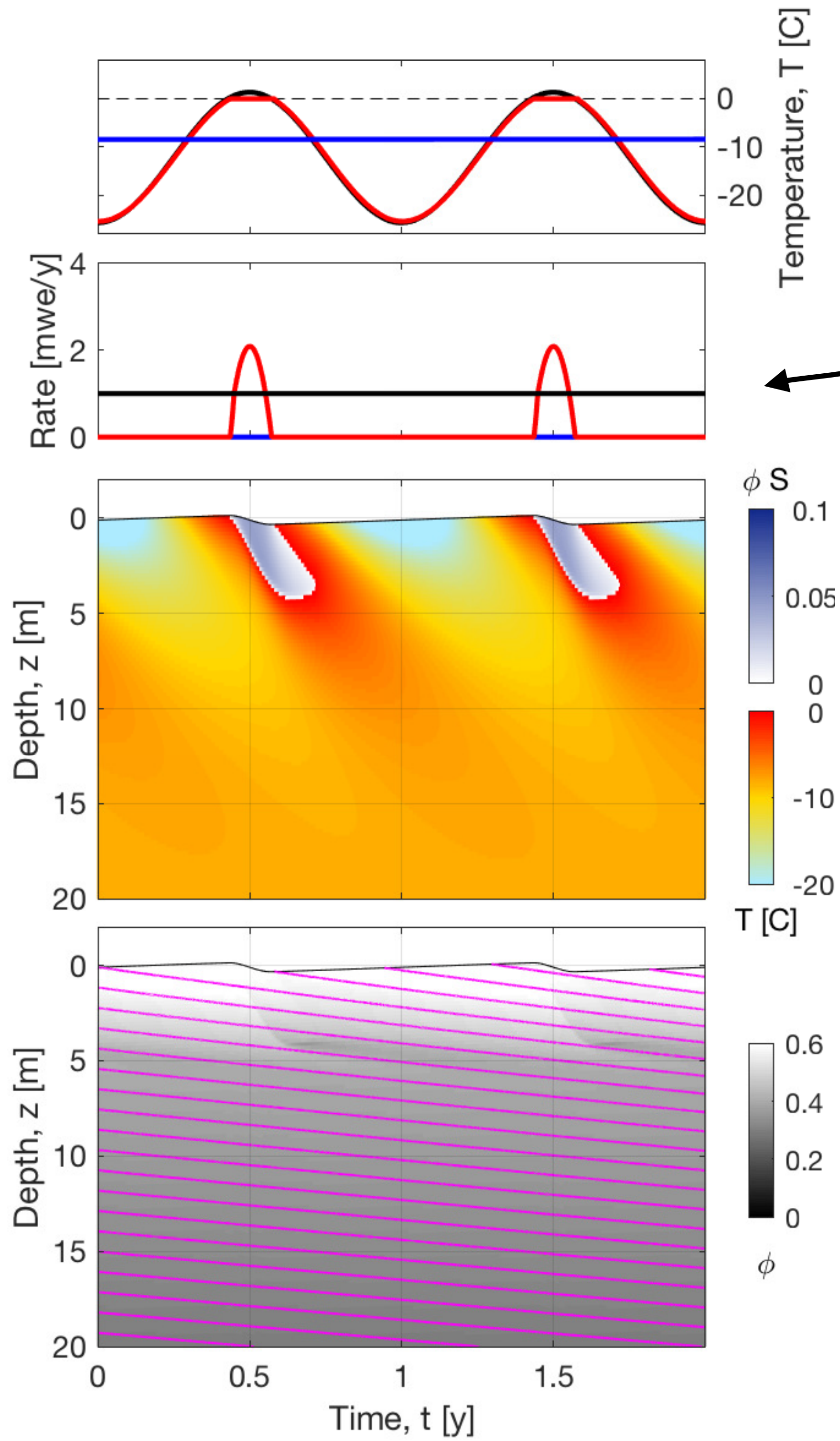
$$h_{\mathrm{eff}}(T_{\mathrm{eff}} - T) = k\frac{\partial T}{\partial z} + m_s L$$



$$Q_S(1-a) + Q_L - \sigma T^4 + h_T(T_a - T) = k\frac{\partial T}{\partial z} + m_s L$$

$$h_{\mathrm{eff}}(T_{\mathrm{eff}} - T)$$

Periodic forcing experiments



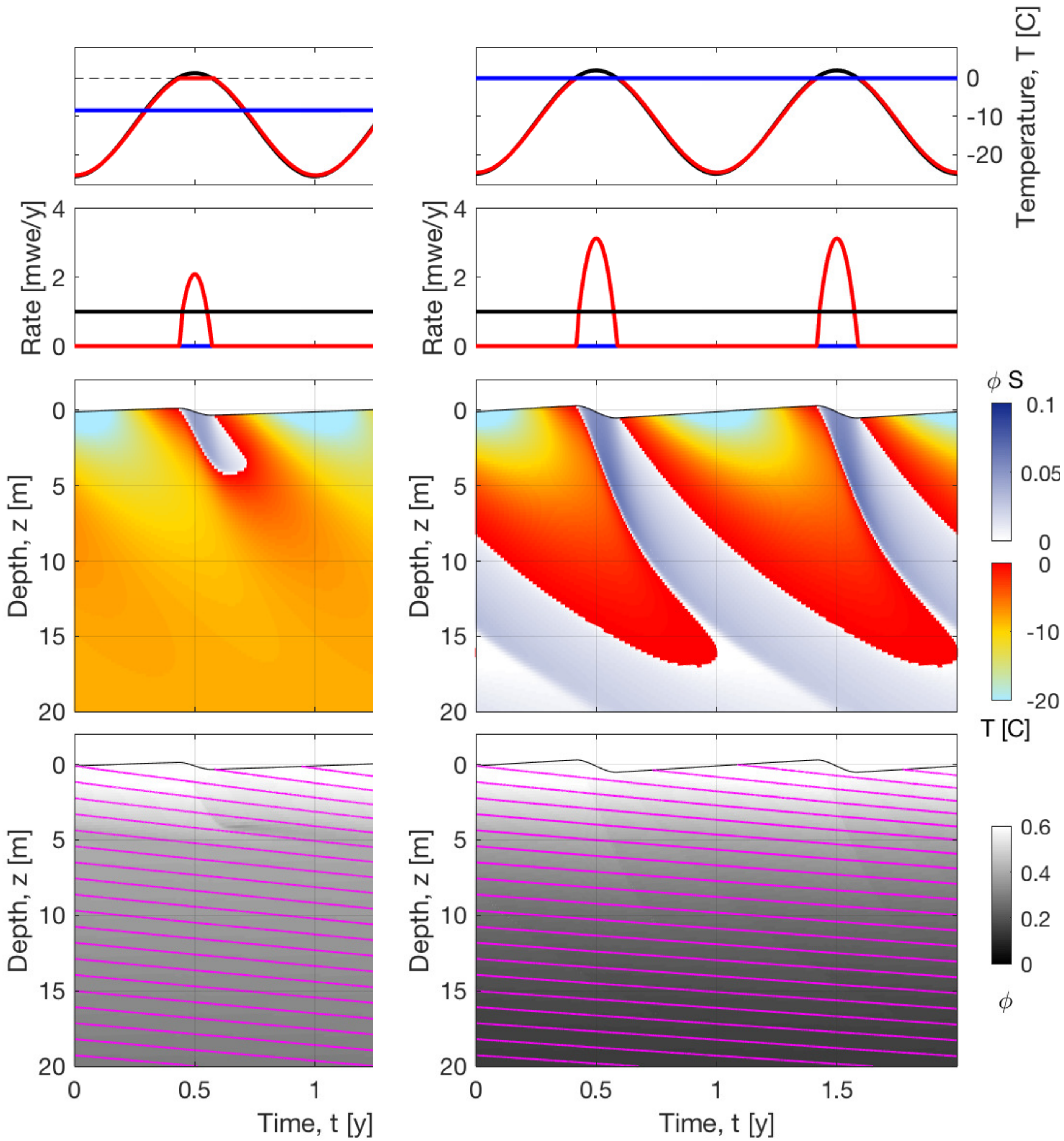
Forcing temp.
Surface temp.
Deep temp.

Melt
Runoff
Accumulation

Temperature / water content

Porosity

Periodic forcing experiments



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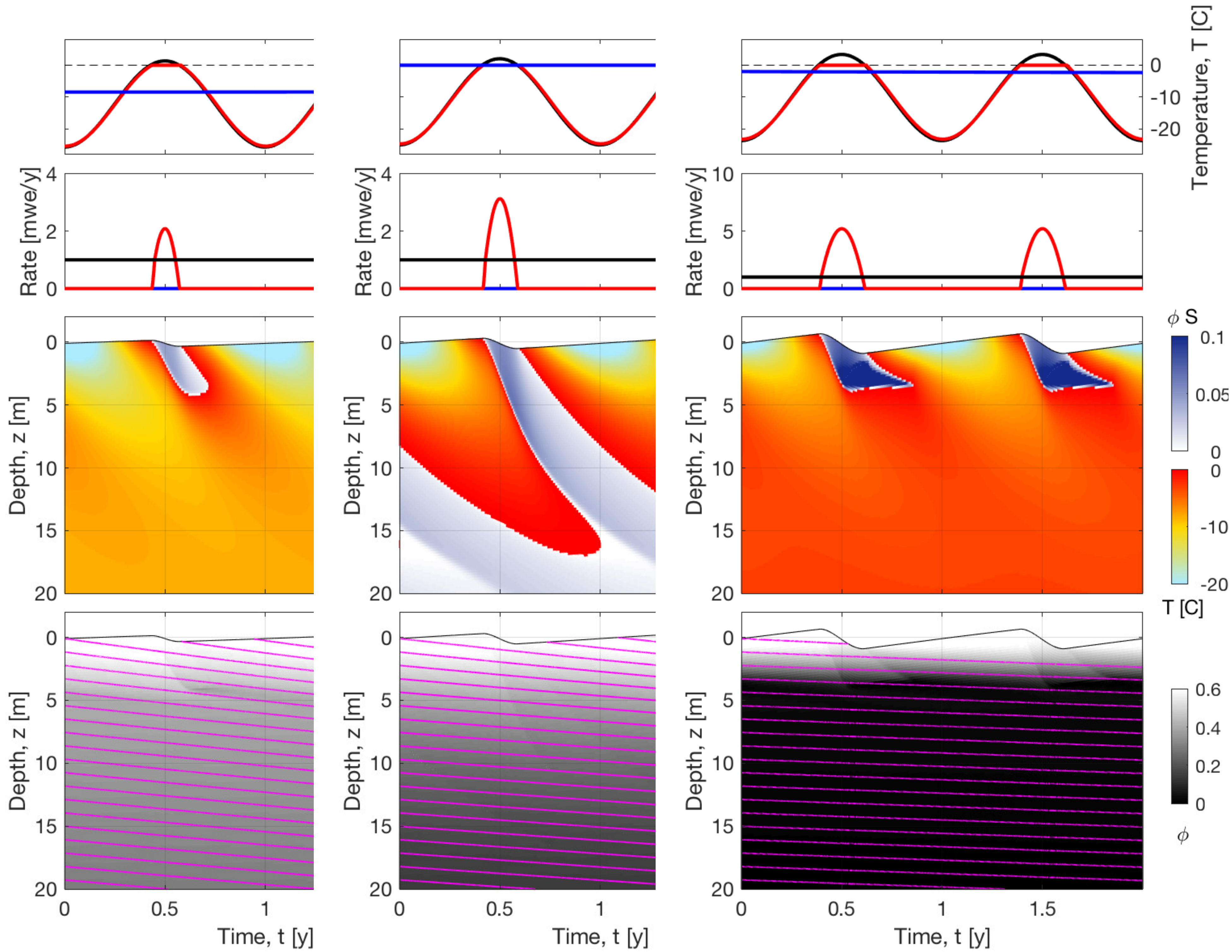

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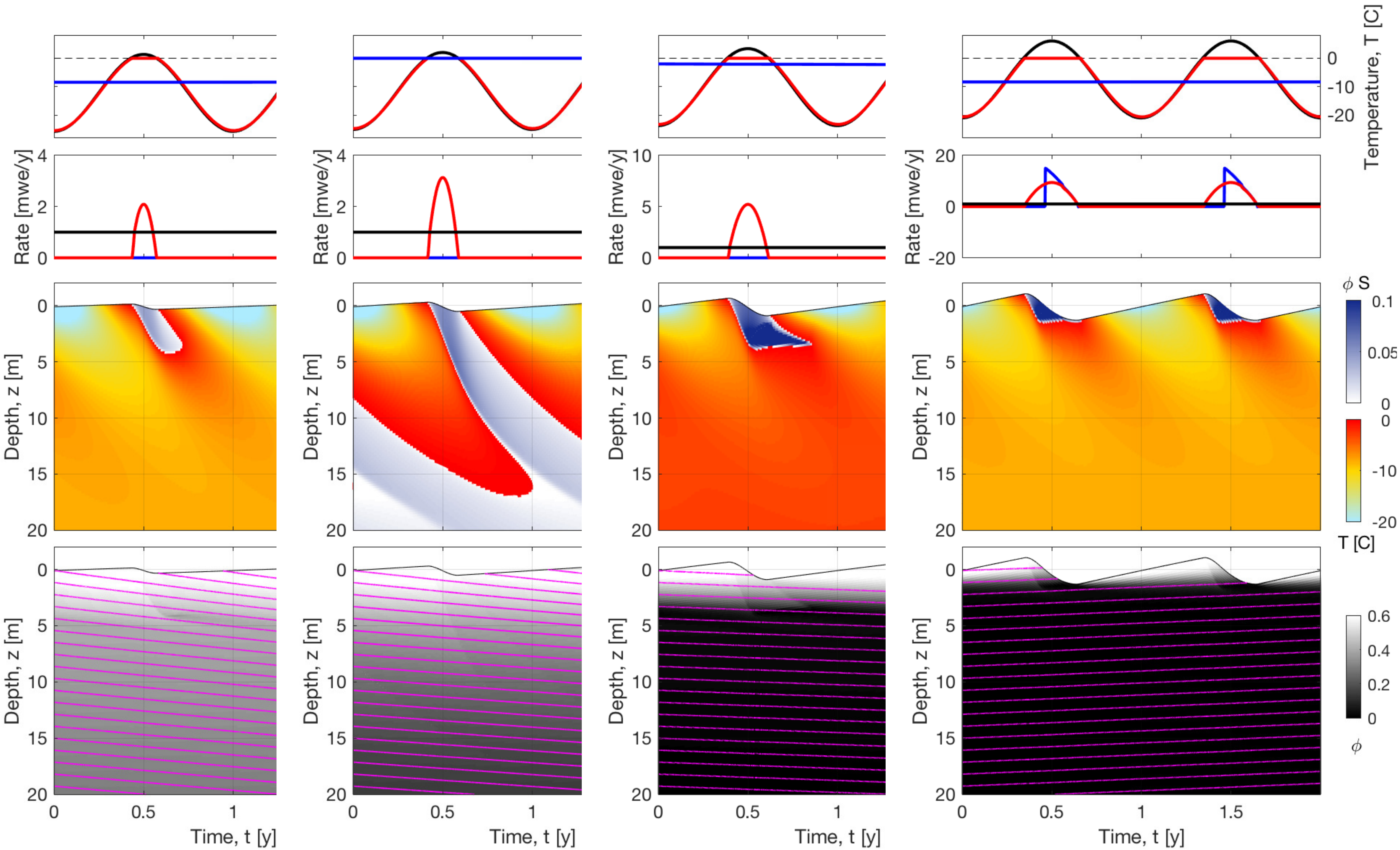
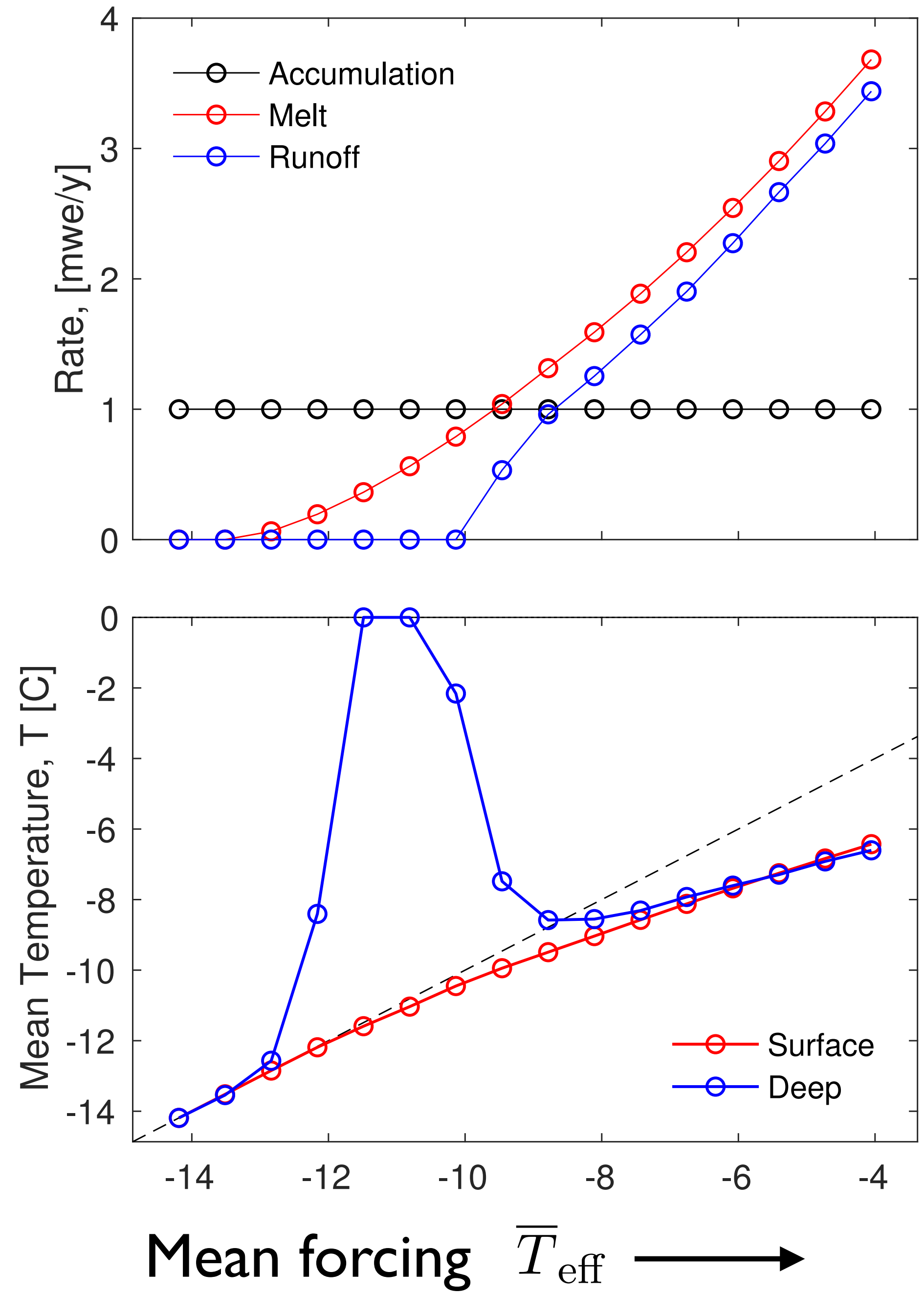


Annual averages (effective conditions)

Mass fluxes

Temperature



- Effective ***surface temperature*** is ***elevated*** and ***runoff*** is ***buffered*** over an intermediate range of thermal forcing. The size and location of that range depend on accumulation rate.

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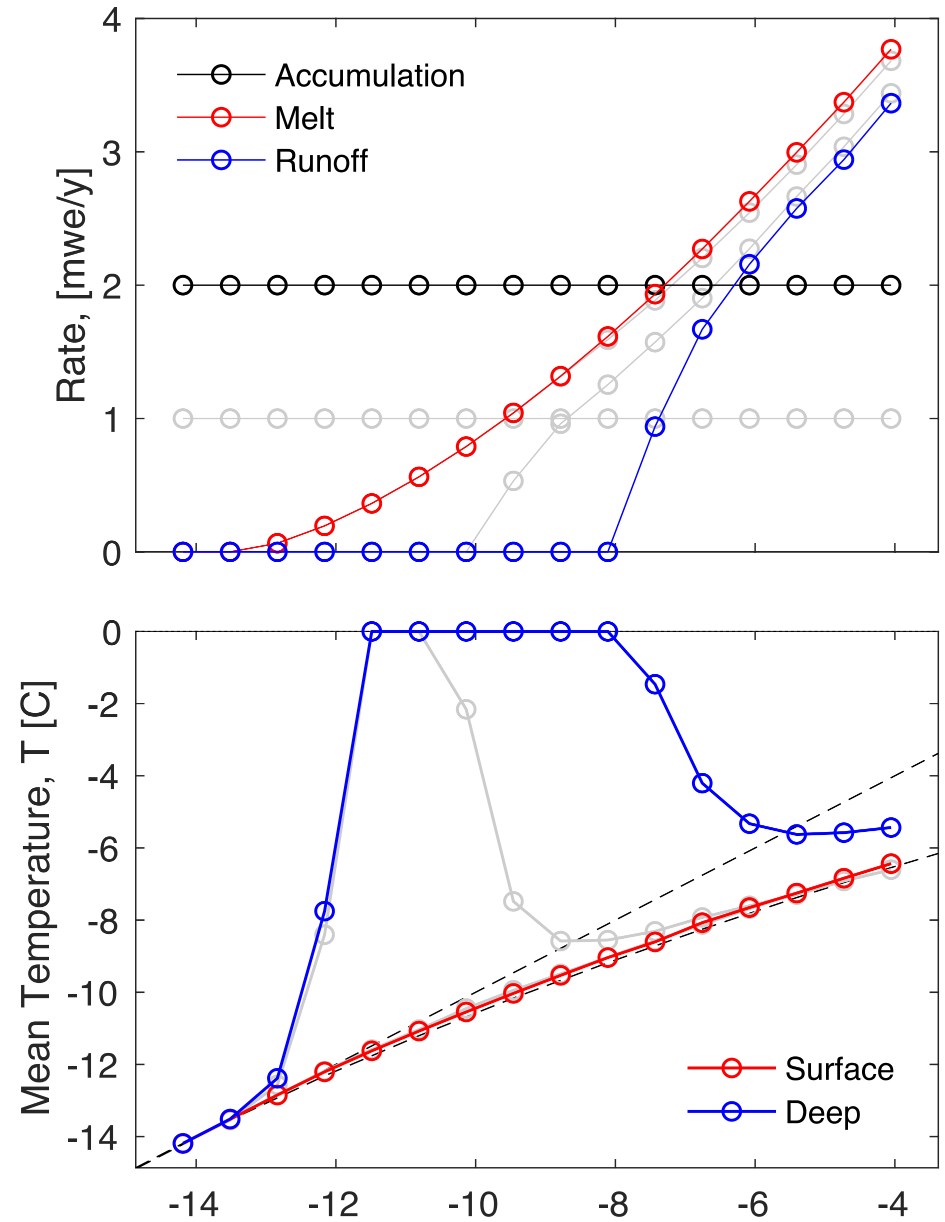

Mean forcing $\overline{T}_{\mathrm{eff}}$ ⟶

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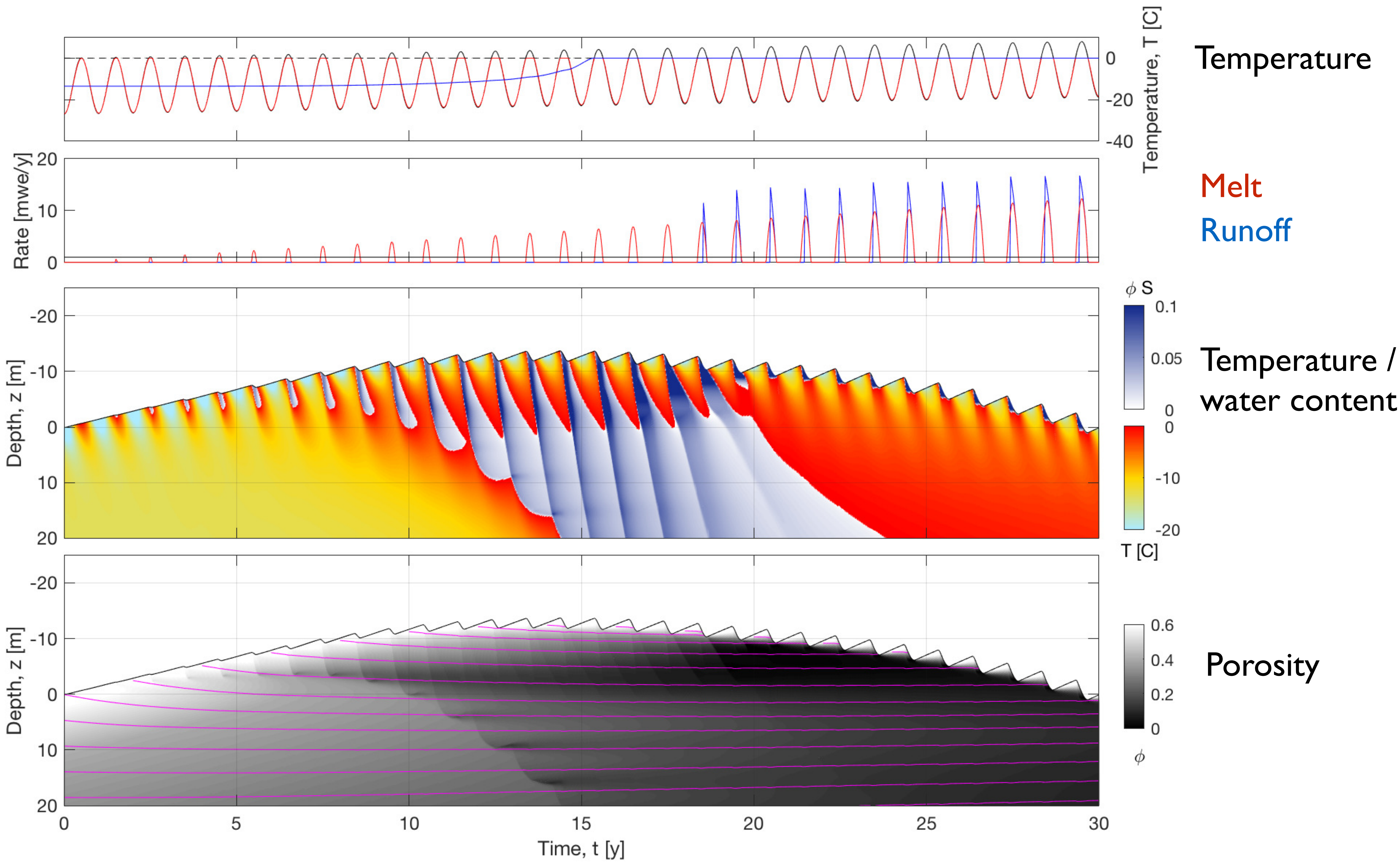
Response to a gradual warming



Summary

We developed a continuum model that describes melt percolation, refreezing and compaction.

We calculate **average properties** (melt, runoff, temperature) for periodic surface forcing.

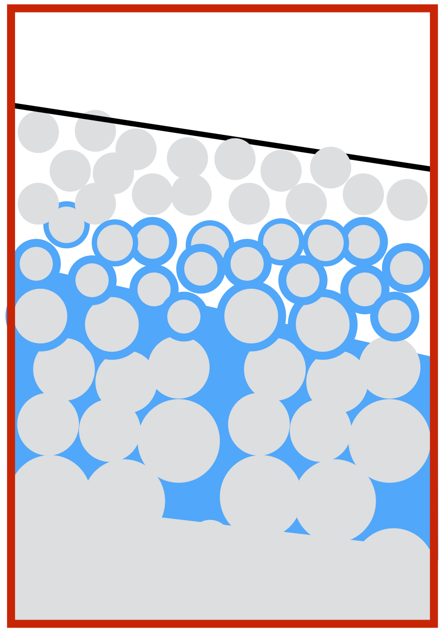
Elevated firn temperature and **buffered runoff** are found for intermediate thermal forcing, depending on accumulation rate (even in steady state).

Gradual changes in forcing produce complicated evolution - water storage and the initiation of runoff are highly sensitive to specifics of the forcing.

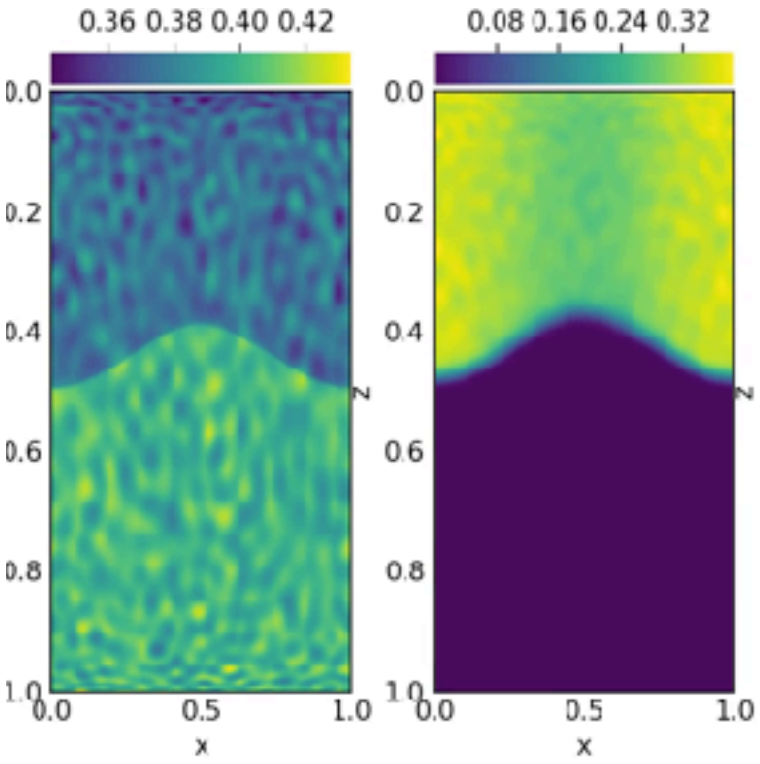
Future directions

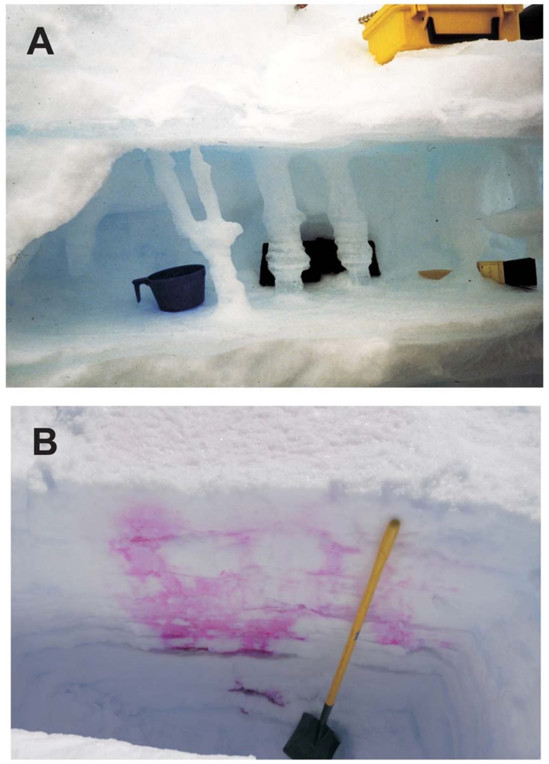
- Compare model with other approaches and with observations.

- Extend to more dimensions
  - lateral flow naturally occurs in this model if the surface or water table is sloped.



- Heterogenous percolation
  - in principle the model can produce high permeability 'pipes'.





Humprey et al 2012

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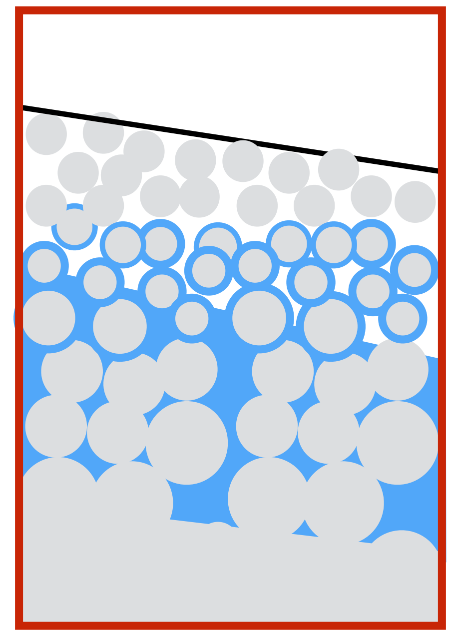

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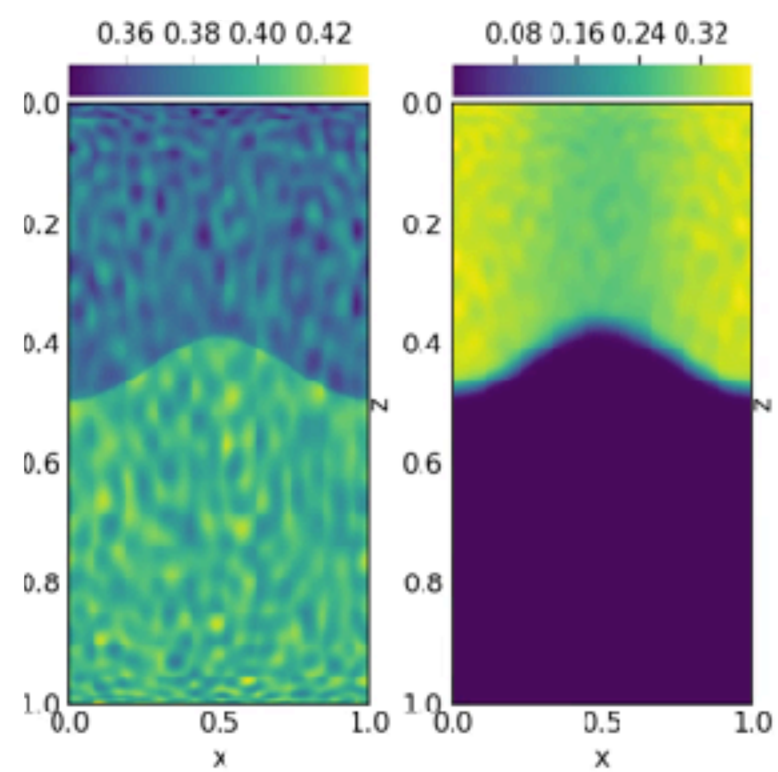

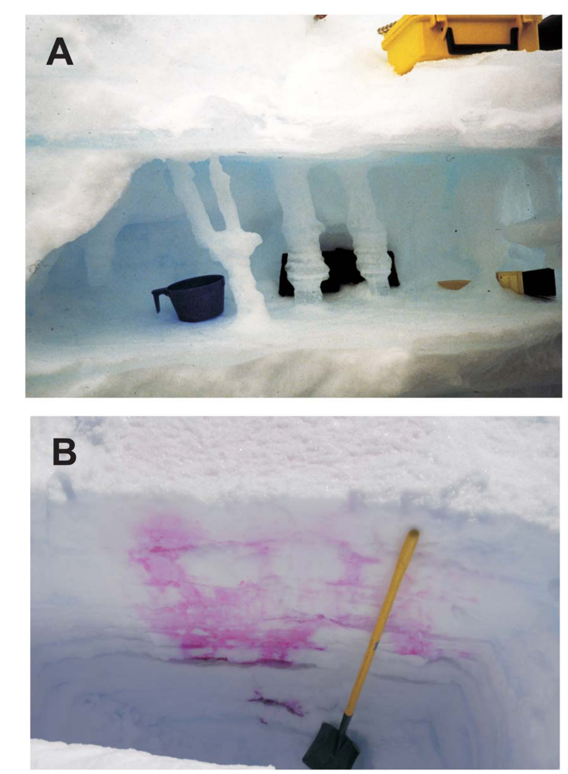

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