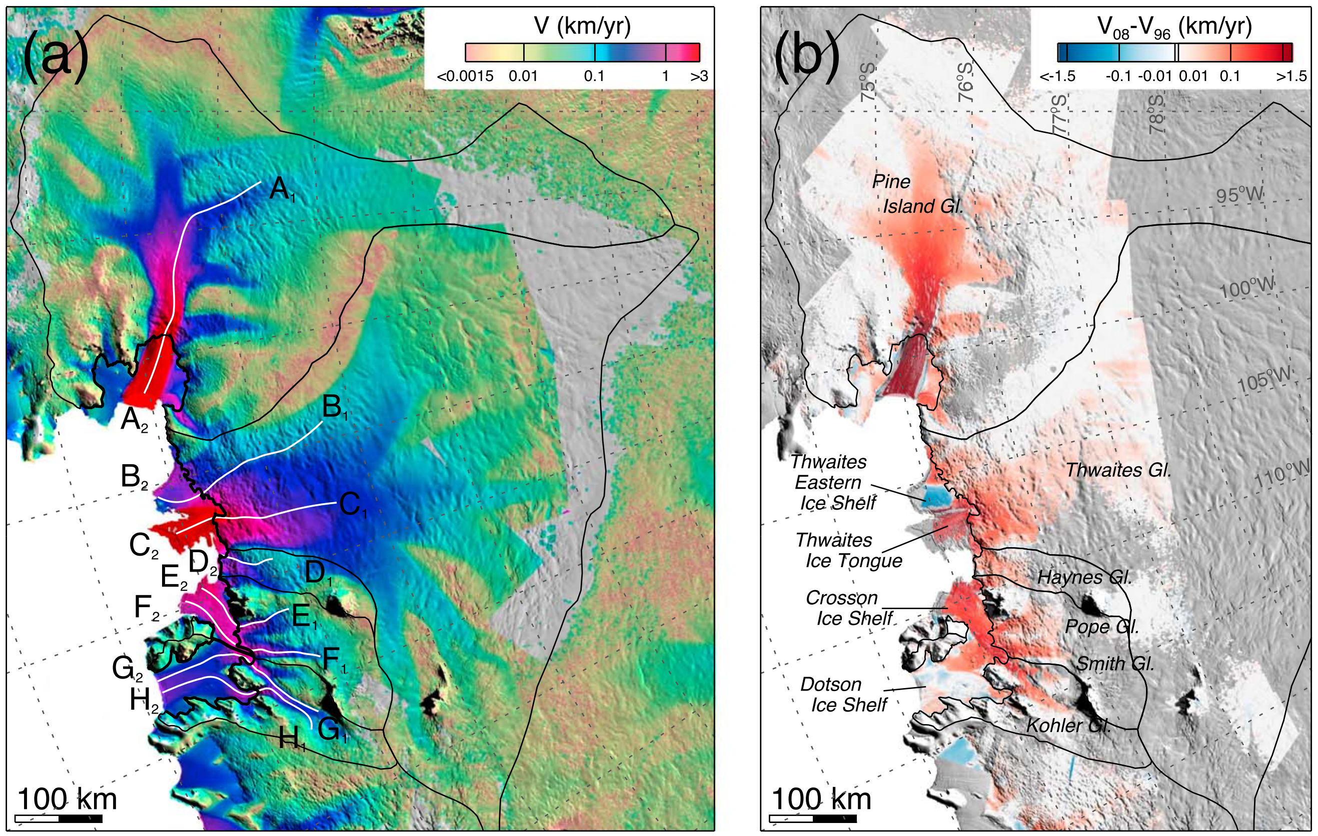
Ice sheets with rapid basal sliding

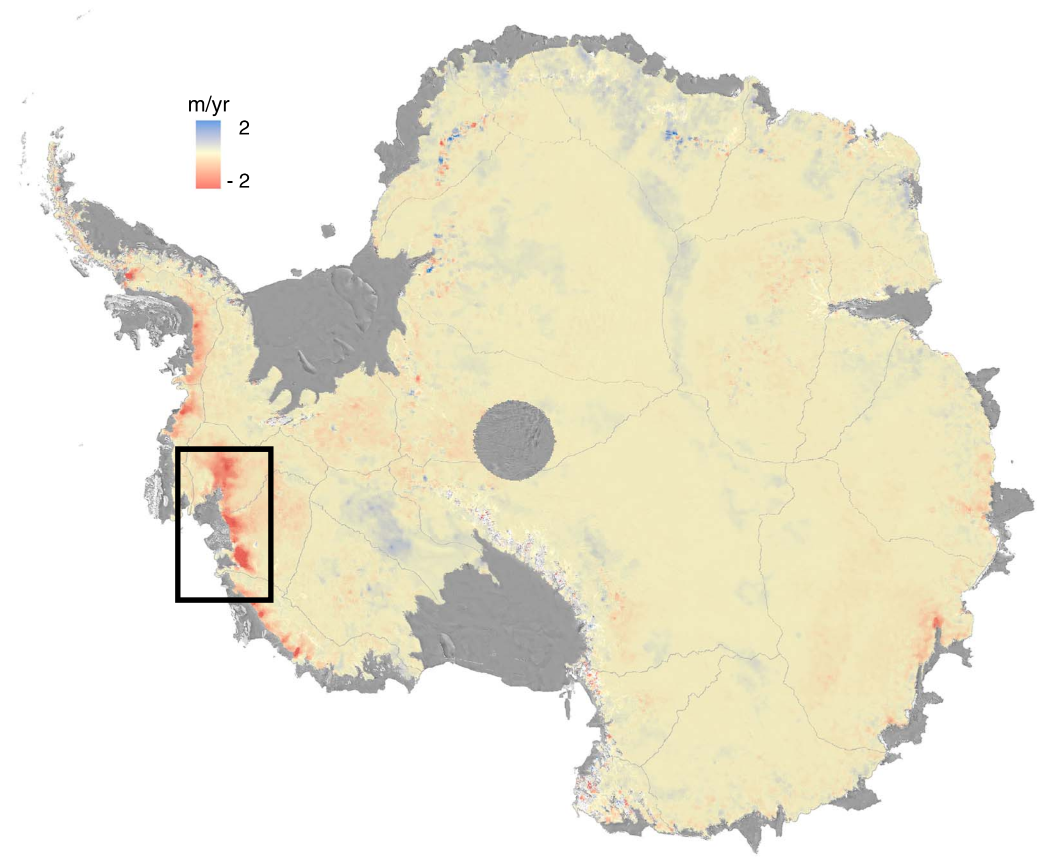
Ian Hewitt, University of Oxford

Antarctic ice sheet

Ice near the margins has accelerated substantially over the last decade



Mouginot et al 2014



McMillan et al 2014

Marine Ice Sheet Collapse Potentially Under Way for the Thwaites Glacier Basin, West Antarctica

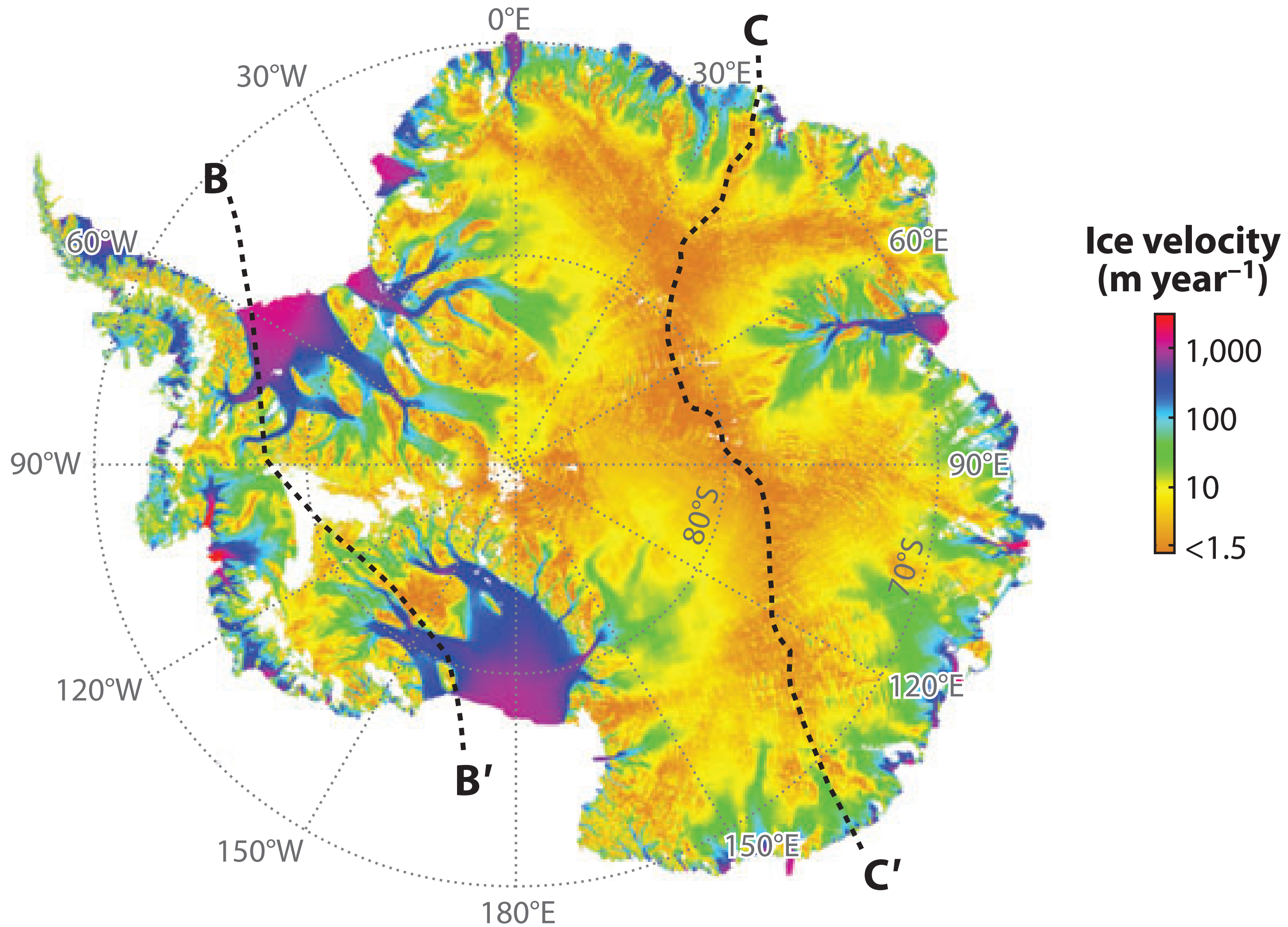
Ian Joughin, Benjamin E. Smith, Brooke Medley

Collapse of the West Antarctic Ice Sheet after local destabilization of the Amundsen Basin

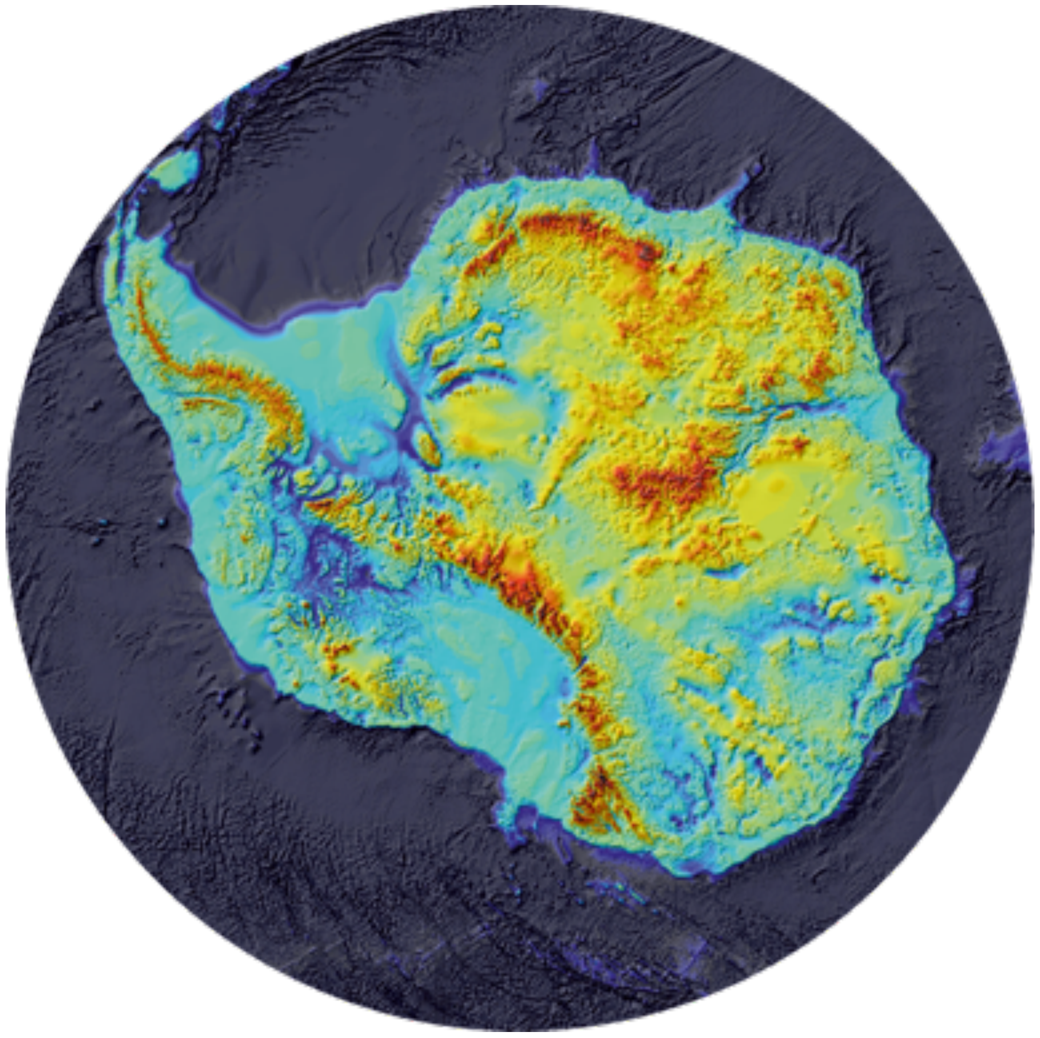
Johannes Feldmann[a,b] and Anders Levermann[a,b,1]

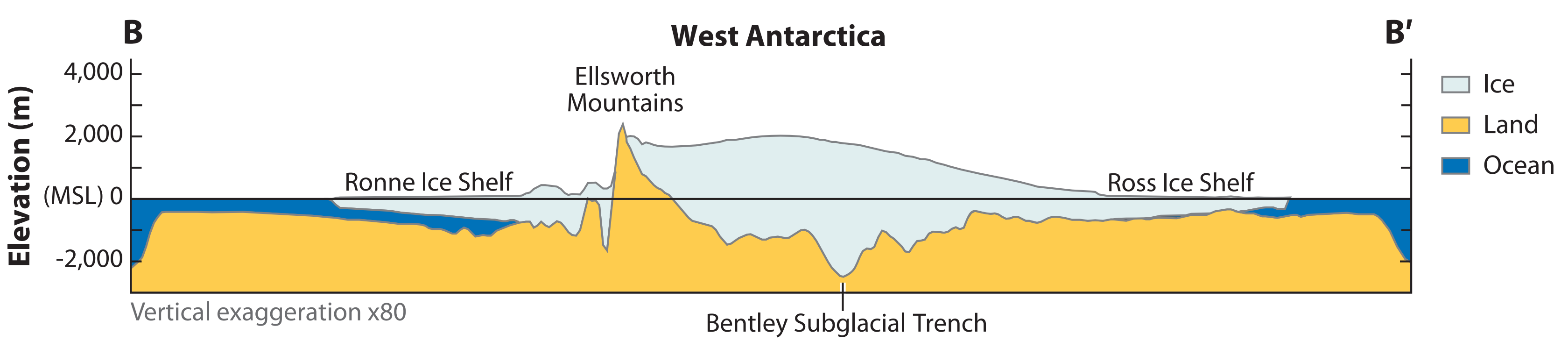
Antarctic ice sheet

Ice speed



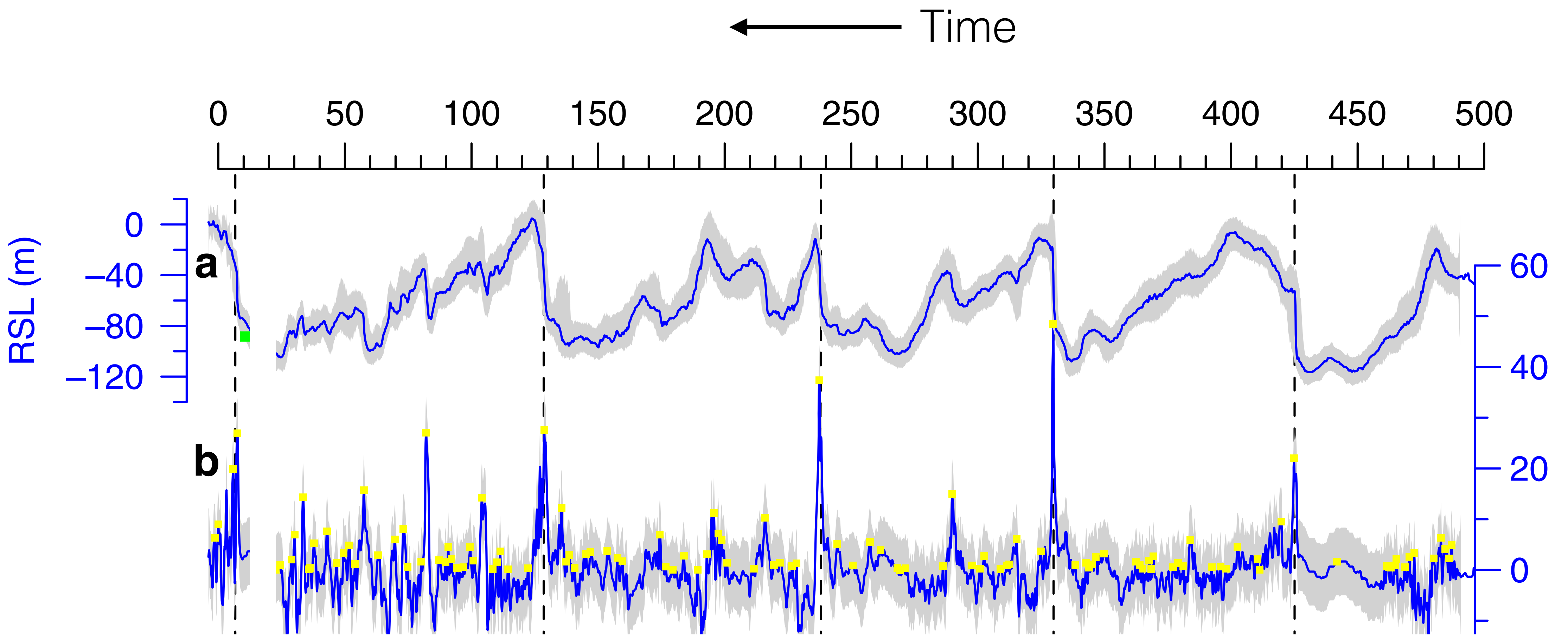
Bed elevation





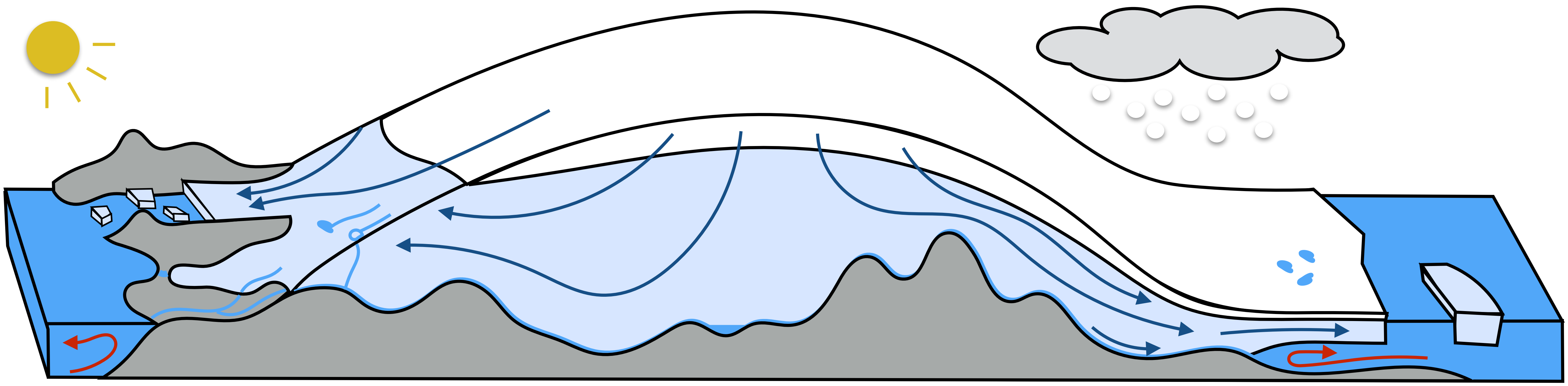
Sea level

The glacial period is punctuated by several periods of rapid sea level rise (~1m/century)



Grant et al 2014

Red Sea Relative Sea Level



Observations show rapid changes in ice dynamics can occur

→ What **mechanisms** cause massive ice loss, and **how rapid**?

The Greenland and Antarctic ice sheets contain ice equivalent to around **65m sea level**

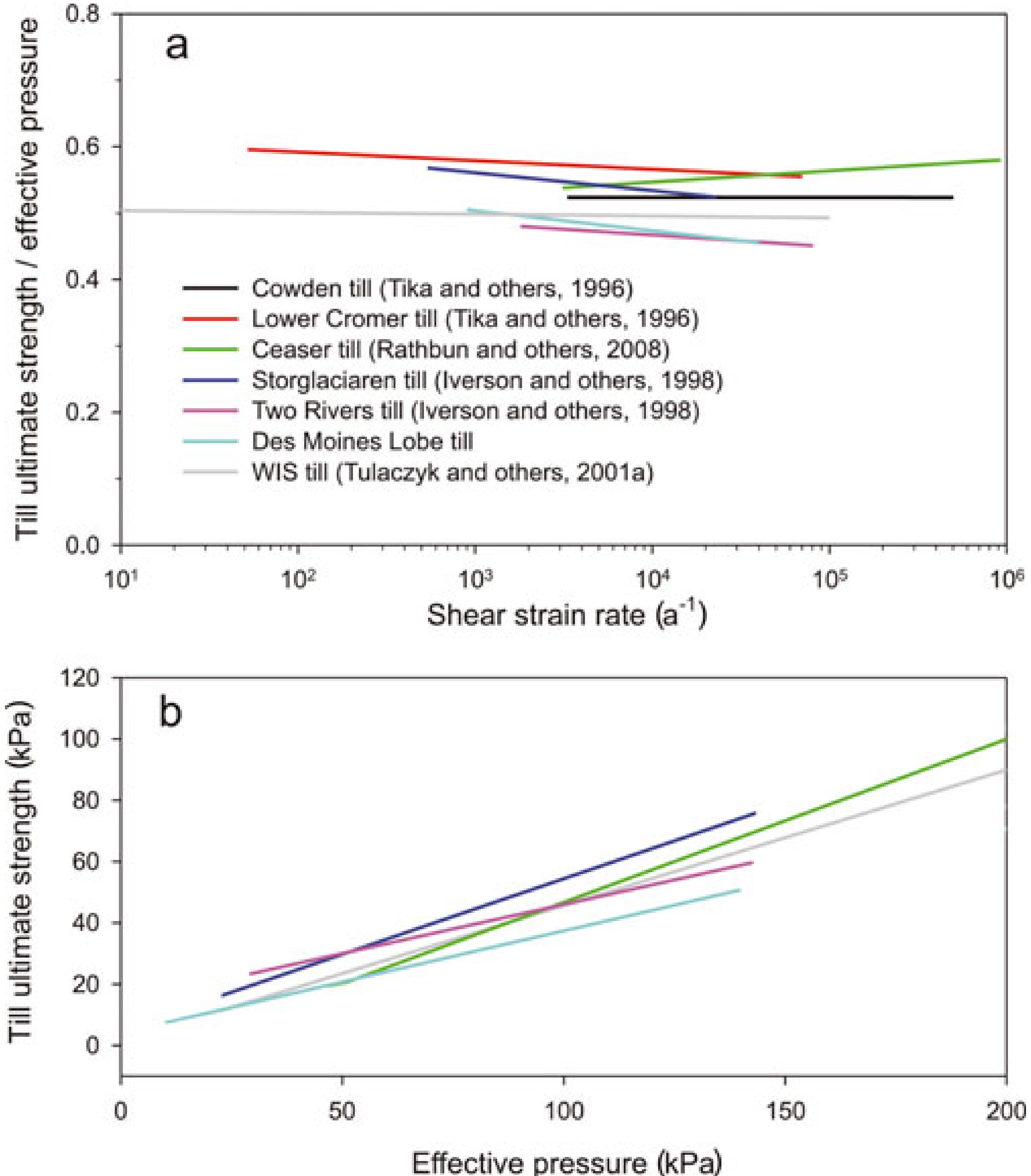
Basal sediments

Most fast-moving ice is thought to be underlain by water-saturated sediments

Laboratory experiments on till samples suggest very weak dependence of stress on strain rate

Shear strength depends on effective pressure (i.e. on pore pressure)

$$\tau_0 = c + p_e \tan\phi$$



Iverson 2010

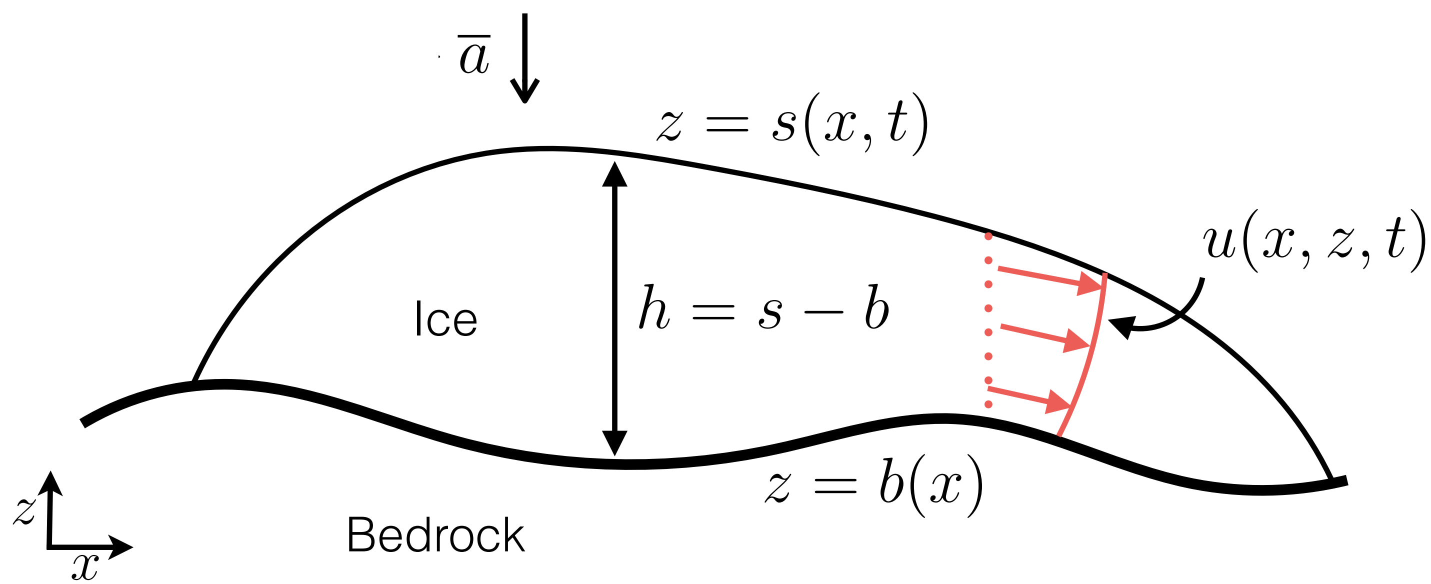
→ **This talk:** Explore the dynamics of an ice sheet with a **perfectly plastic bed**

Glacier flow

Extreme Ice Survey - Time-lapse camera
Khumbu glacier, Nepal

~10,000,000 x real time

A simple ice-sheet model



Net accumulation - melting (climate forcing)

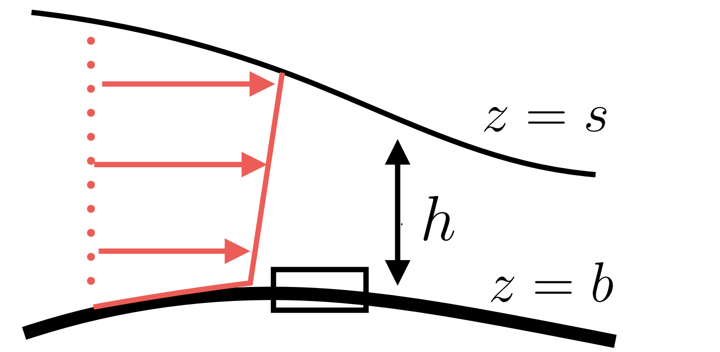
Mass conservation $$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = \overline{a}$$

Ice flux $$q(x,t) = h\overline{u} = \int_b^s u \; \mathrm{d}z$$

Force balance and boundary conditions (Stokes flow) $\longrightarrow$ $u(x,z,t)$

Sliding at the bed

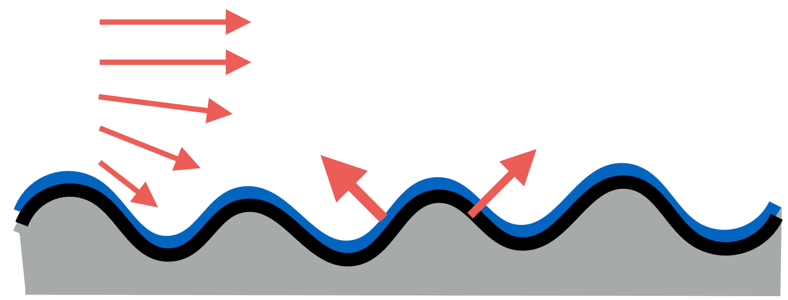
The fastest motion occurs as a plug flow $u(x,z,t) \approx \overline{u}(x,t)$



Two mechanisms for sliding -

- a **thin film of water** between ice and bedrock

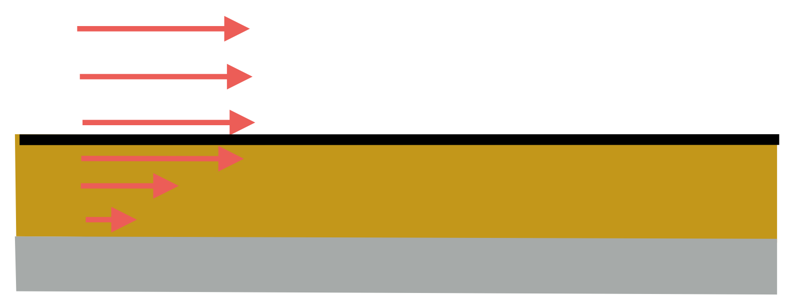
$$\tau_b = Cu$$



Nye 1969

- lubrication by a layer of underlying water-saturated **sediments** (viscous)

$$\tau_b = \frac{\eta_s}{d_s} u$$



From force balance $\tau_b \approx -\rho_i g h \dfrac{\partial s}{\partial x}$

Combining with mass conservation gives a **diffusion equation for ice thickness**

$$\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(h^2 \frac{\partial h}{\partial x}\right) + \overline{a}$$

Plastic bed model

Consider the friction law

$$\tau_b \leq \tau_0 \quad u = 0$$

$$\tau_b = \tau_0 \quad u \geq 0$$

Assuming deformation occurs, force balance becomes an equation for ice thickness

$$-\frac{\partial h}{\partial x} = \frac{\partial b}{\partial x} + \frac{\tau_0}{\rho_i g h} \qquad h = 0 \quad \text{at} \quad x = x_m$$

So **ice thickness** is (almost) determined without reference to mass conservation or velocity

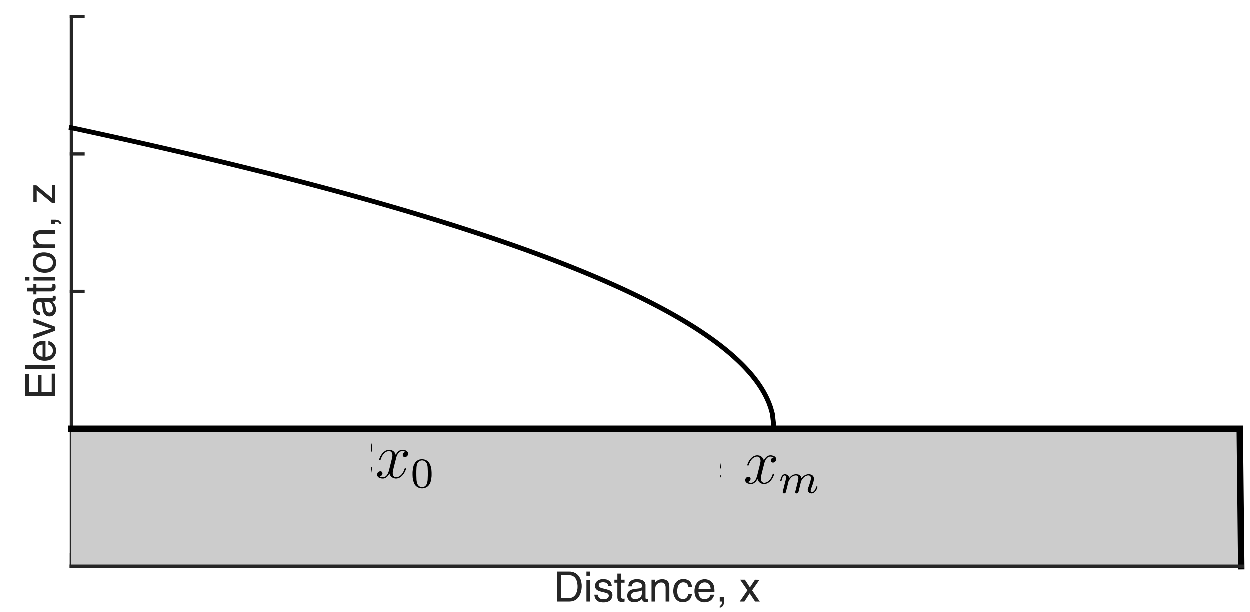
Global mass conservation $\dot{V} = \int_0^{x_m} \overline{a}\,\mathrm{d}x$ Ice volume $V = \int_0^{x_m} h\,\mathrm{d}x$

Example

Eg. Flat bed $\longrightarrow$ $$h = \sqrt{2h_0}(x_m - x)^{1/2}$$ $$h_0 = \frac{\tau_0}{\rho_i g}$$

Position-dependent accumulation

$$\overline{a} = \lambda(x_0 - x)$$



Global mass conservation

$$\sqrt{2h_0}x_m^{1/2}\,\dot{x}_m = \lambda\left(x_0 - \frac{1}{2}x_m\right)x_m$$

$\longrightarrow$ Stable steady state $x_m = 2x_0$

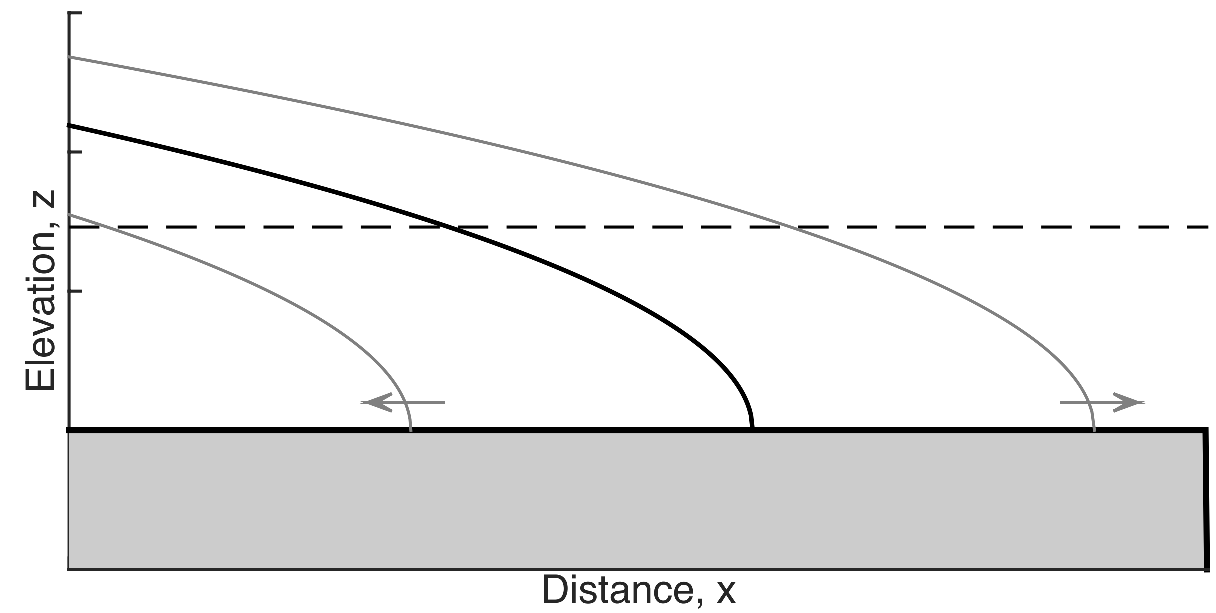
‘accumulation = ablation’

Example

Eg. Elevation-dependent accumulation

$$h = \sqrt{2h_0}(x_m - x)^{1/2} \qquad h_0 = \frac{\tau_0}{\rho_i g}$$

$$\overline{a} = \lambda(s - s_0)$$



Global mass conservation

$$\sqrt{2h_0}x_m^{1/2}\,\dot{x}_m = \lambda\left(\frac{2}{3}\sqrt{2h_0}\,x_m^{3/2} - s_0 x_m\right)$$

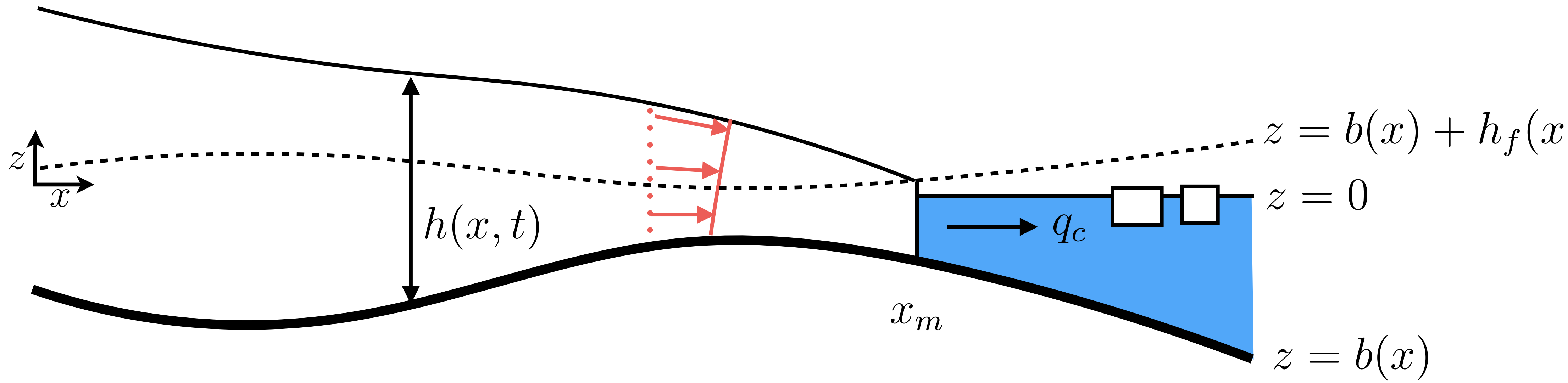
→ **Unstable** steady state $x_m = \frac{9}{8}\frac{s_0^2}{h_0}$

→ Ice sheet shrinks to nothing, or fills continental shelf

Marine-terminating glaciers

Extreme Ice Survey - Time-lapse camera
Columbia Glacier, Alaska

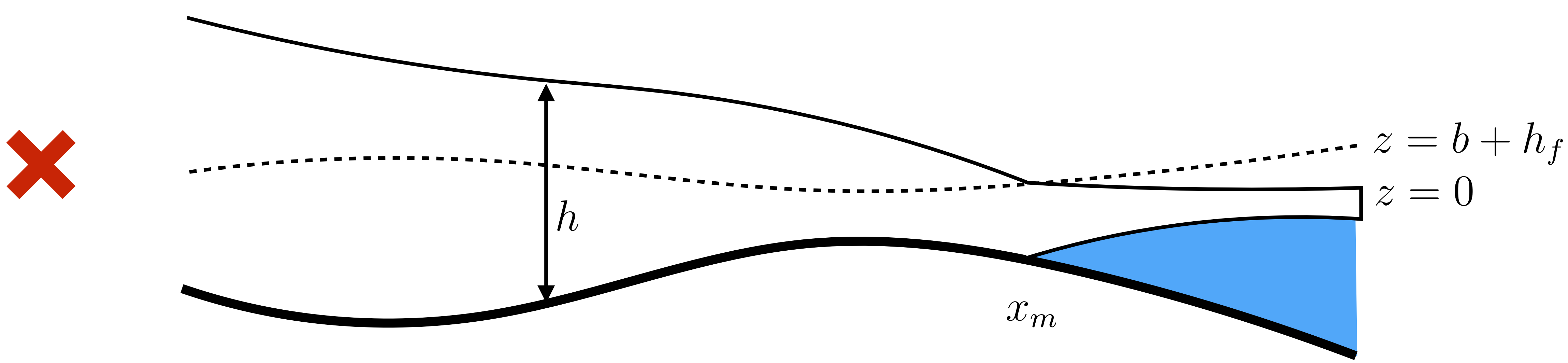
Marine-terminating glaciers



Flotation thickness $h_f(x) = -\frac{\rho_o}{\rho_i} b(x)$

Mass conservation $q_m - h_m \dot{x}_m = q_c$

calving rate (includes ocean-driven melting)



Approximate force balance

Full force balance

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z}$$

$$0 = -\frac{\partial p}{\partial z} + \cancel{\frac{\partial \tau_{xz}}{\partial x}} + \frac{\partial \tau_{zz}}{\partial z} - \rho_i g$$

Small aspect ratio $\frac{[h]}{[x]} \sim 10^{-3}$

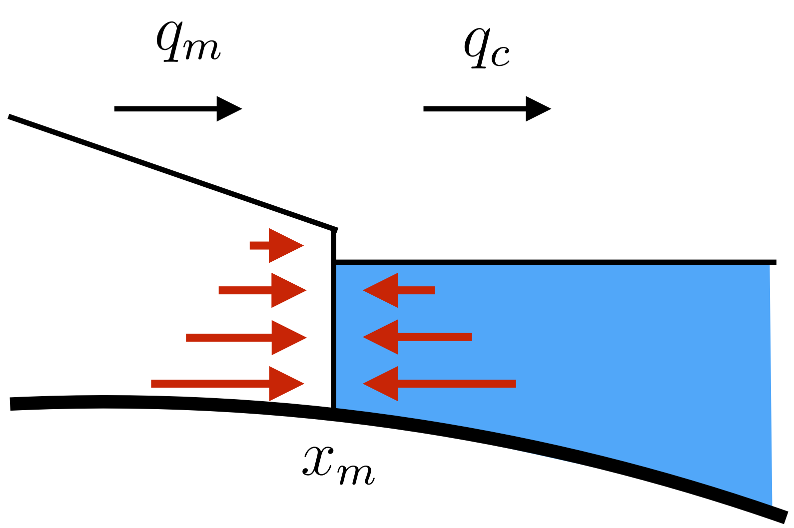
→ vertical balance approximately hydrostatic $p - \tau_{zz} = \rho_i g(s - z)$

→ depth integrate horizontal balance, with $\overline{\tau}_{xx} \approx 2\eta_i \frac{\partial u}{\partial x}$

$$\boxed{\frac{\partial}{\partial x}\left(4\eta_i h \frac{\partial u}{\partial x}\right) - \tau_b - \rho_i g h \frac{\partial s}{\partial x} = 0}$$

Note also, depth-integrated horizontal stress $\int_b^s -p + \tau_{xx}\,\mathrm{d}x = -\frac{1}{2}\rho_i g h^2 + 4\eta_i h \frac{\partial u}{\partial x}$

Conditions at the marine margin



Continuity of longitudinal stress at the margin

$$-\frac{1}{2}\rho_i g h^2 + 4\eta_i h \frac{\partial u}{\partial x} = -\frac{1}{2}\rho_o g b^2 \quad \text{at} \quad x = x_m$$

Some models for calving prescribe a 'rate' $q_c = q_m - h\dot{x}_m$

Others prescribe an equilibrium-like 'criteria' - e.g. flotation condition $h = h_f \equiv -\frac{\rho_o}{\rho_i} b$

Full model

Non-dimensional equations

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = \overline{a}$$

$$\varepsilon \frac{\partial}{\partial x}\left(4h\frac{\partial u}{\partial x}\right) - \tau_* - h\frac{\partial (b+h)}{\partial x} = 0$$

$$r = \frac{\rho_o}{\rho_i} \approx 1.1$$

$$\varepsilon = \frac{\eta_i [a]}{[\tau][x]} \ll 1$$

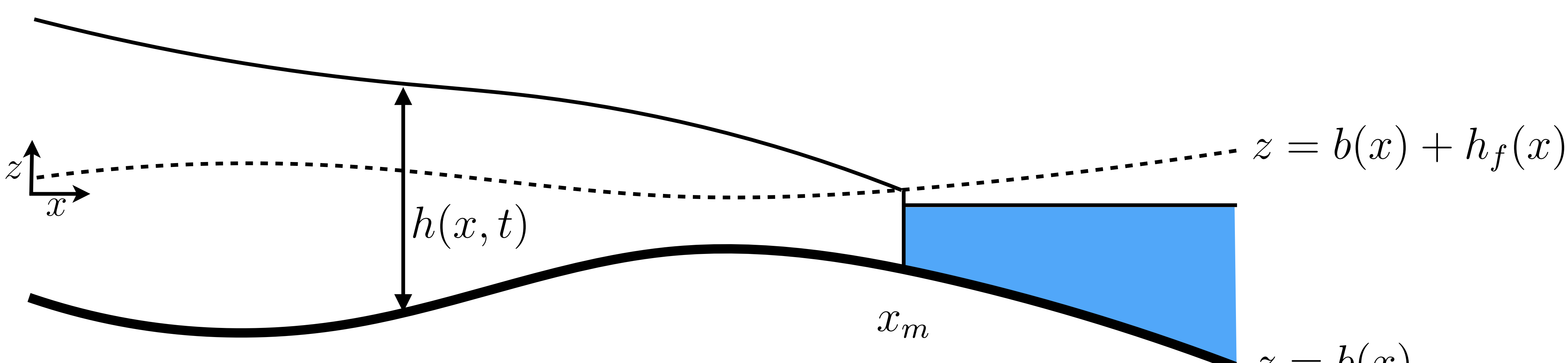
$$\tau_* = 1$$

Boundary conditions

$$q = 0 \quad \text{at} \quad x = 0$$

$$h = h_f \qquad \varepsilon 4h\frac{\partial u}{\partial x} = \frac{1}{2}(h^2 - h_f^2/r) \quad \text{at} \quad x = x_m(t)$$

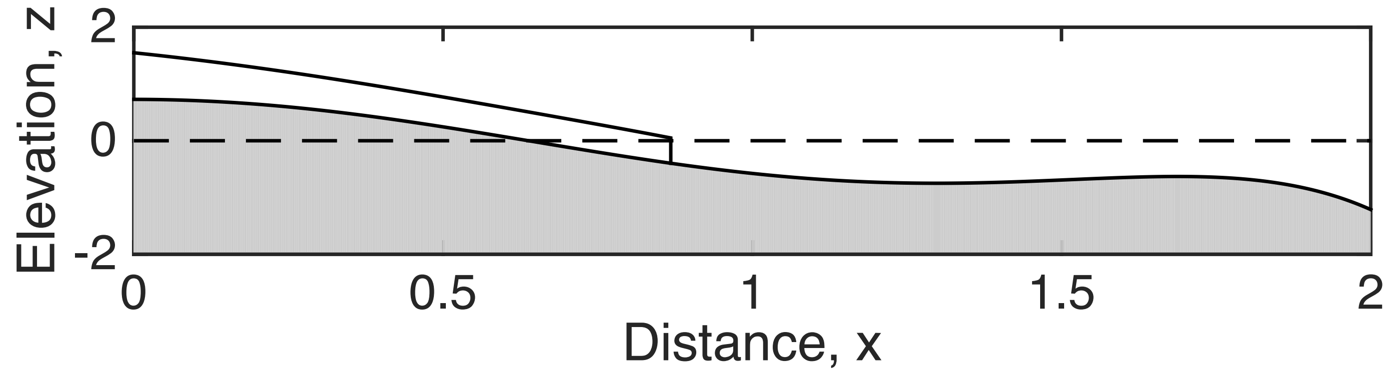
flotation condition



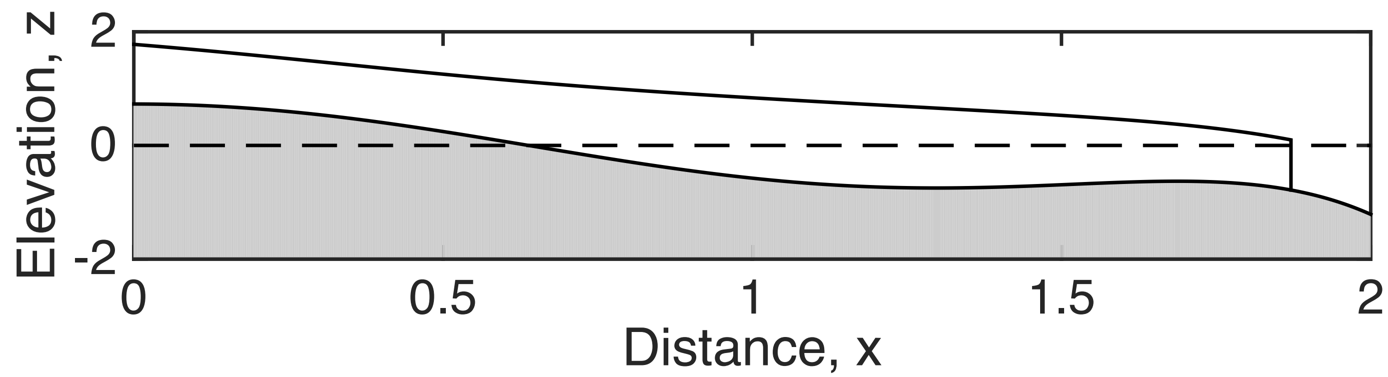
Numerical solutions

Steady states for constant accumulation

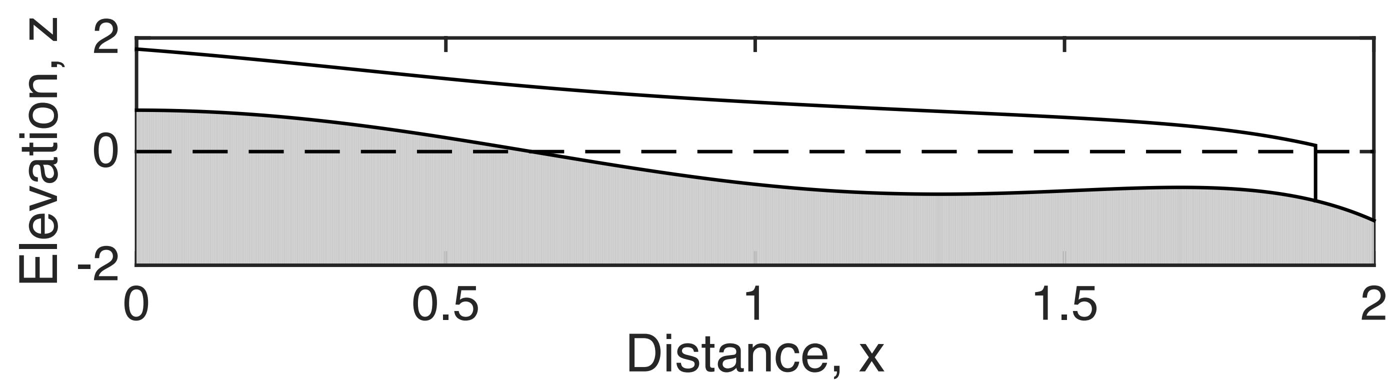
$\overline{a} = 0.1$



$\overline{a} = 1$



$\overline{a} = 2$



Margin boundary layer

Longitudinal stress most important near margin

→ rescale $h = \varepsilon^{1/4} H$ $u = \varepsilon^{-1/4} U$ $x_m(t) - x = \varepsilon^{1/2} X$ $H_f = \varepsilon^{-1/4} r b(x_m)$

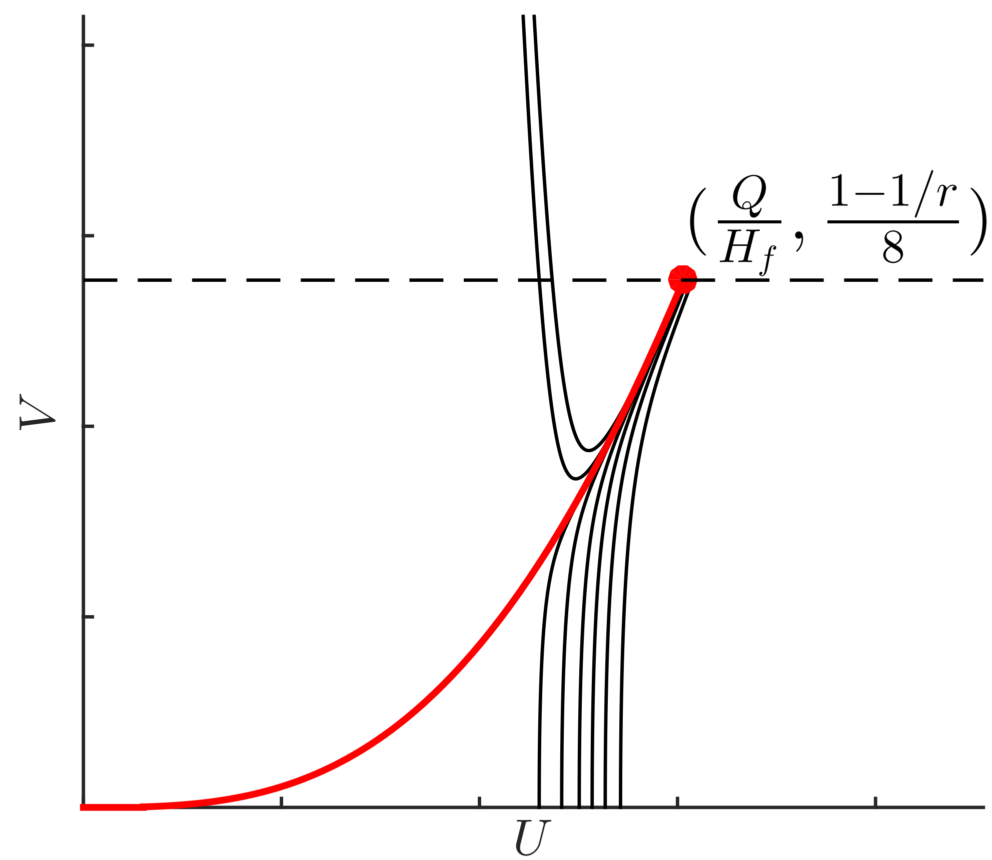
Equations become

$$\frac{\partial}{\partial X}(HU) = 0 \quad \rightarrow \quad HU = Q$$

$$\frac{\partial}{\partial X}\left(4H\frac{\partial U}{\partial X}\right) - \tau_* + H\frac{\partial H}{\partial X} = 0$$

$$U_X = -V$$

$$V_X = -\frac{\tau_* U}{4Q} + \frac{Q}{4U^2}V - \frac{V^2}{U}$$



Boundary / matching conditions

$$U = \frac{Q}{H_f} \quad V = \frac{1}{8}(1 - 1/r)H_f \quad \text{at} \quad X = 0$$

$$U \to 0 \quad V \to 0 \quad \text{as} \quad X \to \infty$$

→ $$Q = \mathcal{Q}_m(H_f) \approx \frac{(1-1/r)}{8\tau_*} H_f^4$$

Only one value of Q allows the required trajectory

Reduced model

Away from the margin, ignore longitudinal stress

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = \overline{a}$$

$$-\tau_* - h\frac{\partial h}{\partial x} = 0 \qquad h = 0 \quad \text{at} \quad x = x_m$$

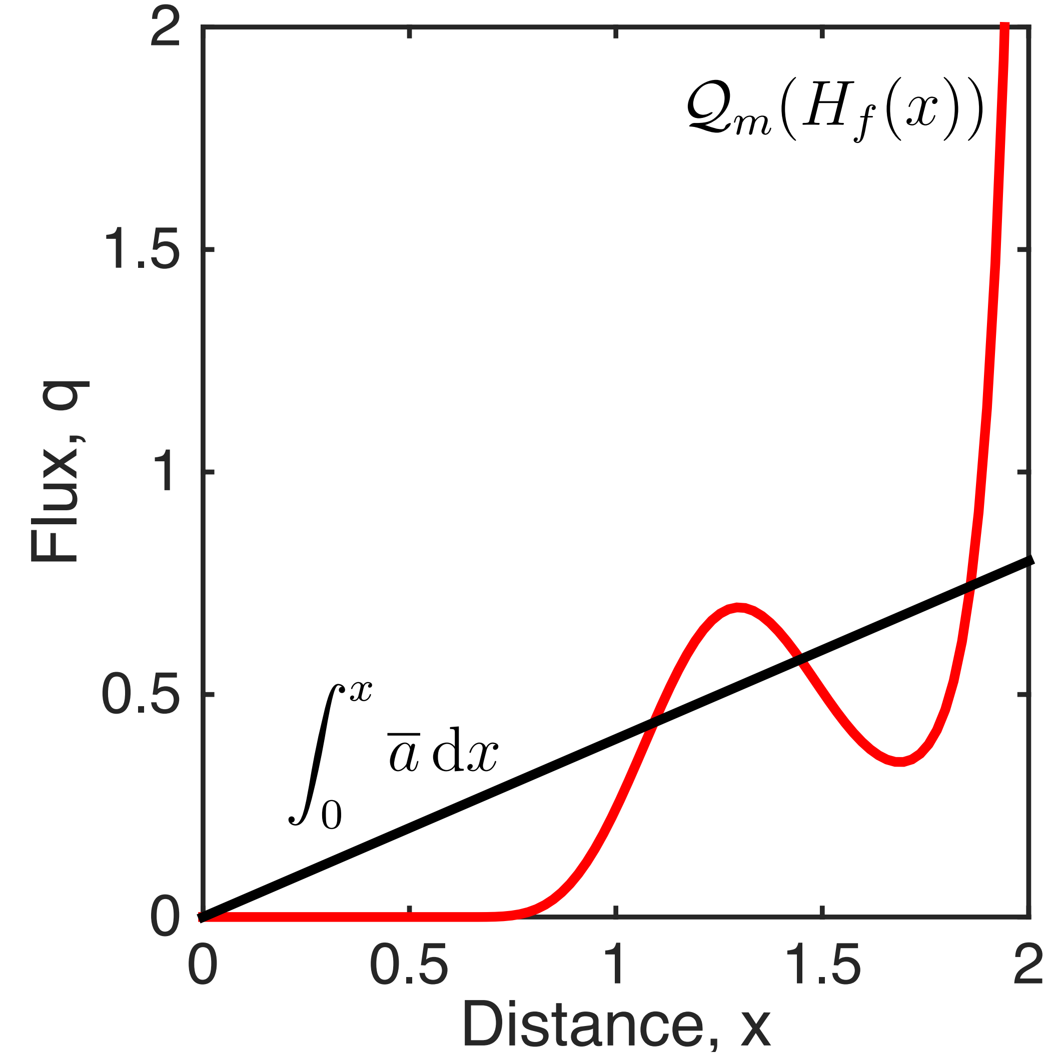
$$b(x) = O(\varepsilon^{1/4})$$

$$h_f = \varepsilon^{1/4} H_f$$

$$\longrightarrow \quad h = \sqrt{2\tau_*}(x_m - x)^{1/2}$$

Integrating mass conservation

$$\sqrt{2\tau_*}x_m^{1/2}\dot{x}_m = \int_0^{x_m} \overline{a}\,\mathrm{d}x - \mathcal{Q}_m(H_f(x_m))$$

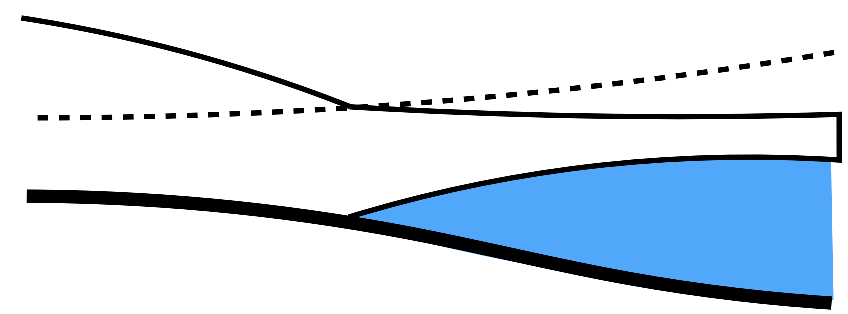


Multiple steady states depending on bed slope
(analogous to grounding lines, Schoof 2007)

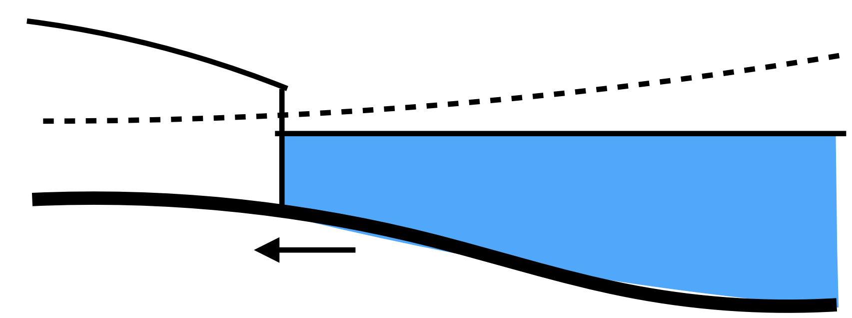
More about calving

The role of calving in this model was to evacuate ice delivered to the margin

If the processes responsible for calving cannot keep up, a floating ice shelf will form



But if calving is more efficient, it may result in margin thickness above flotation



→ Replace flotation condition with $h_m \dot{x}_m = q_m - q_c$

The boundary layer analysis can be generalised to find $q_m = \mathcal{Q}_m(h_m, h_f)$

$$h_m \dot{x}_m = \mathcal{Q}_m(h_m, h_f) - q_c$$

Reduced model II

Away from the margin, ignore longitudinal stress

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = \overline{a}$$

$$-\tau_* - h\frac{\partial h}{\partial x} = 0 \qquad h = 0 \quad \text{at} \quad x = x_m$$

$$b(x) = O(\varepsilon^{1/4})$$

$$h_f = \varepsilon^{1/4} H_f$$

$$\longrightarrow \quad h = \sqrt{2\tau_*}(x_m - x)^{1/2}$$

Integrating mass conservation

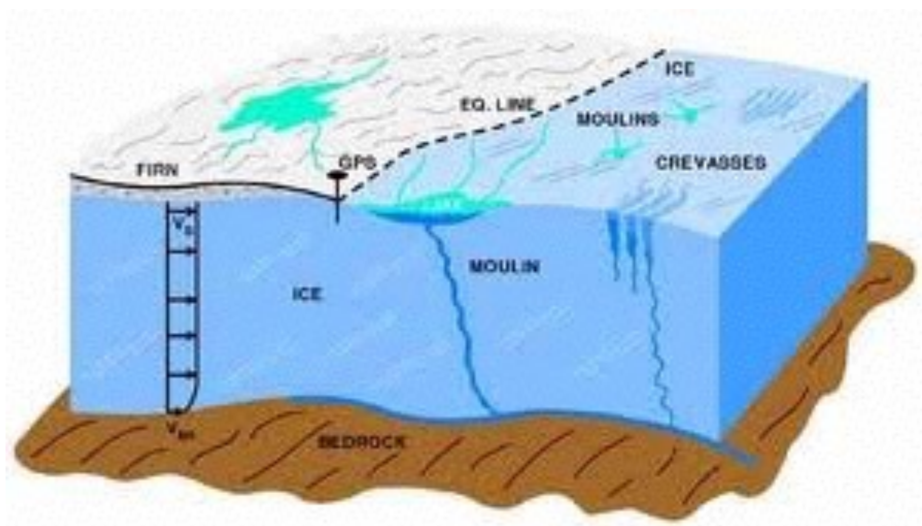
$$\sqrt{2\tau_*}x_m^{1/2}\dot{x}_m = \int_0^{x_m} \overline{a}\,\mathrm{d}x - q_c$$

$$\mathcal{Q}_m(H_m, H_f) = q_c$$

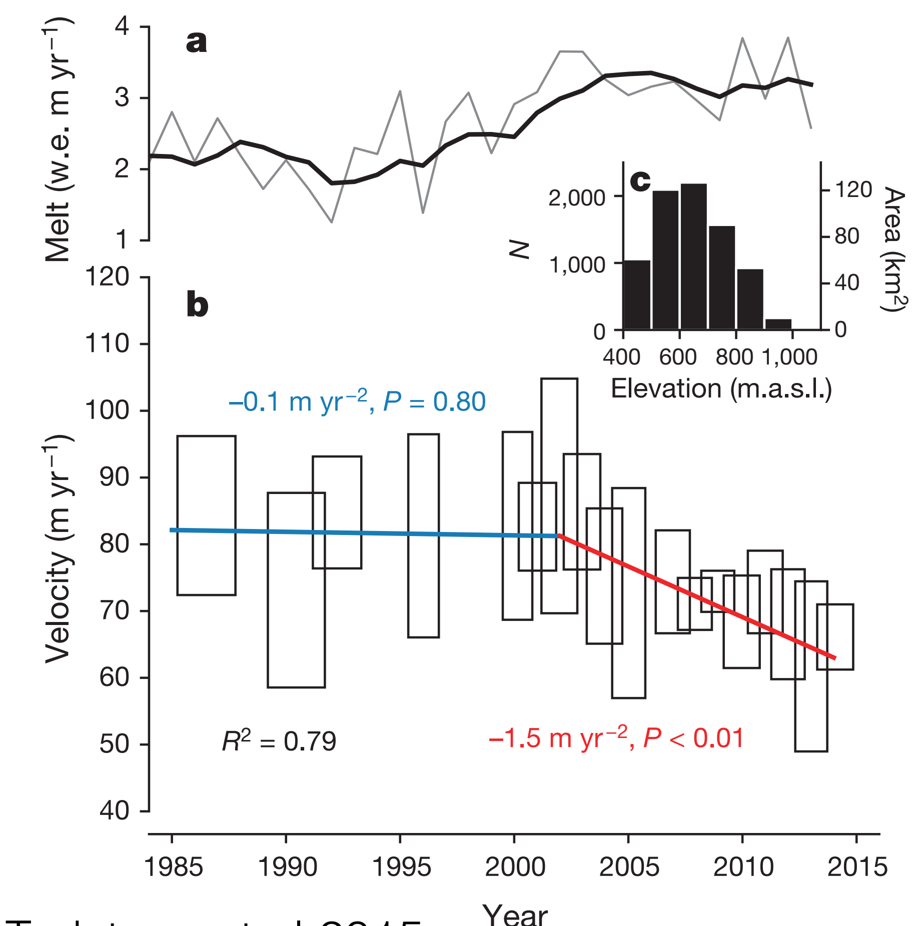
Height above flotation adjusts to balance calving rate
(Hindmarsh 2012)

Could meltwater-induced ice-sheet slow-down be a greater problem than acceleration?

Recent observations suggest surface meltwater penetrating to the bed may slightly reduce, rather than increase, ice velocities.



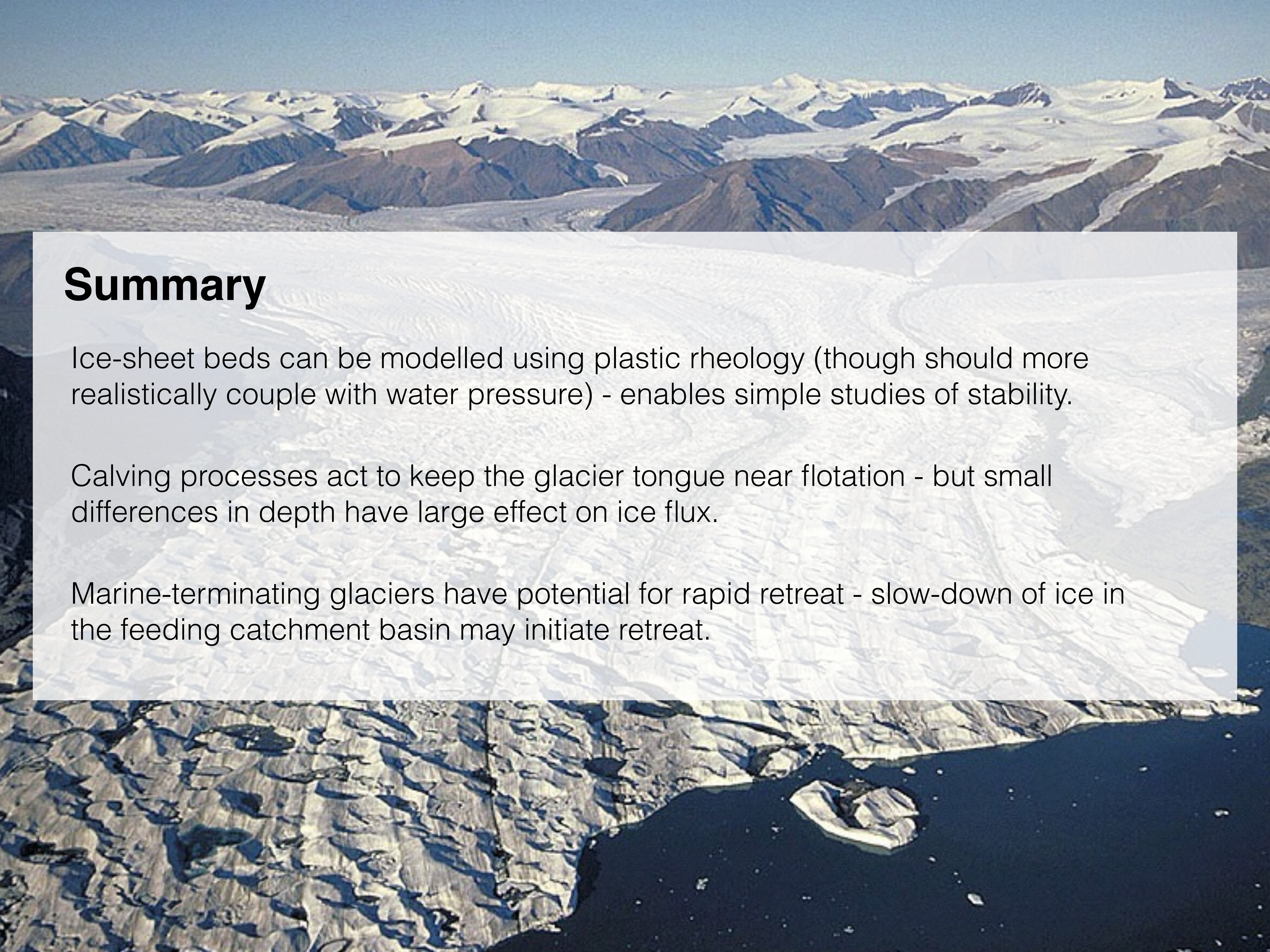
Zwally et al 2002



Tedstone et al 2015

Changes in accumulation are the primary driver of the marine ice-sheet instability in this model.

For outlet glaciers, ‘accumulation’ includes inflow of ice from catchment basin - slow-down of surrounding ice will reduce ice supply, with potential rapid retreat.



Summary

Ice-sheet beds can be modelled using plastic rheology (though should more realistically couple with water pressure) - enables simple studies of stability.

Calving processes act to keep the glacier tongue near flotation - but small differences in depth have large effect on ice flux.

Marine-terminating glaciers have potential for rapid retreat - slow-down of ice in the feeding catchment basin may initiate retreat.