Subglacial lubrication, ice-sheet dynamics and mass loss

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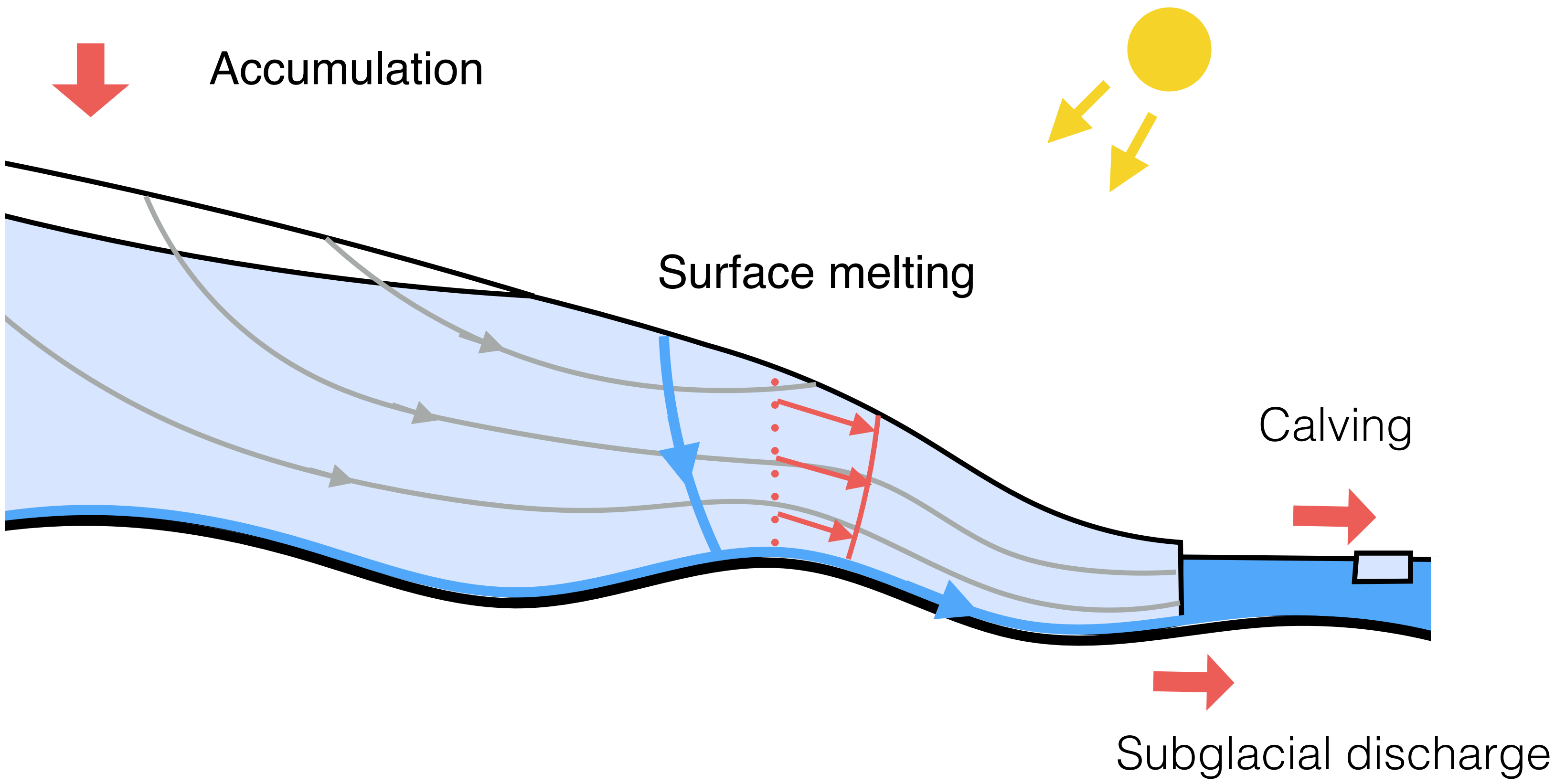
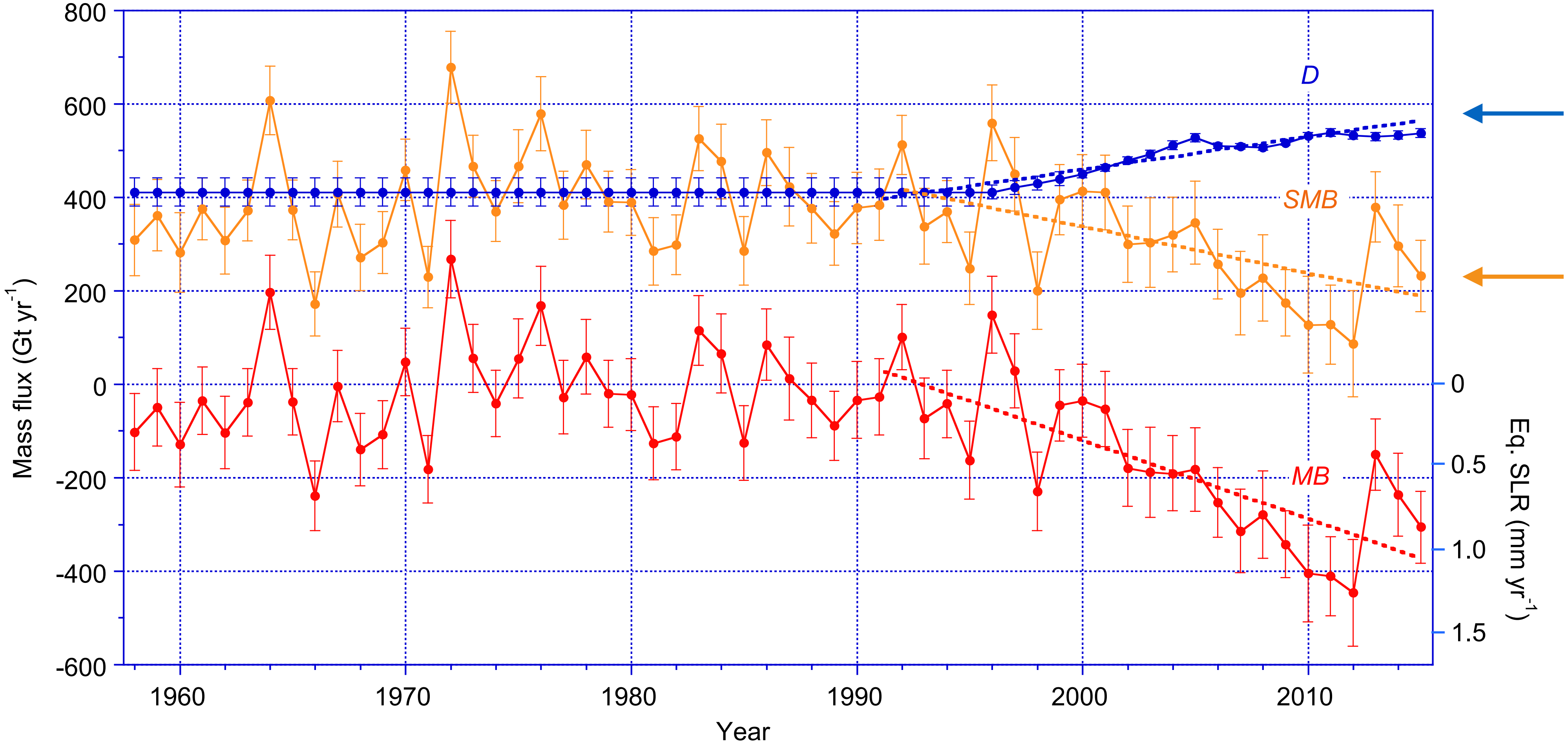
Greenland, *NASA*



How does meltwater penetrating to the bed affect ice velocities?

What effect does this have on ice loss (sea level)?

Greenland ice sheet mass balance



Discharge

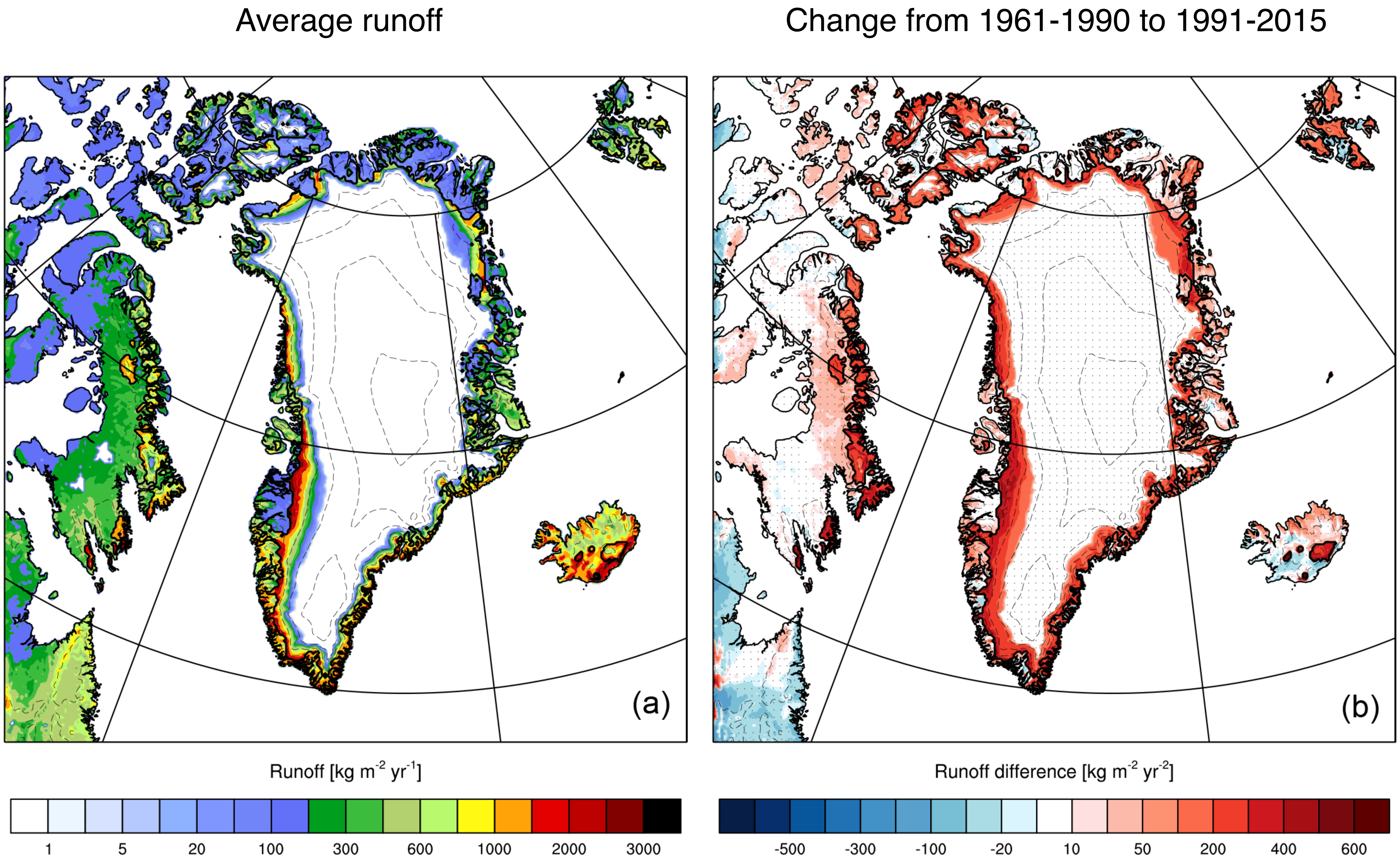
calving

Surface balance

accumulation - runoff

van den Broeke et al 2016

Greenland ice sheet surface melt

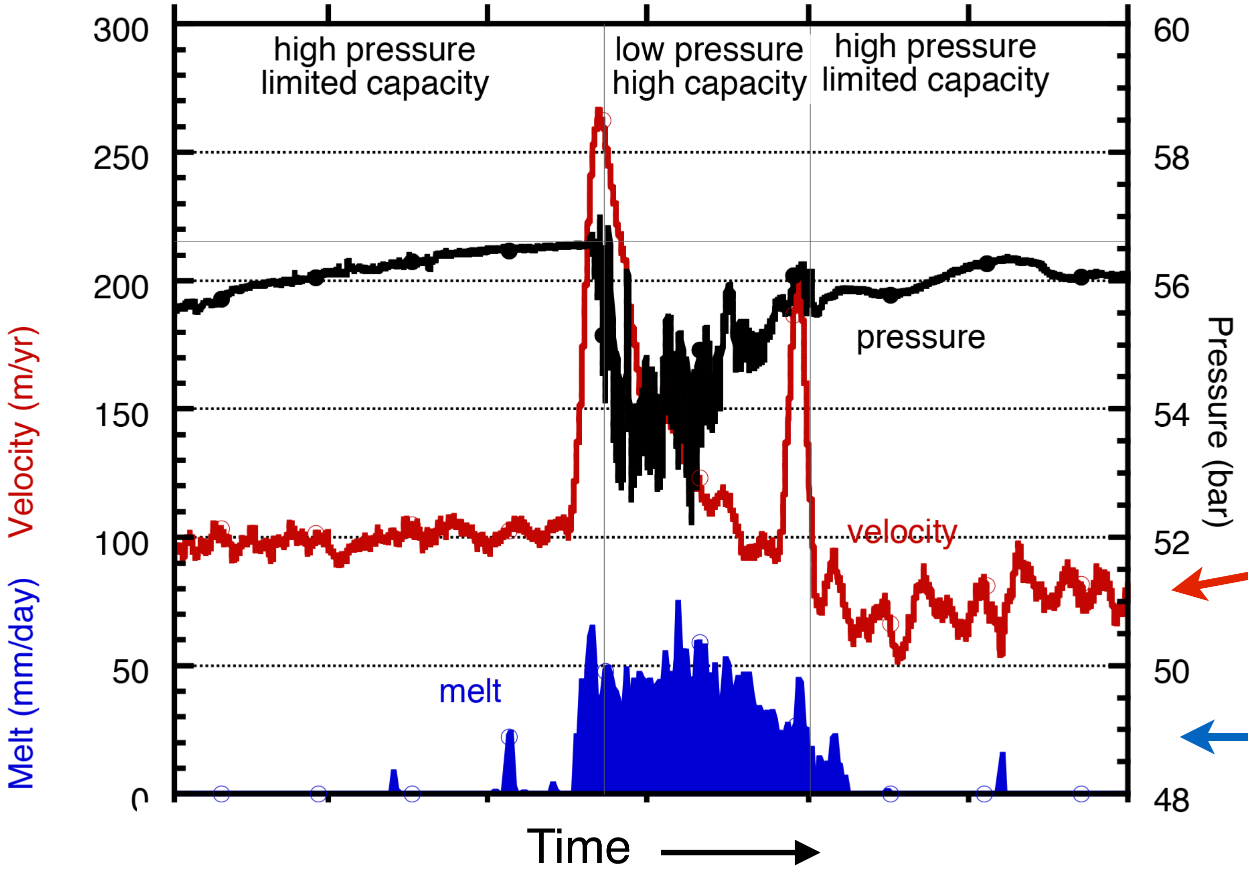


van den Broeke et al 2016

Greenland ice sheet velocities

Summer drainage of surface meltwater to the bed causes large fluctuations in ice speed.

→ suggests potential for significant changes in ice velocity



100%

95%

90%

Ice speed (GPS)

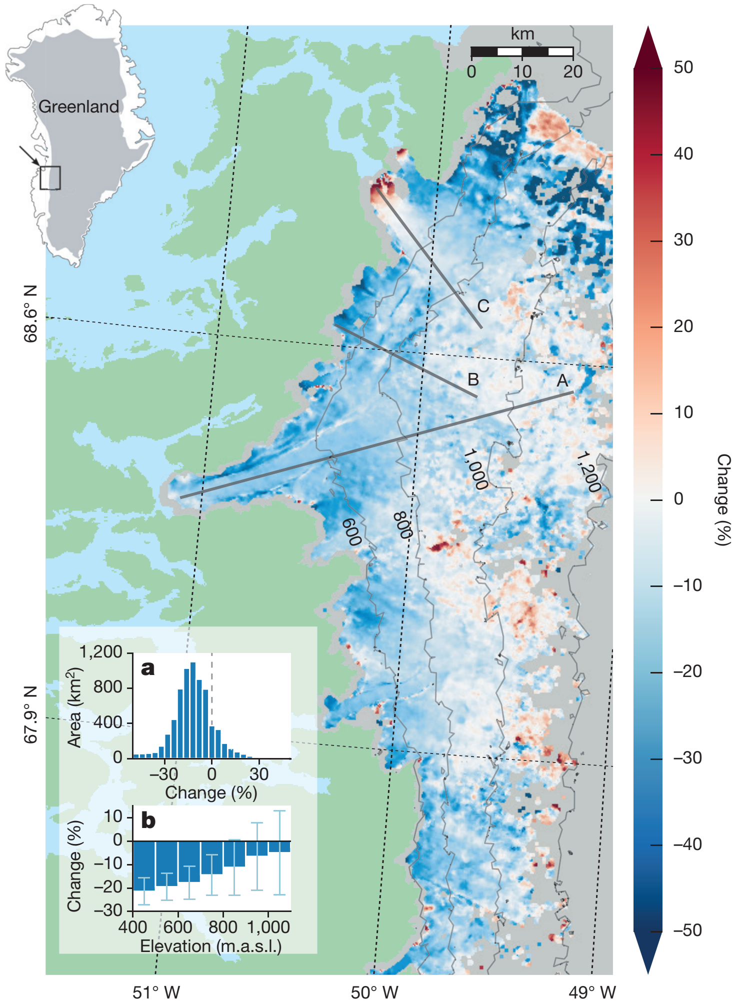
Runoff

van de Wal et al 2015

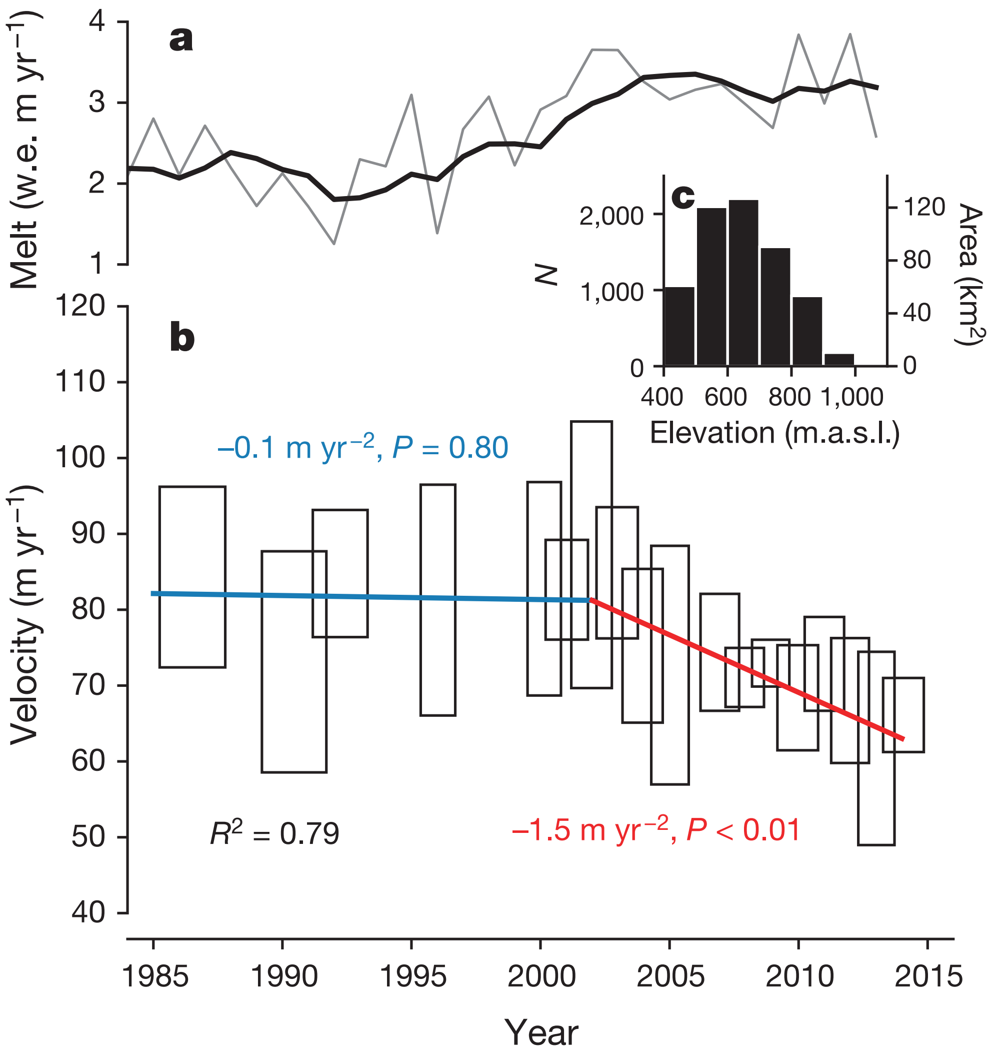
Greenland ice sheet velocities

Longer term measurements show a slight **decreasing** trend in average velocity, while runoff shows an increasing trend.

→ suggests possible negative relationship between runoff and average velocity?

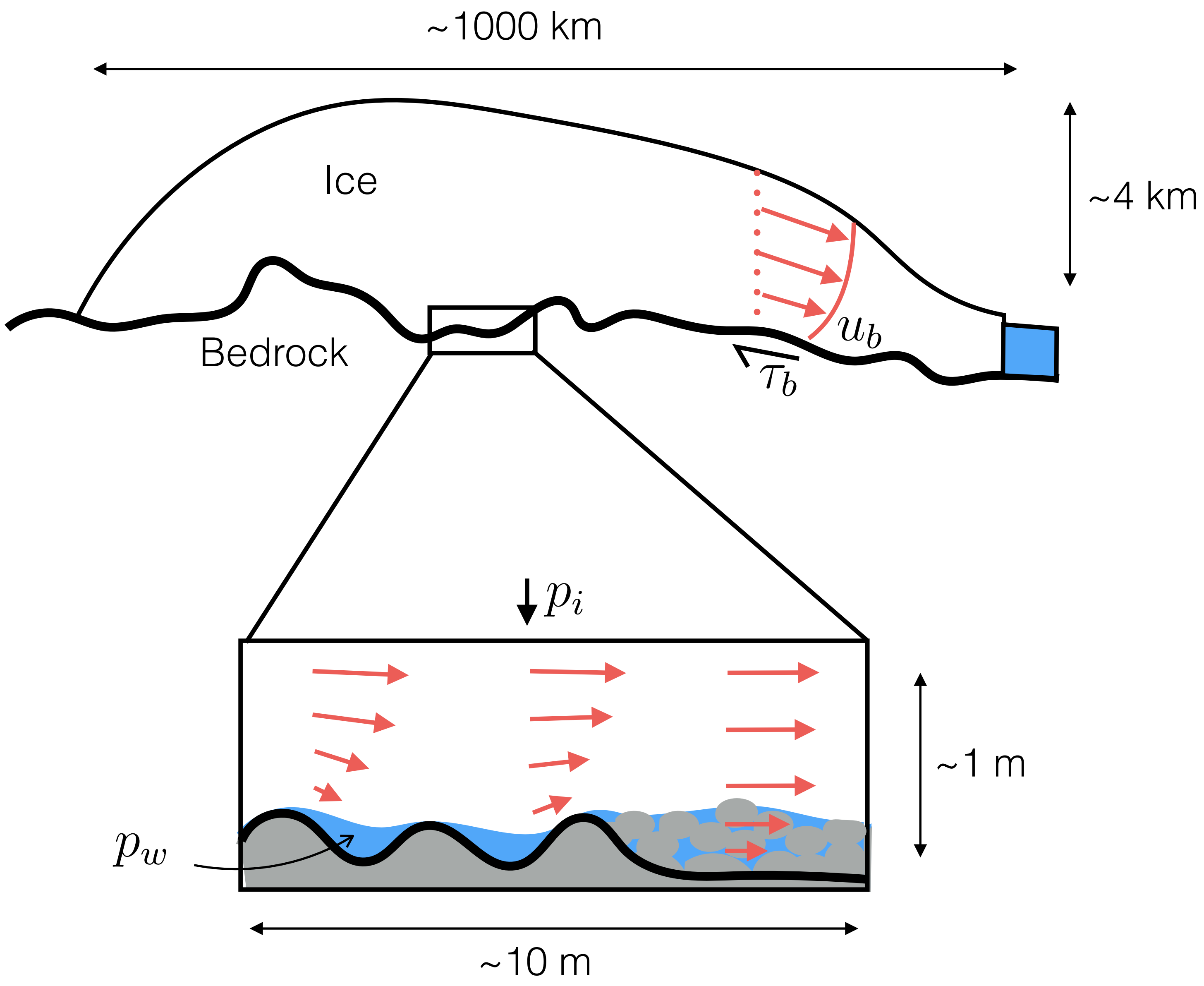


Tedstone et al 2015

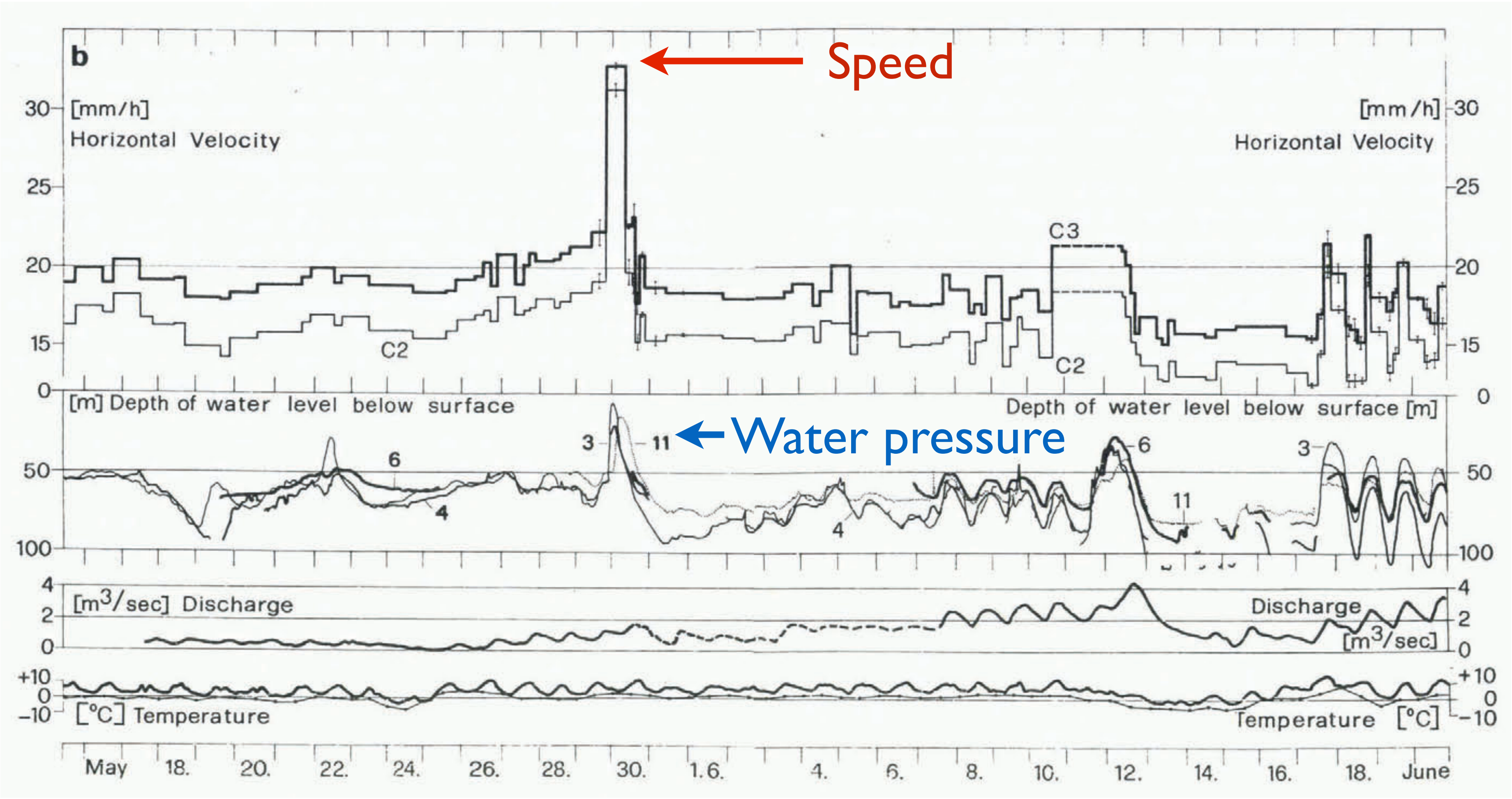


Mount Robson, Canada

Theoretical framework



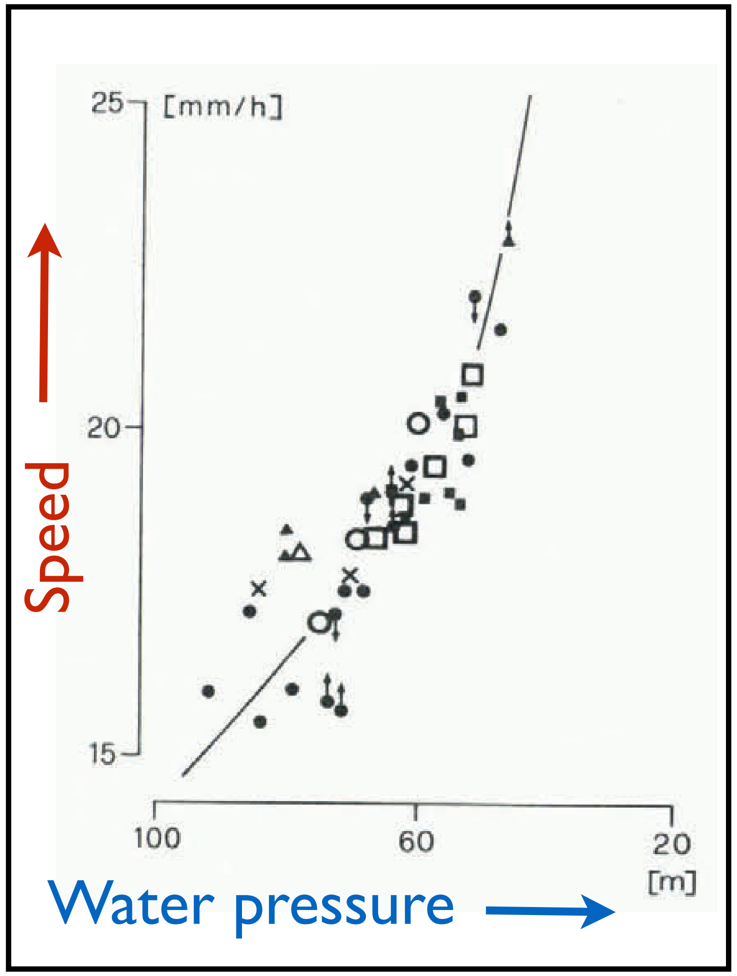
Sliding and basal water pressure



Speed

Water pressure

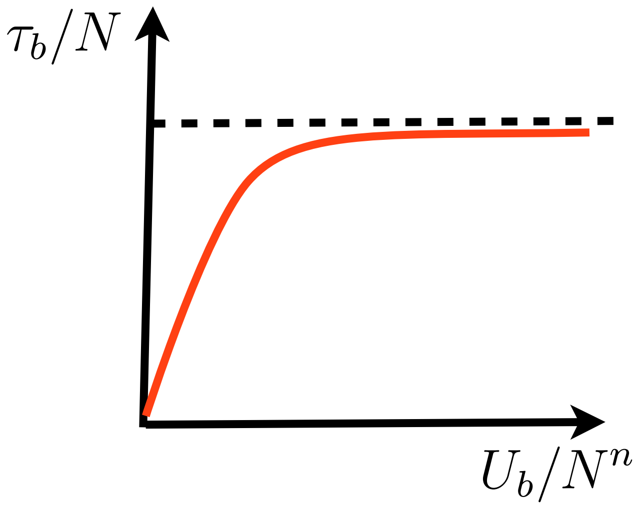
Iken & Bindschadler 1986



Speed

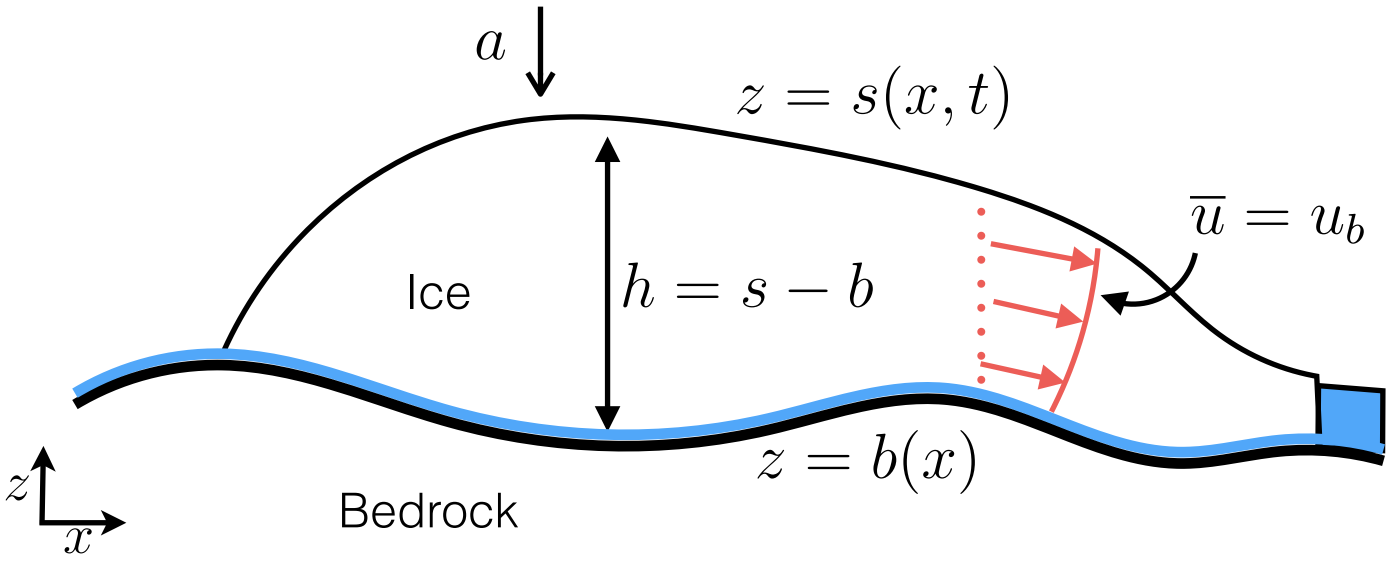
Water pressure

Some theory and *some* measurements suggest a friction law of the form $\tau_b = f(u_b, N)$



$$N = p_i - p_w$$

Mathematical model



- Vertically-integrated mass conservation

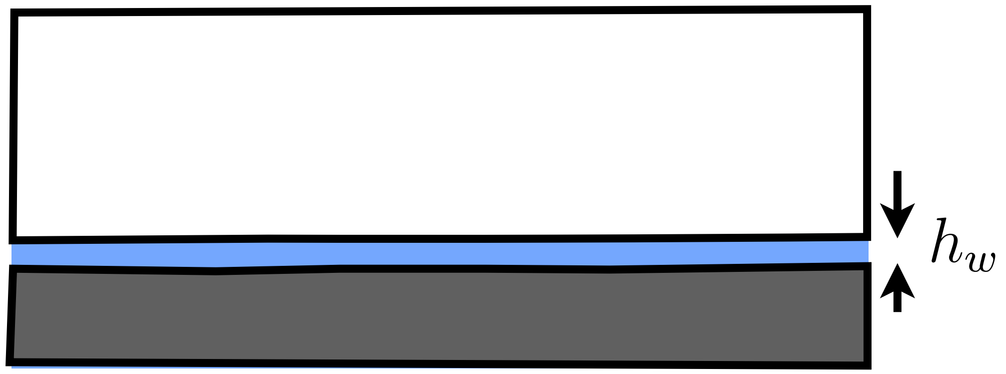
$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = a \qquad q = h\overline{u}$$

a = net accumulation - melting

- Force balance $0 = \nabla \cdot \boldsymbol{\sigma} + \rho_i \mathbf{g}$

$$p_i = \rho_i g (s - z) \qquad \tau_b = -\rho_i g h \frac{\partial s}{\partial x} + \frac{\partial}{\partial x}\left(4 h \eta_i \frac{\partial \overline{u}}{\partial x}\right) \qquad \tau_b = f(\overline{u}, N) \qquad N = p_i - p_w$$

Subglacial hydraulic model: poro-elastic sheet



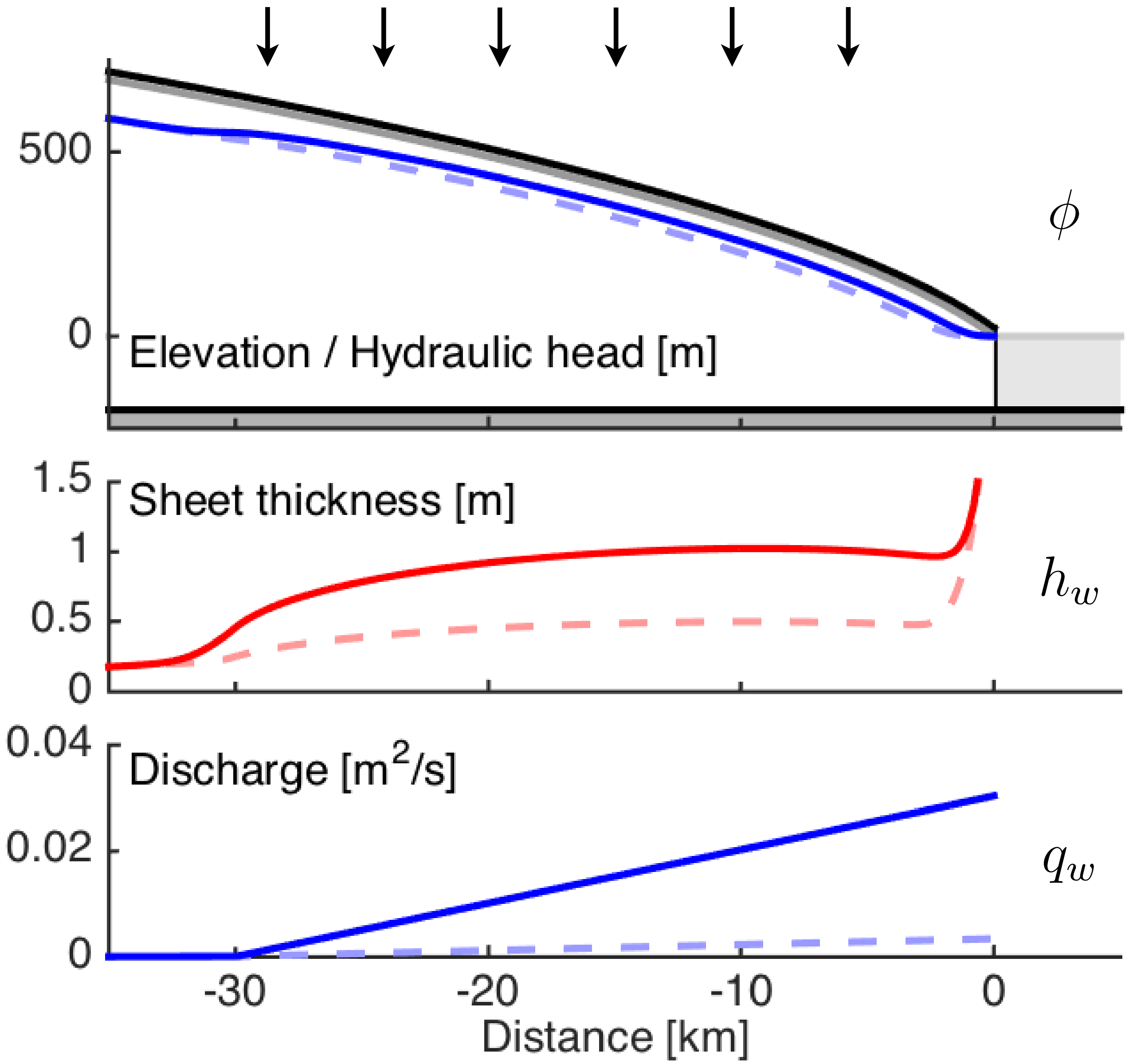
$$h_w = f(N) \qquad N = p_i - p_w$$

Water conservation

$$\frac{\partial h_w}{\partial t} + \frac{\partial q_w}{\partial x} = r \qquad q_w = -k h_w^{\alpha} \frac{\partial \phi}{\partial x}$$

Hydraulic potential (hydraulic head)

$$\phi = \rho_w g b + p_w$$

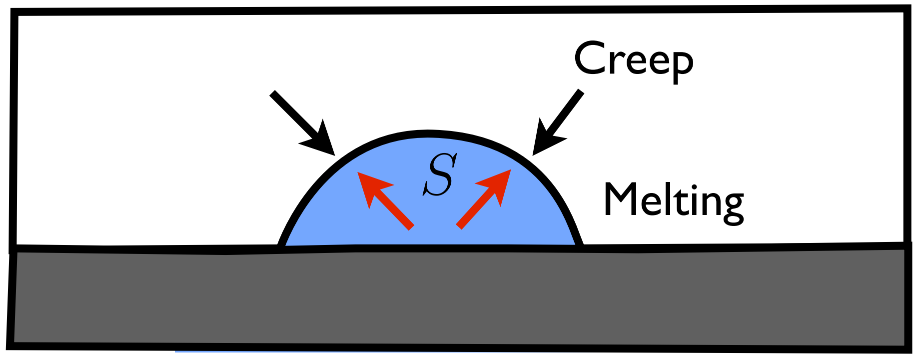


Uniform input to subglacial layer over 30km

dashed = low input, solid = high input

Vernagtferner, Austria

Subglacial hydraulic system: channel



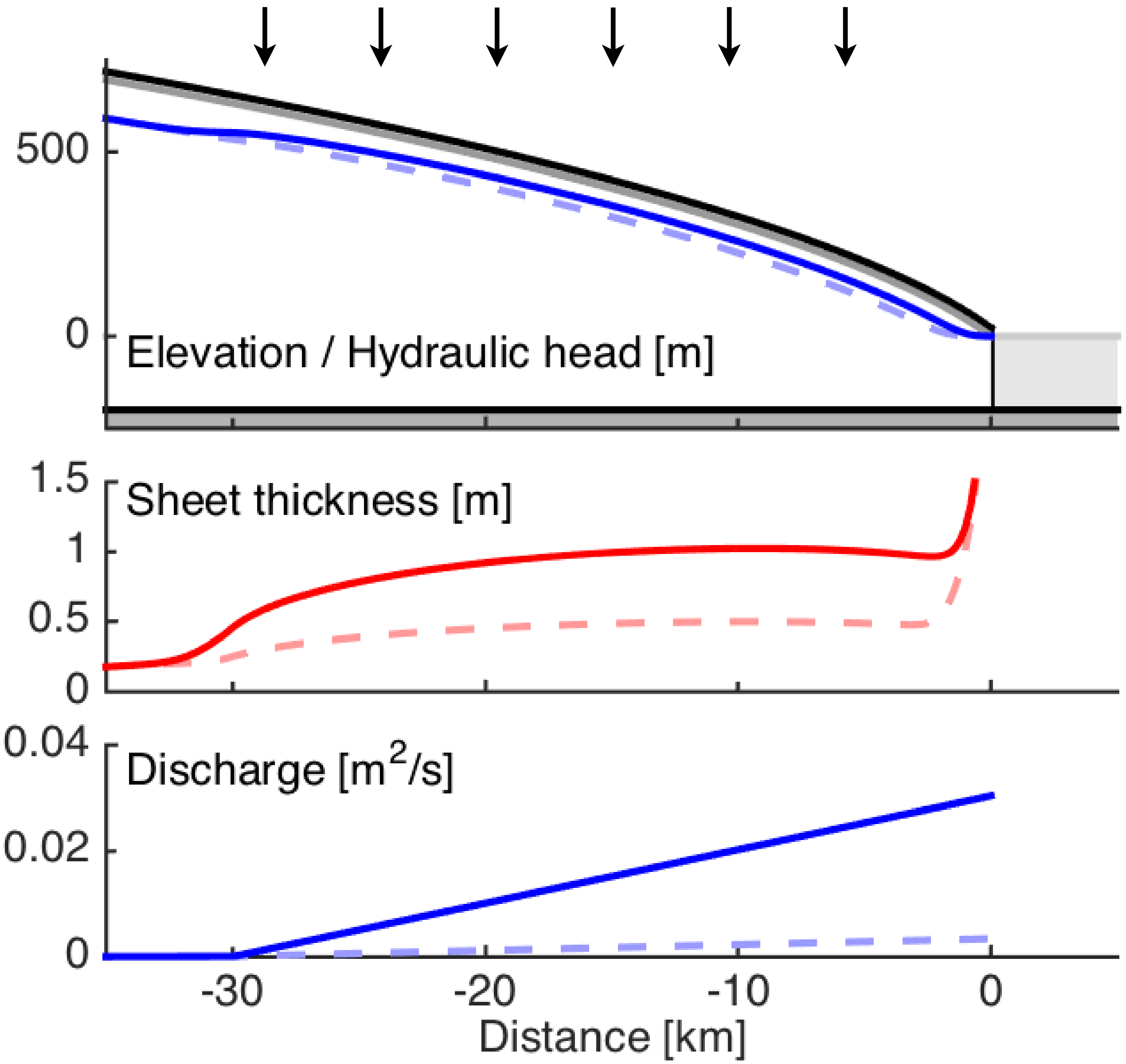
$$\frac{\partial S}{\partial t} = \frac{1}{\rho_i L}\left|Q\frac{\partial \phi}{\partial x}\right| - \frac{SN}{\eta_i}$$

Water conservation

$$\frac{\partial S}{\partial t} + \frac{\partial Q}{\partial x} = r \qquad Q = -KS^{4/3}\left|\frac{\partial \phi}{\partial x}\right|^{1/2}$$

Hydraulic potential (hydraulic head)

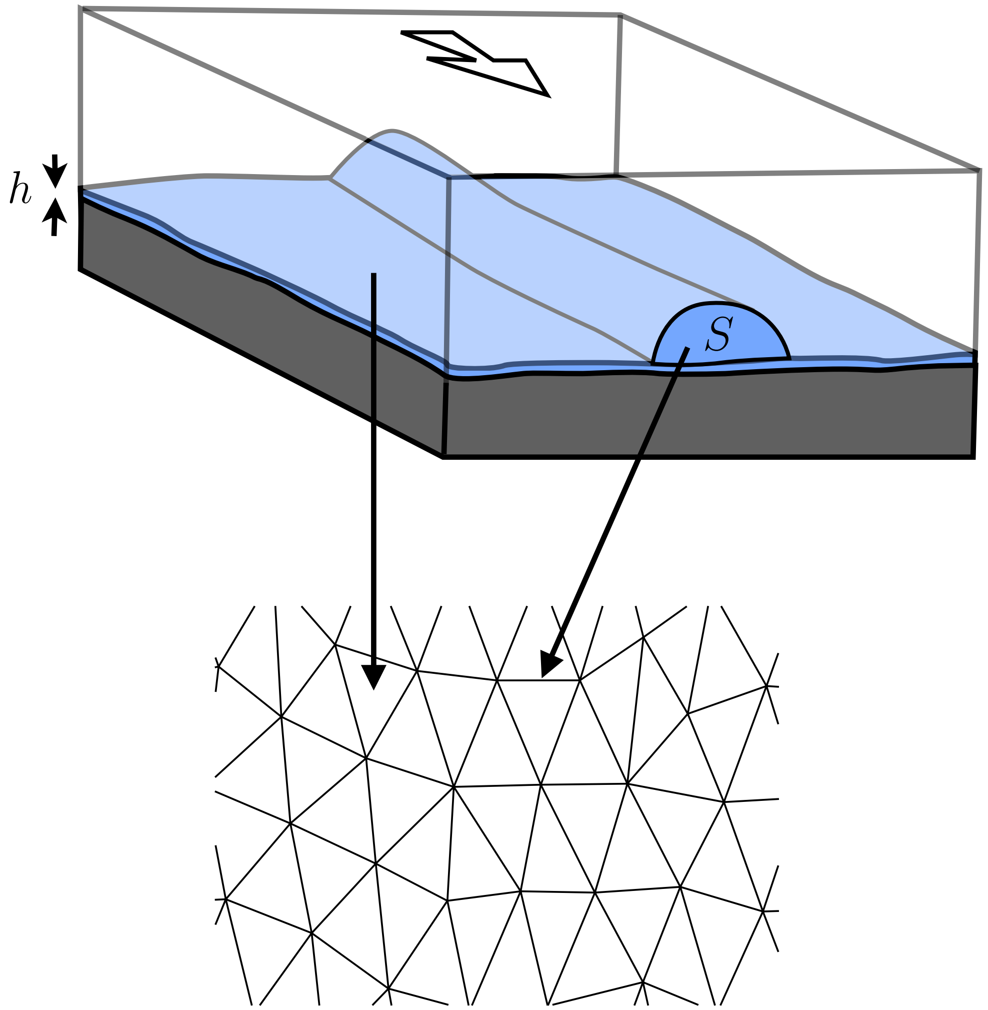
$$\phi = \rho_w g b + p_w$$



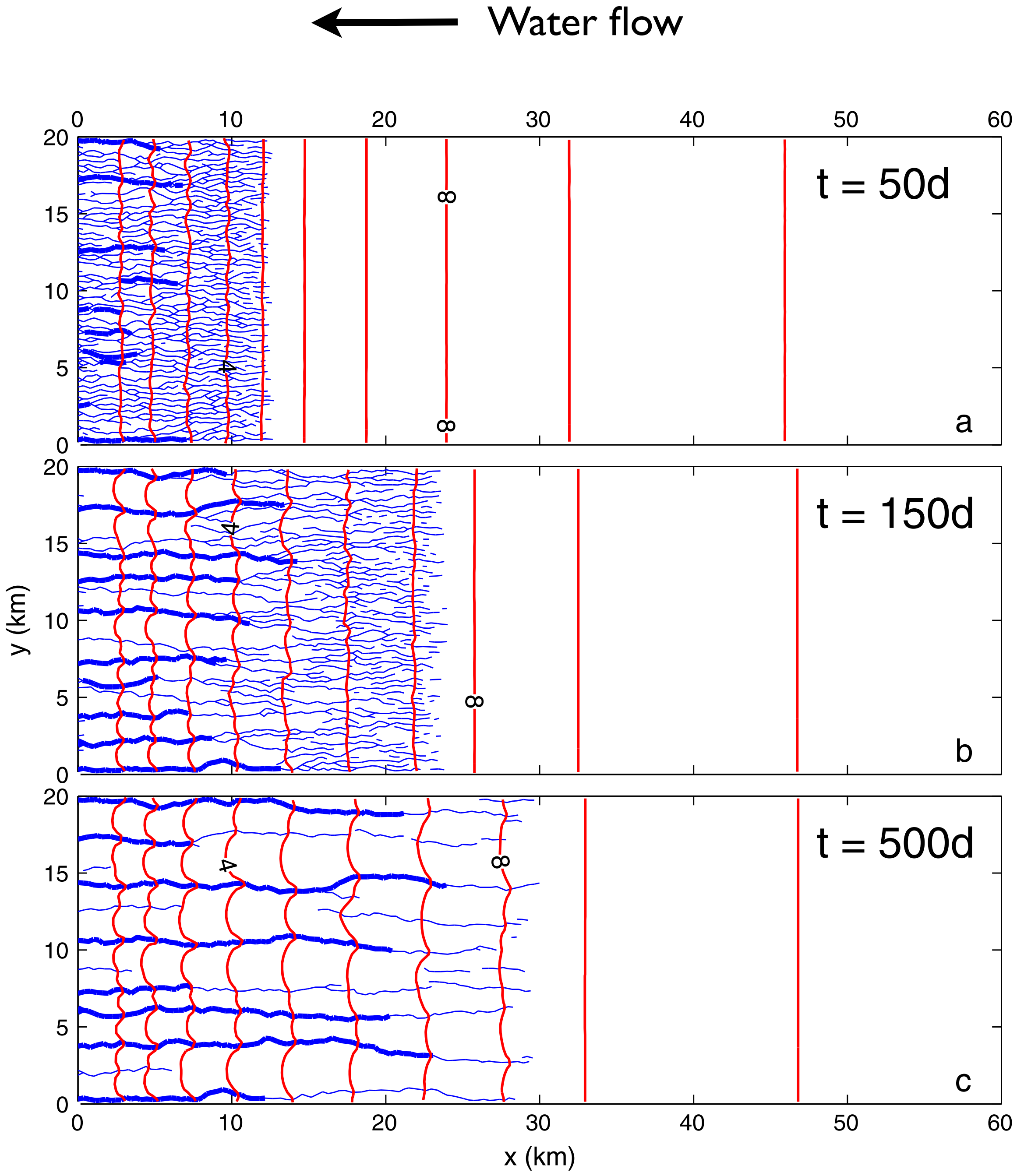
Uniform input to subglacial channel over 30km

dashed = low input, solid = high input

Network-based numerical model

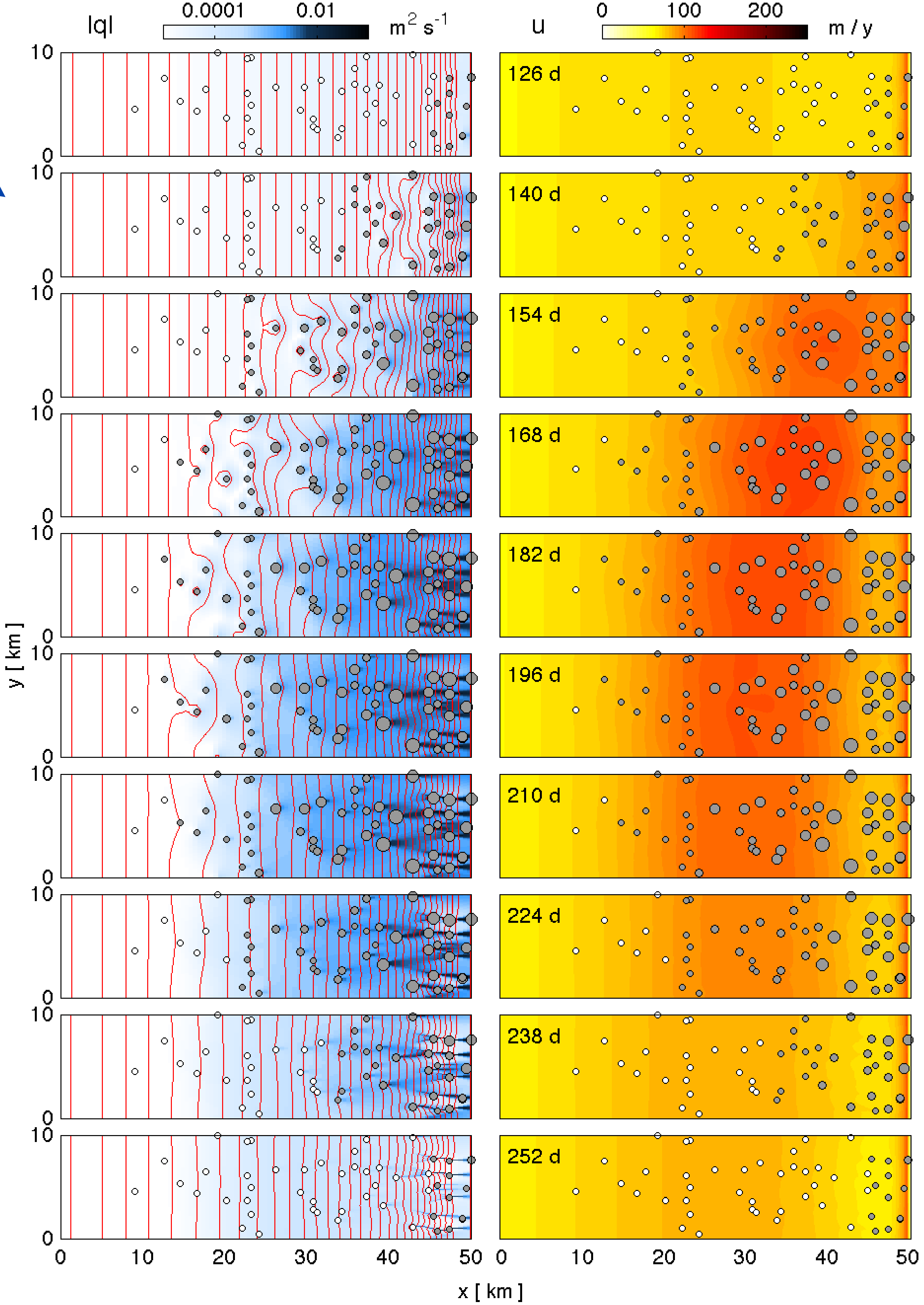


Channel segments connected on a planar graph, coupled to a continuum ‘sheet’.



Werder et al 2013

Subglacial discharge
(areal m^2/s)



Ice speed

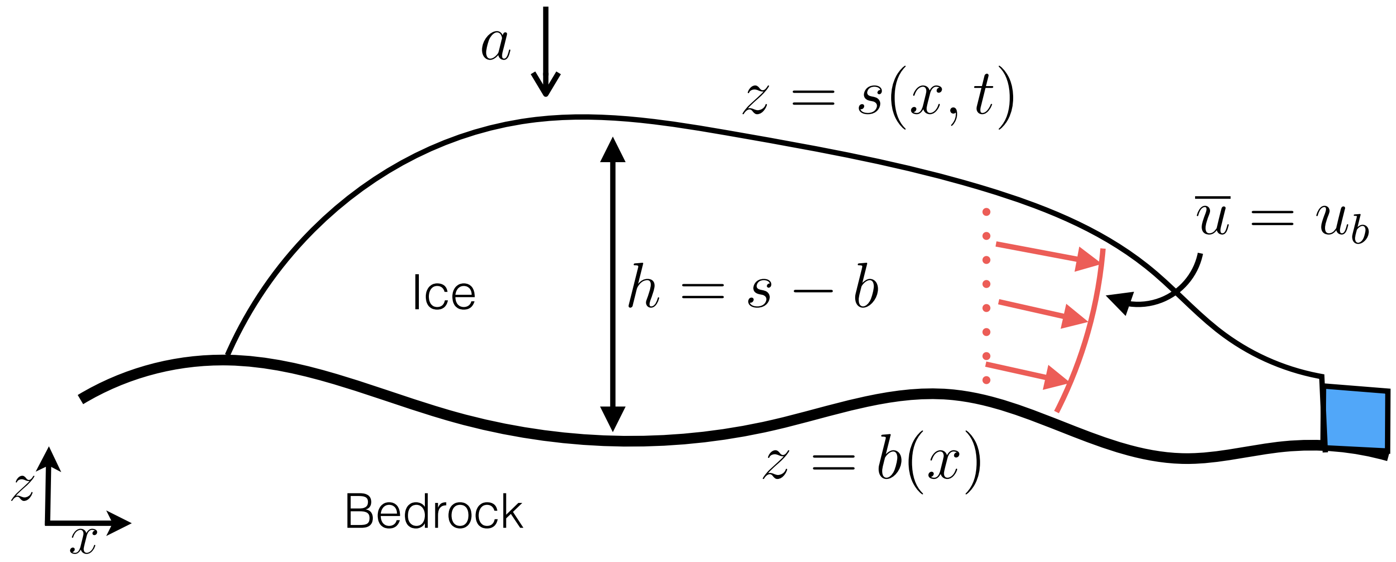
$$\tau_b = \mu N u_b$$

Time

Hewitt 2013, EPSL

Mathematical model

annually averaged



- Vertically-integrated mass conservation

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = a \qquad q = h\overline{u}$$

$a = \lambda(s - s_e)$ net accumulation - melting

s_e equilibrium line altitude (ELA)

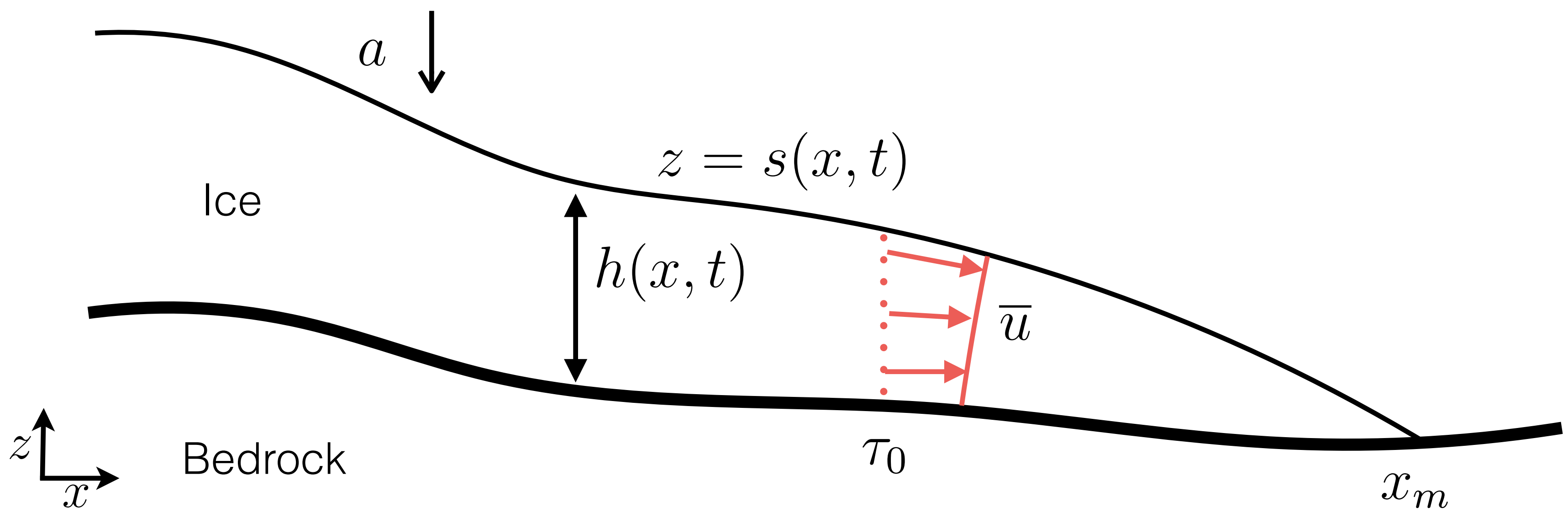
- Force balance + friction parameterisation

$$\tau_b = -\rho_i g h \frac{\partial s}{\partial x} + \frac{\partial}{\partial x}\left(4h\eta_i \frac{\partial \overline{u}}{\partial x}\right)$$

$\tau_b = \mu \langle N \rangle = \tau_0$ bed ‘strength’

Goal: consider effect of a long-term change in τ_0

Land-terminating glacier



- Boundary conditions $h = 0, \quad q = 0 \qquad \text{at} \qquad x = x_m(t)$

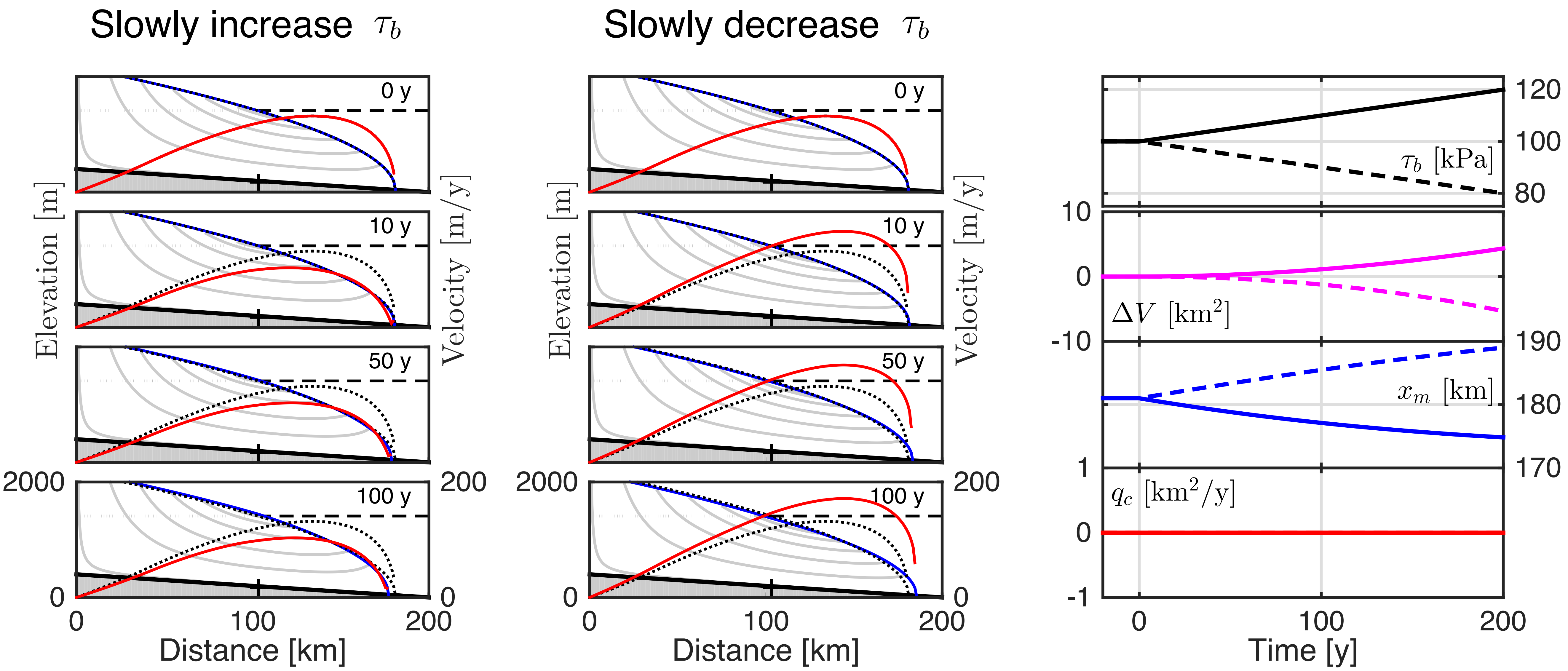
Force balance $\tau_0 = -\rho_i g h \dfrac{\partial s}{\partial x}$ → critical ‘yielding’ geometry + $V = \displaystyle\int_0^{x_m} h \, \mathrm{d}x$

e.g. for a flat bed profile $h = \sqrt{\dfrac{2\tau_0}{\rho_i g}}(x_m - x)^{1/2}$

Mass conservation → $\dfrac{\mathrm{d}V}{\mathrm{d}t} = \displaystyle\int_0^{x_m} a \, \mathrm{d}x$ + ice velocity / flux $q = \displaystyle\int_0^{x} \left(a - \dfrac{\partial h}{\partial t} \right) \mathrm{d}x$

cf. Nye 1951, Weertman 1961, 1976

Land-terminating glacier

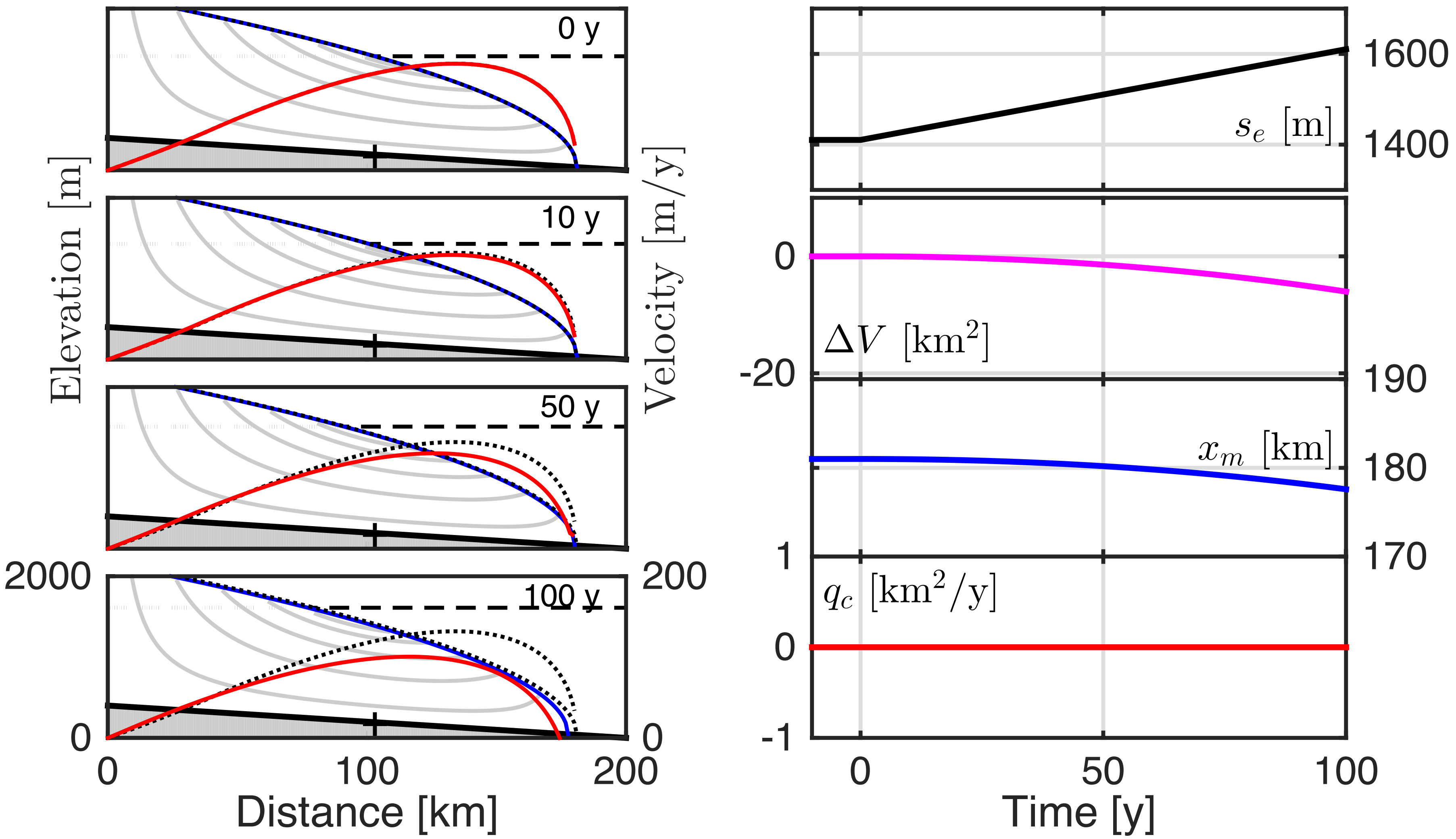


→ Increasing the bed strength causes **decrease in velocities** and **mass gain**

Decreasing the bed strength causes the opposite

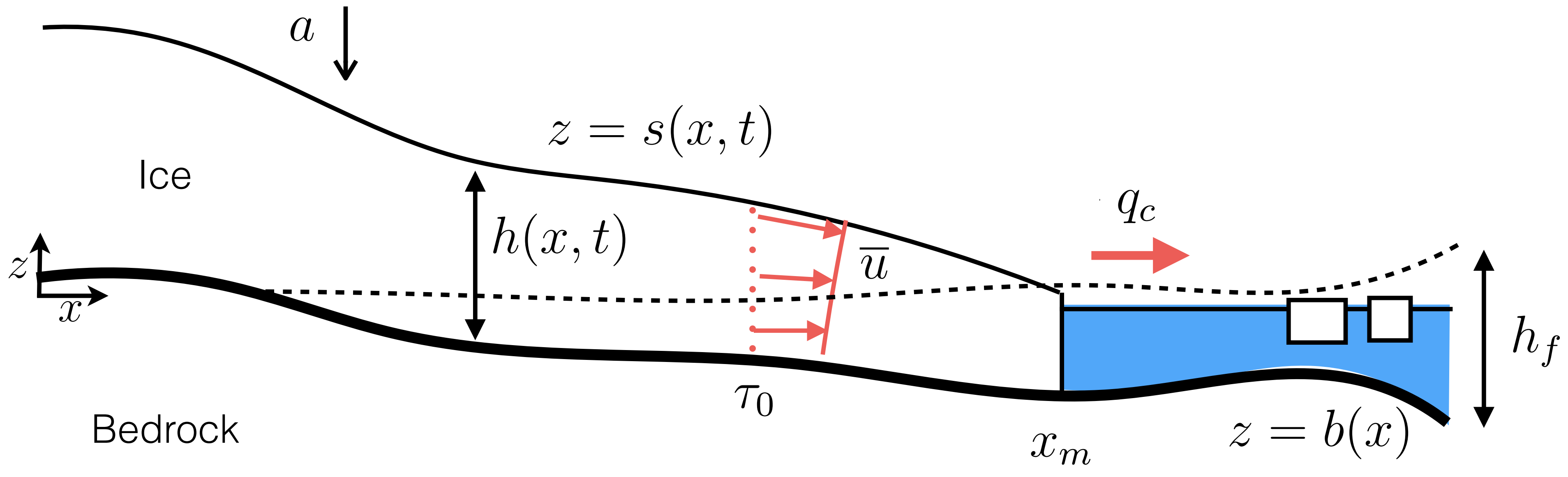
Land-terminating glacier

Net accumulation $a = \lambda(s - s_e)$ s_e equilibrium line altitude (ELA)



→ Raising the ELA causes increased melting and decreased ice velocities

Marine-terminating glacier



- Boundary conditions $h\dot{x}_m = q - q_c \qquad 4h\eta_i \dfrac{\partial \overline{u}}{\partial x} = \dfrac{1}{2}\left(\rho_i g h^2 - \rho_o g b^2\right) \qquad \text{at} \qquad x = x_m(t)$

  + calving condition (prescribed ice depth) $h = f h_f(x_m)$ $\qquad f \geq 1$ flotation factor

  $h_f(x) = -\dfrac{\rho_o}{\rho_i} b(x)$ flotation thickness

- A near-terminus boundary-layer analysis determines a relation between ice depth and calving flux (cf. grounding-line flux).

$$\longrightarrow \quad \boxed{q \approx q_c = \frac{\rho_i g}{\eta_i \mu} \hat{Q}(f) h_f^3}$$

Time-lapse movie

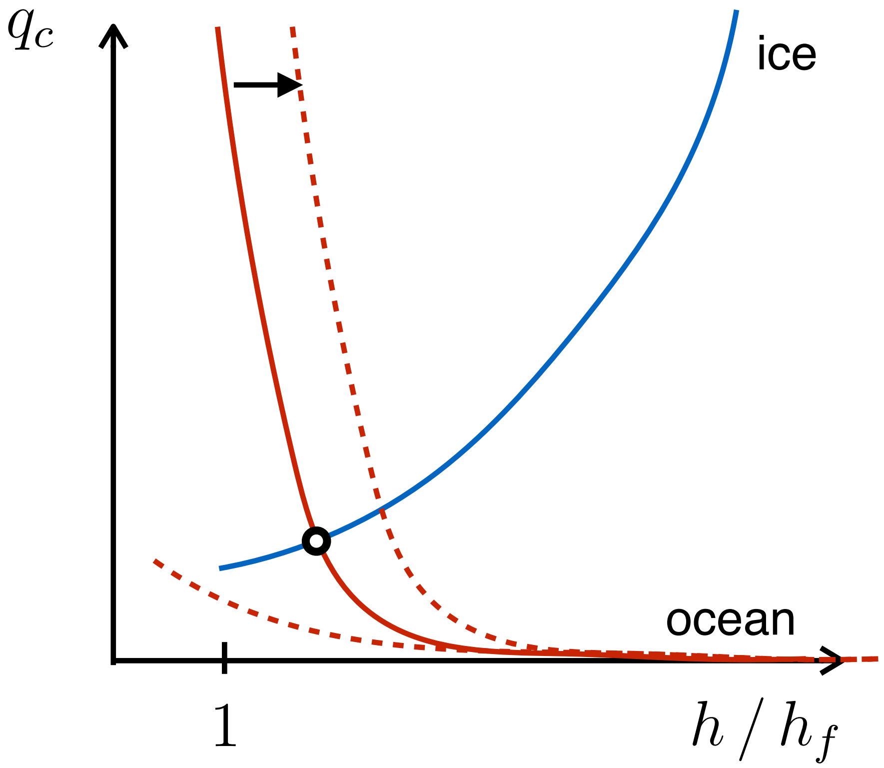
Extreme Ice Survey - Time-lapse camera
Columbia Glacier, Alaska

More on calving

- Dominant balance in kinematic condition $h_m \dot{x}_m = q - q_c$ is $q \approx q_c$

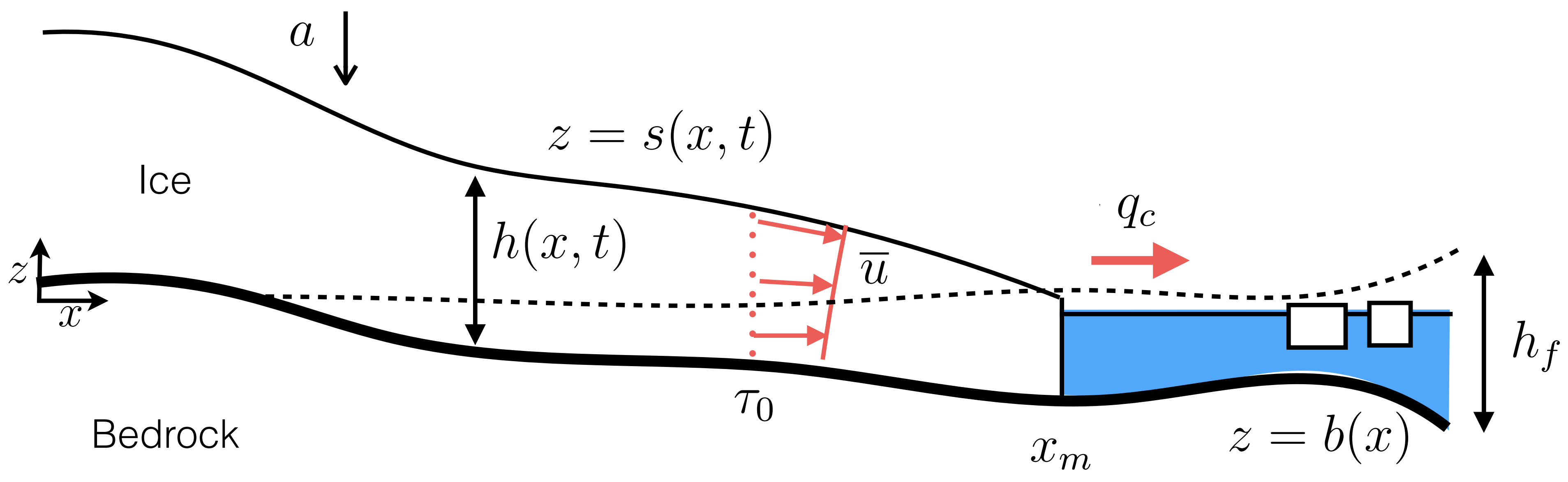
- Ice dynamics give a relationship between terminus ice depth and flux.

  (?) Ocean / calving dynamics provide a similar relationship - expect a rapid increase near flotation due to buoyancy and flexure.



- Intersection determines calving rate. (If no intersection, an ice shelf forms)

Marine-terminating glacier

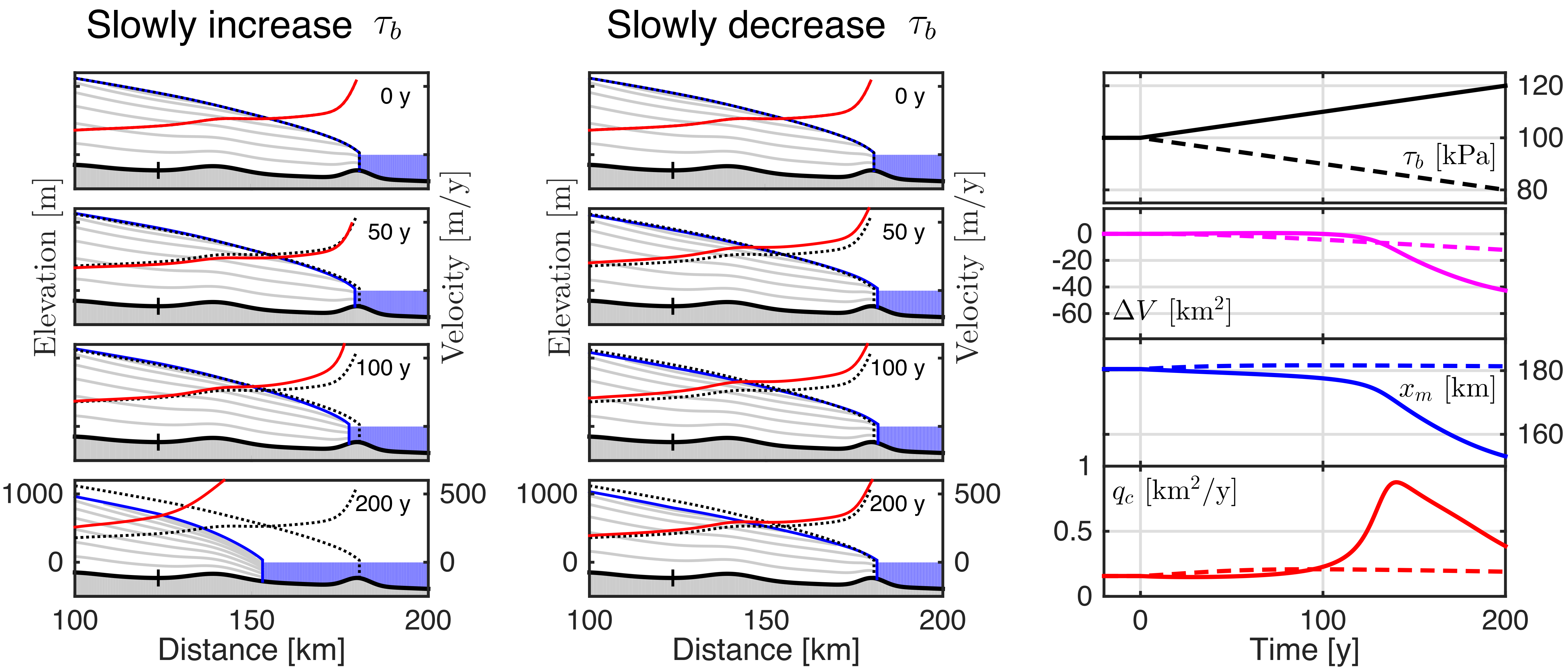


Force balance $\tau_0 = -\rho_i g h \dfrac{\partial s}{\partial x}$ ⟶ geometry + volume $V = \displaystyle\int_0^{x_m} h \, \mathrm{d}x$

Mass conservation ⟶ $\dfrac{\mathrm{d}V}{\mathrm{d}t} = \displaystyle\int_0^{x_m} a \, \mathrm{d}x - q_c$ $\qquad q_c = \mathcal{Q}(f, h_f)$

an ODE for the evolution of margin position

Marine-terminating glacier


Slowly increase τ_b

Slowly decrease τ_b

→ Increasing the bed strength causes **decrease in velocities** but can initiate margin retreat that results in **mass loss** (i.e. tidewater / marine-ice-sheet instability)

Summary

- Large seasonal velocity fluctuations driven by surface melt water can be qualitatively explained by changes in basal water pressure.
- Ice-sheet models should concentrate on fitting longer-term average velocities.
- A Coulomb friction law implies long-term ice velocity is kinematically controlled
- Slow-down of marine-terminating glaciers is not necessarily advantageous, since it tends to initiate retreat into over-deepenings.