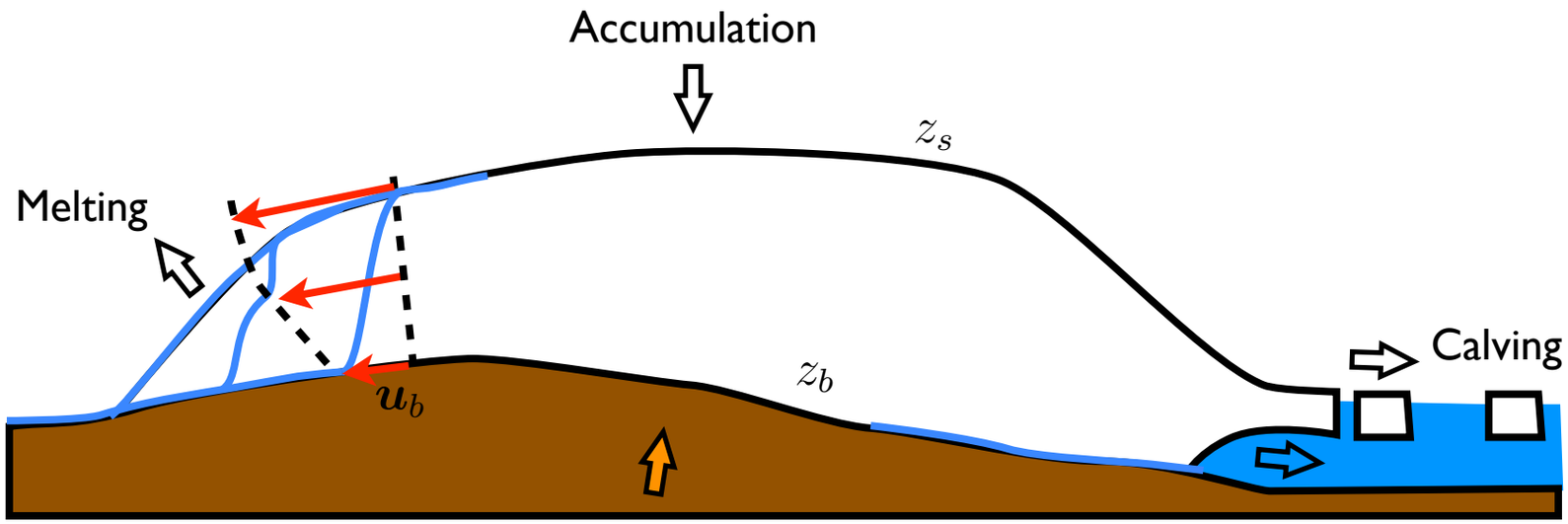
Modelling subglacial hydrology

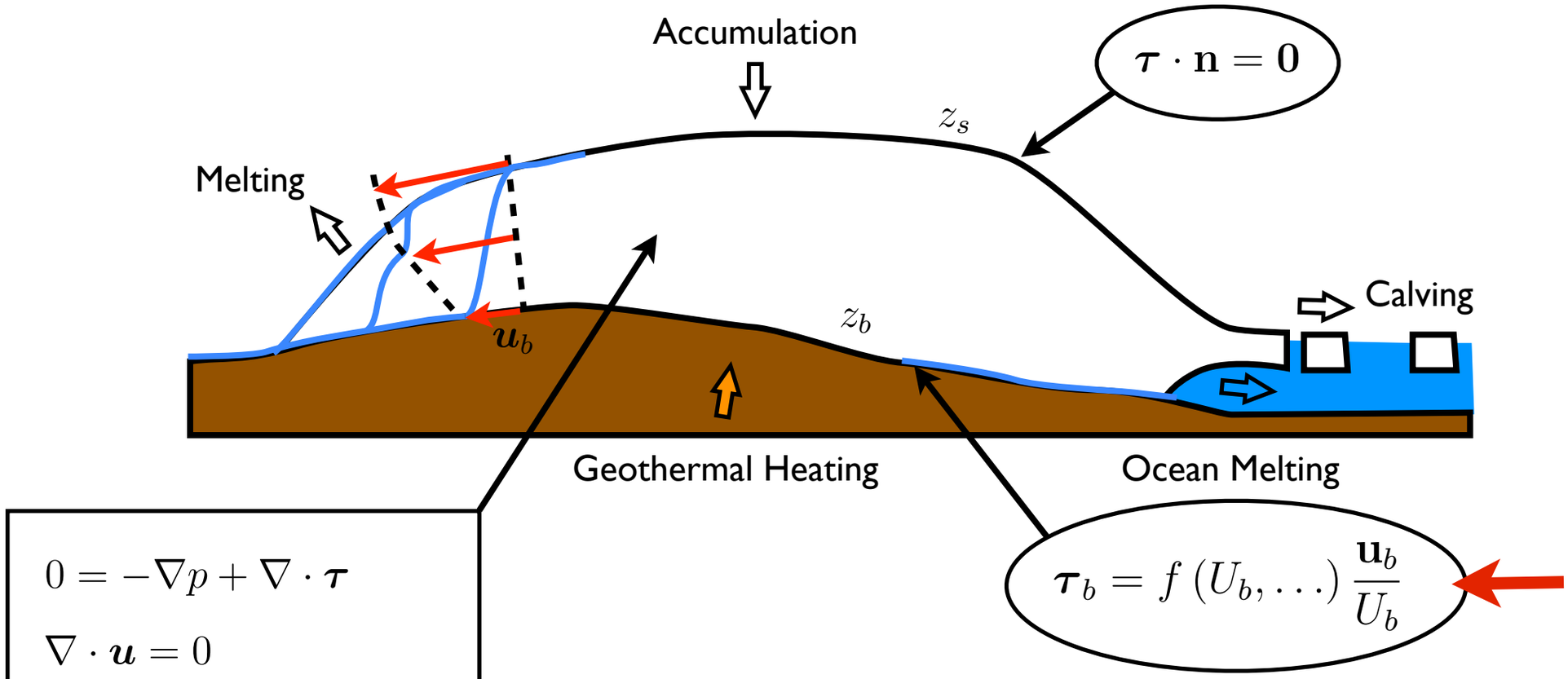
Ian Hewitt, University of Oxford



How does surface melting affect ice flow?

- A few observations
- Background subglacial hydrology
- Modelling melt water lubrication
- Discussion

Modelling ice flow



Current ice-sheet models

Fit spatially varying roughness to observed surface velocities ⇨ $f(x, y)$

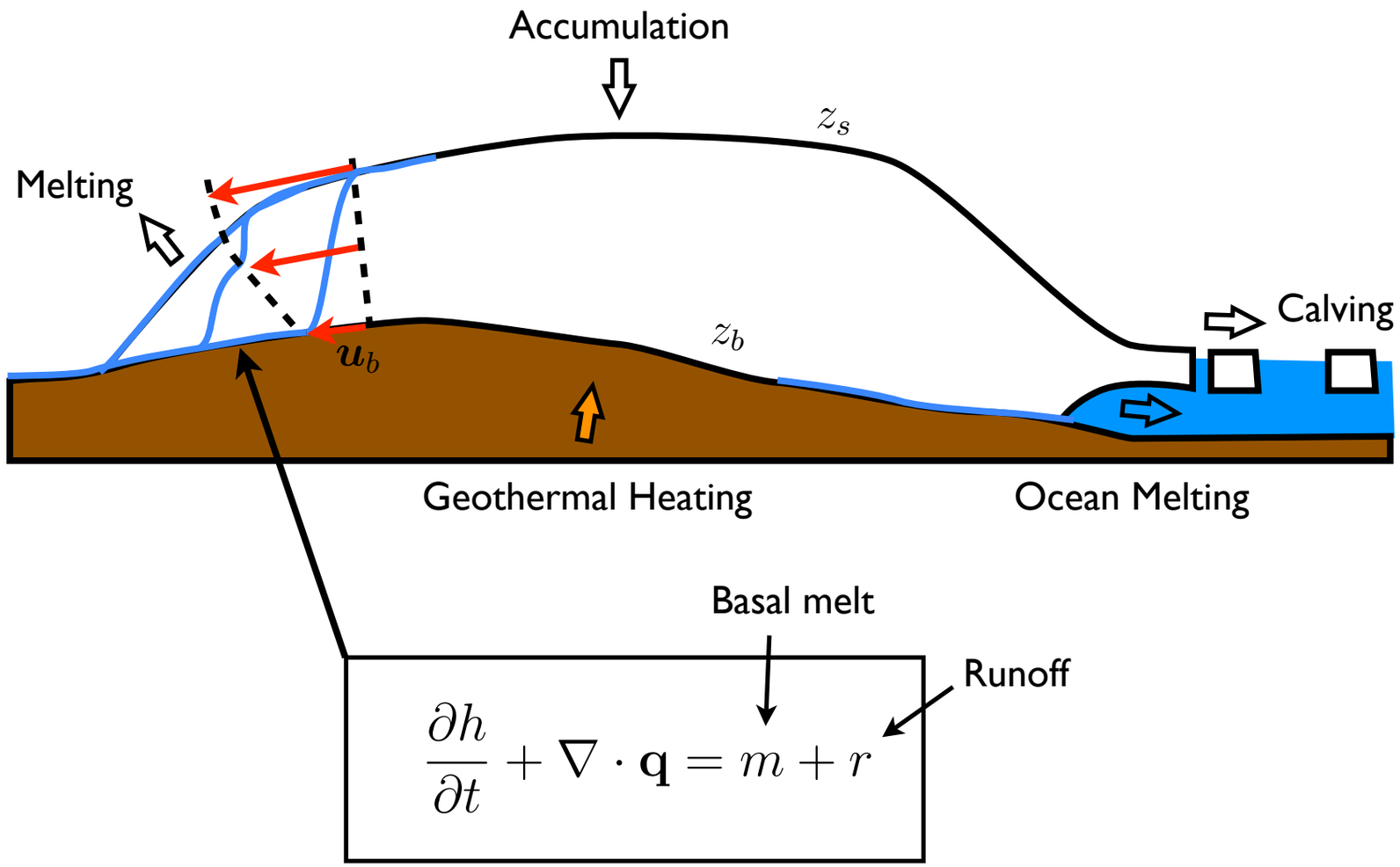
Drawback - no dynamics!

Lubrication changes can be parameterized Parizek & Alley 2004, Shannon et al 2013

Drainage model needed to calculate the … for friction law

eg effective pressure for $\tau_b = f(U_b, N)$

Modelling subglacial hydrology



Water depth h

Water flux $\mathbf{q}$

Effective pressure $N = p_i - p_w$

Need some physics to relate these

Ice flow

Supraglacial stream

Moulin

Erosional deposit

Subglacial channel

Crevasse

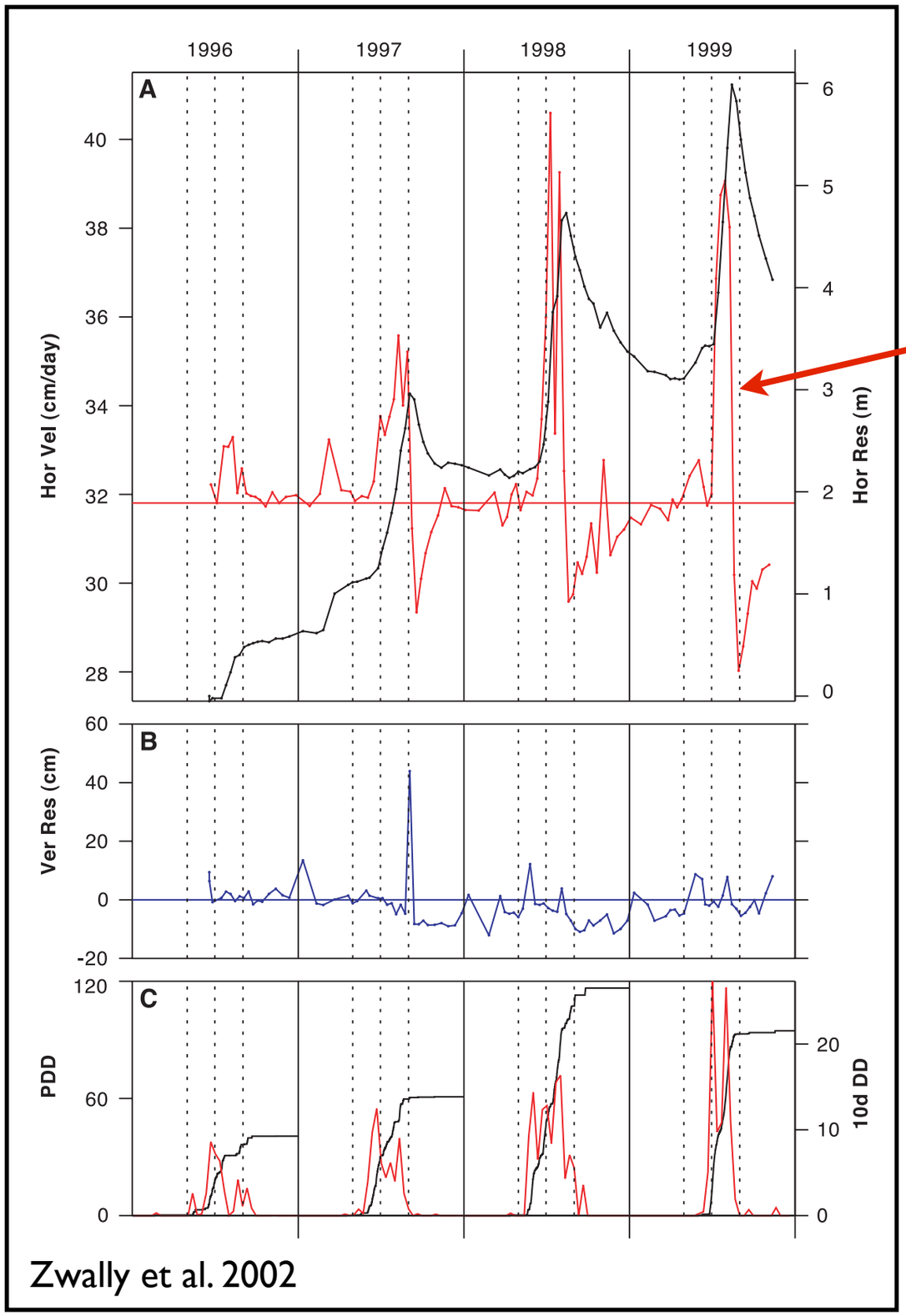
Greenland

Net mass loss 215 [157-274] Gt/yr for 2002-2011 (around 0.6 mm/yr sea level rise)

- part due to increased surface melting
- part due to dynamic thinning (i.e. ice movement)

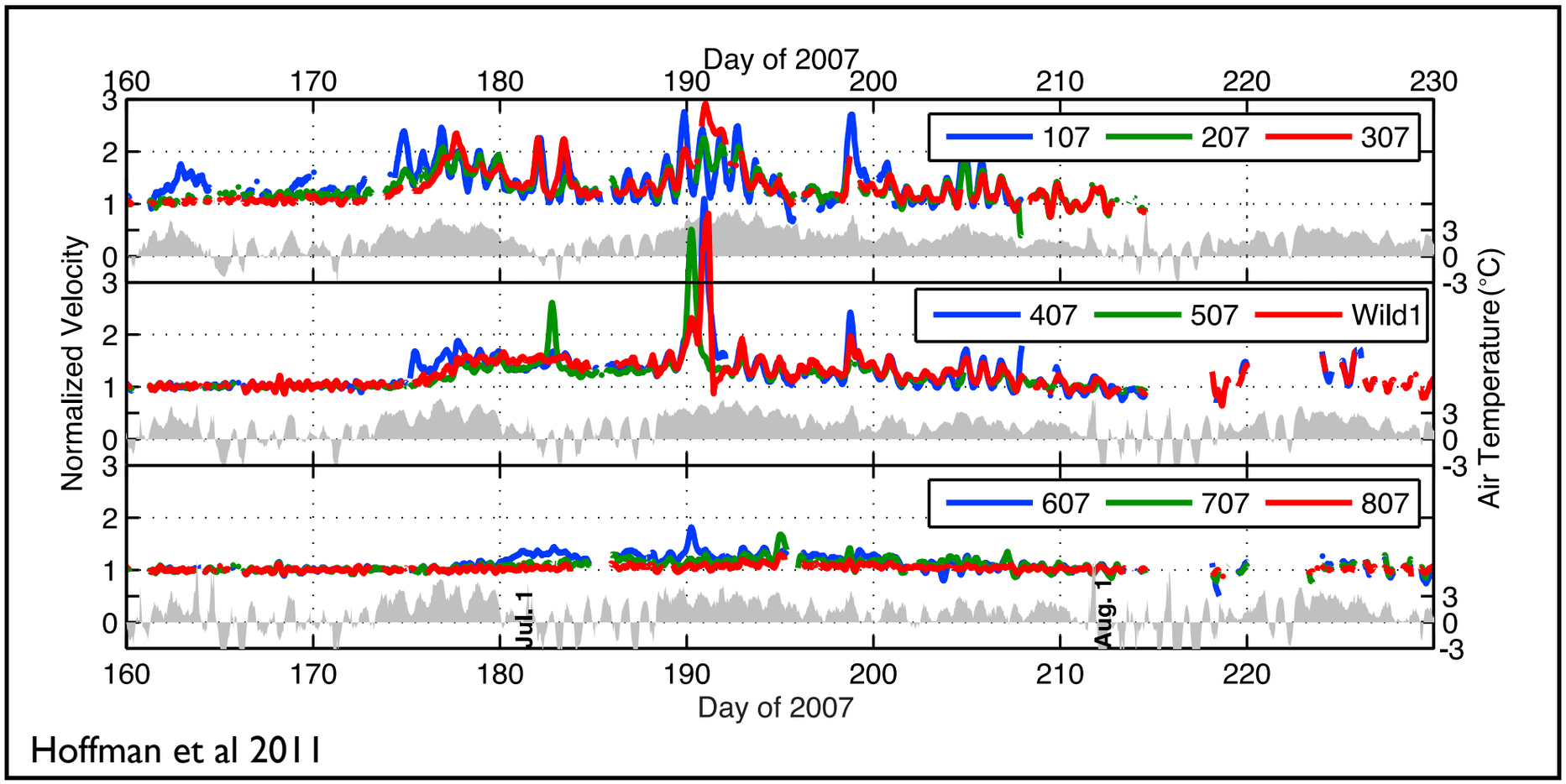
Greenland surface velocity

Annual acceleration and deceleration due to surface melt water



Zwally et al. 2002

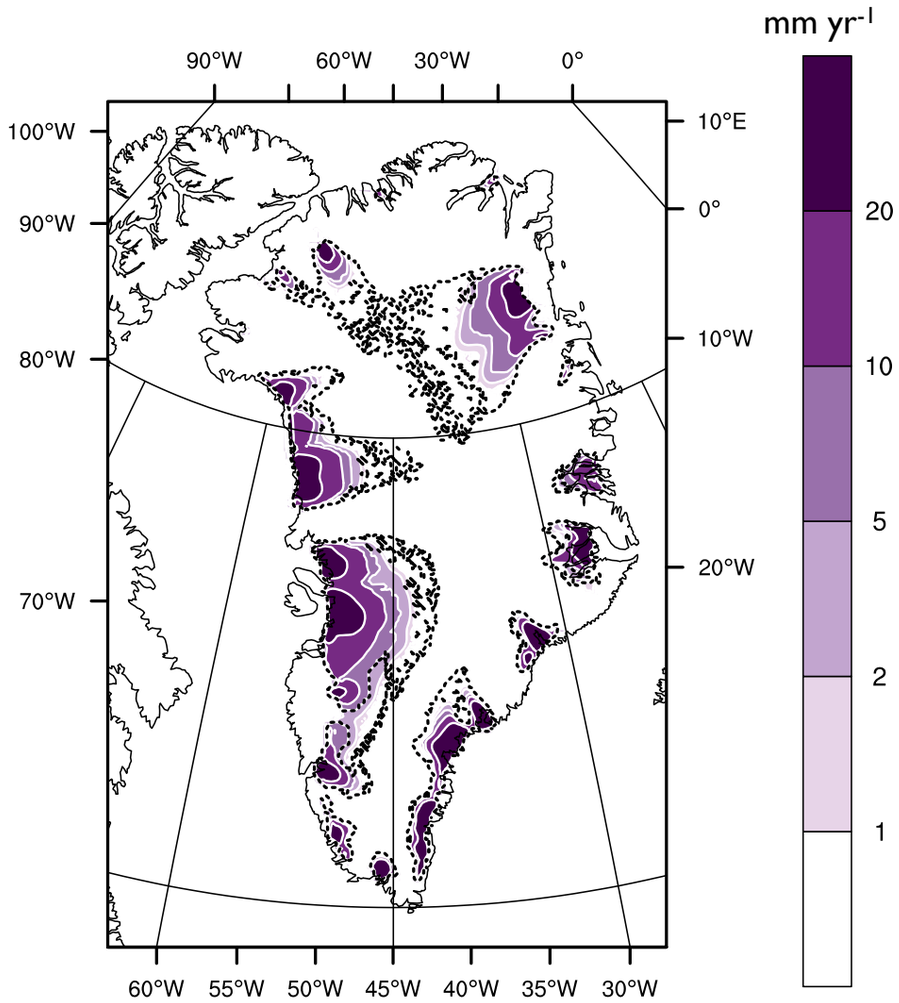
Greenland surface velocity



van de Wal et al 2008, Joughin et al 2008, Shepherd et al 2009, Bartholomew et al 2010, Sundal et al 2011, Sole et al 2013

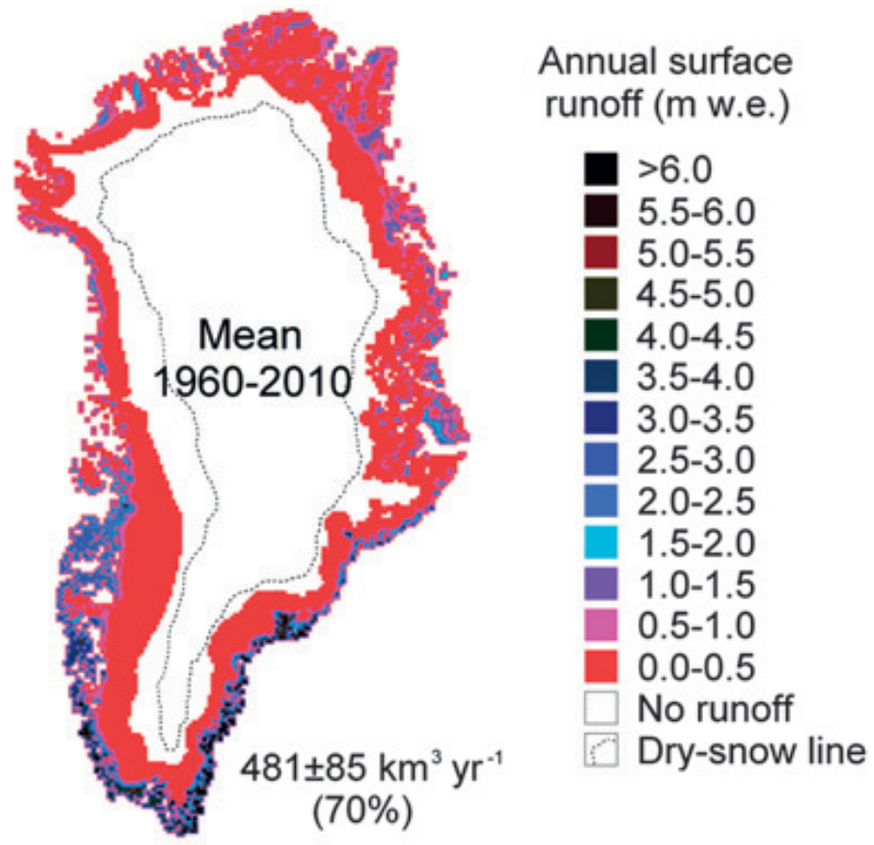
Water sources in Greenland

Basal melting ~5 mm yr^{-1}



Aschwanden et al 2012

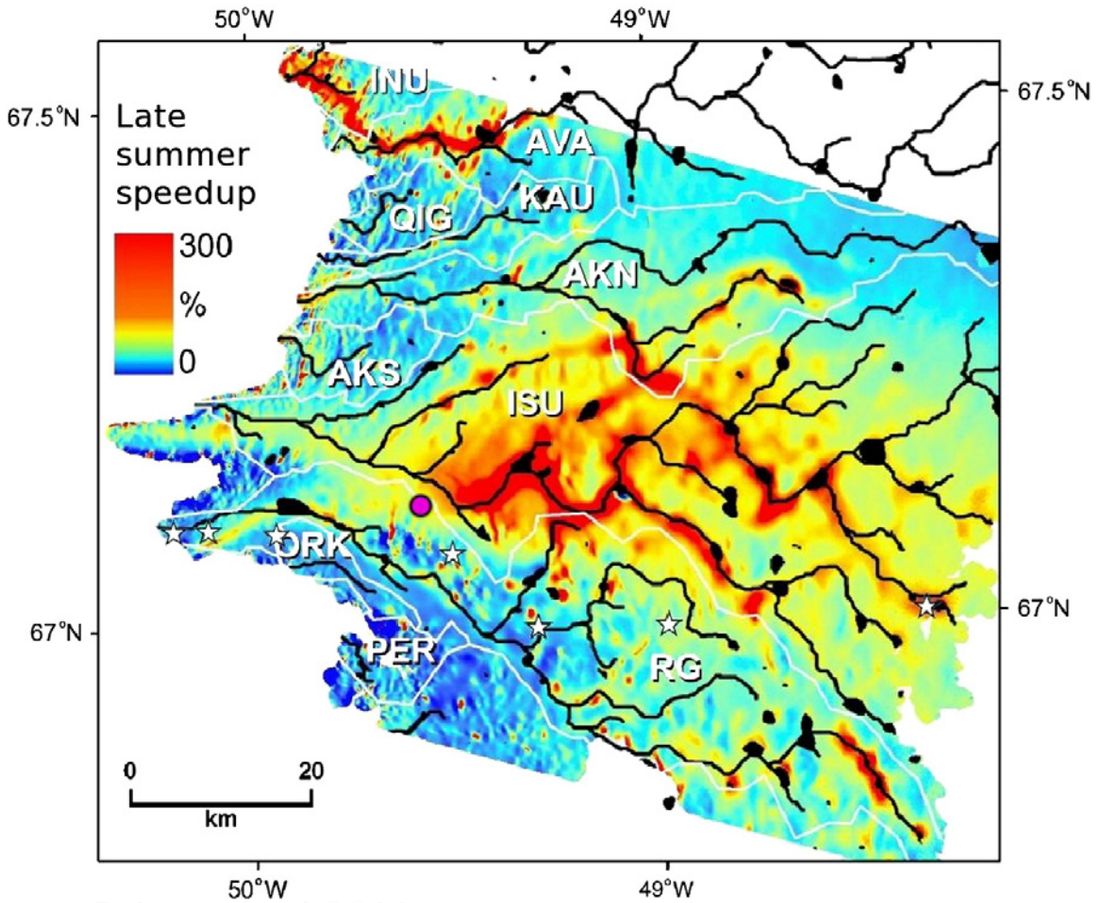
Surface runoff ~1000 mm yr^{-1}



Mernild & Liston 2012

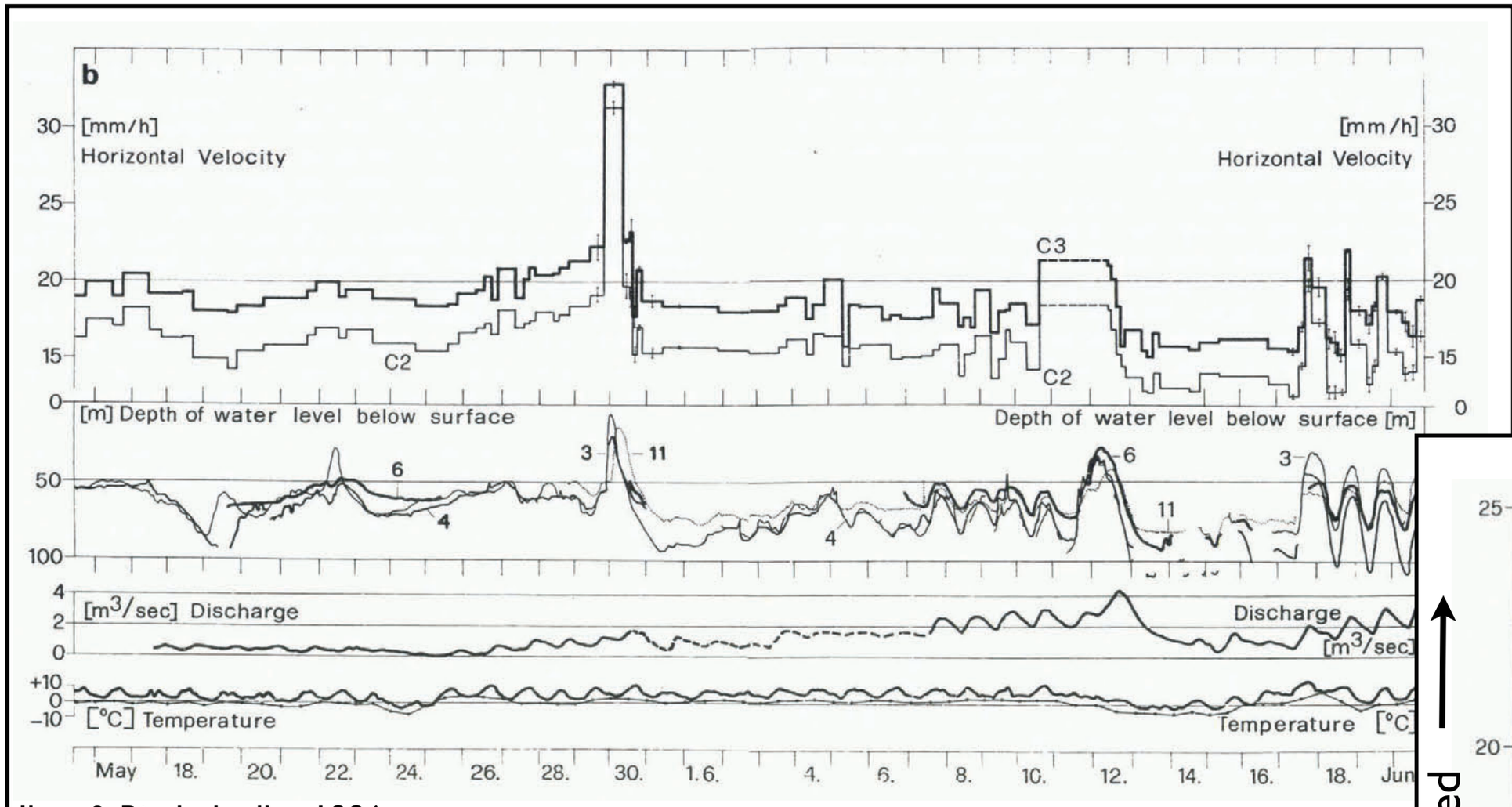
Quite different from Antarctica

Greenland summer speed-up



Palmer et al 2011

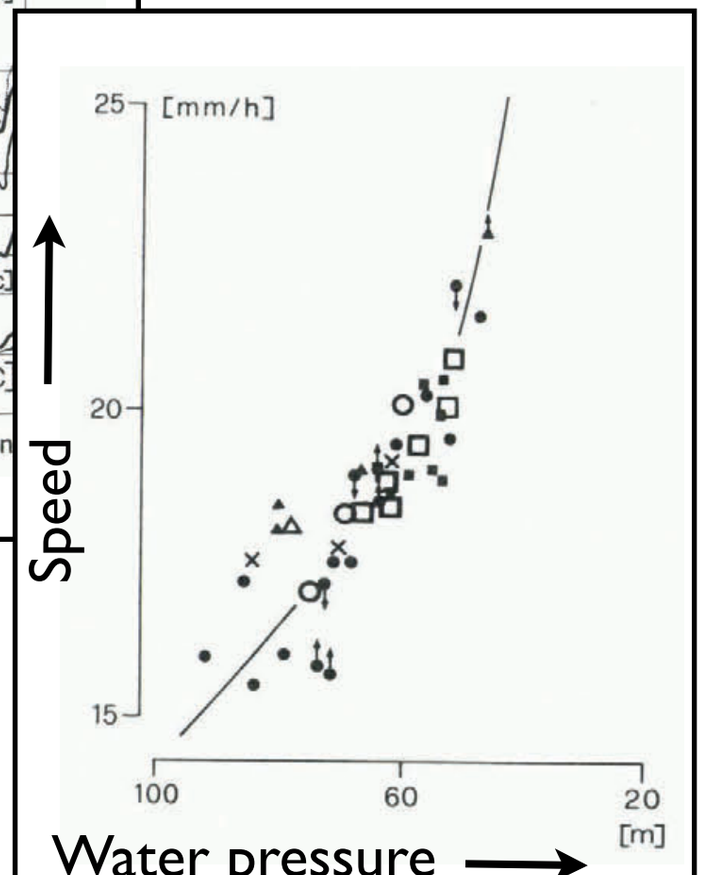
Sliding and water pressure



Iken & Bindschadler 1986

Speed

Water pressure



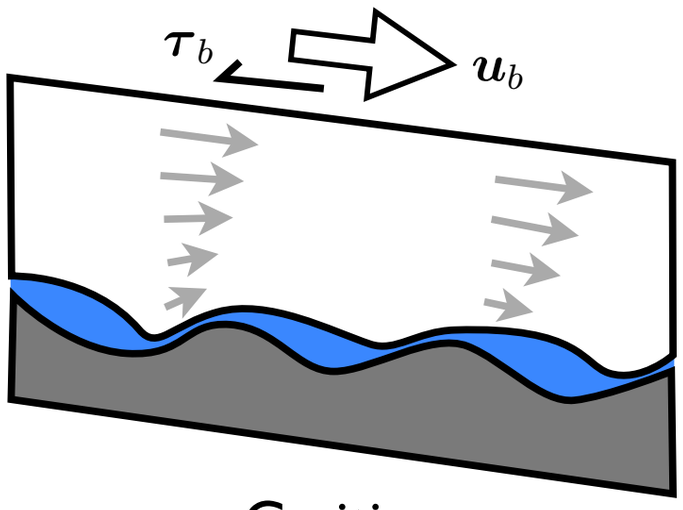
Iken & Bindschadler 1986

Relationship between ice speed and borehole water pressure

Basal friction laws

Sliding depends on *effective pressure* $N = p_i - p_w$

Lower effective pressure ⇨ Less drag, More sliding



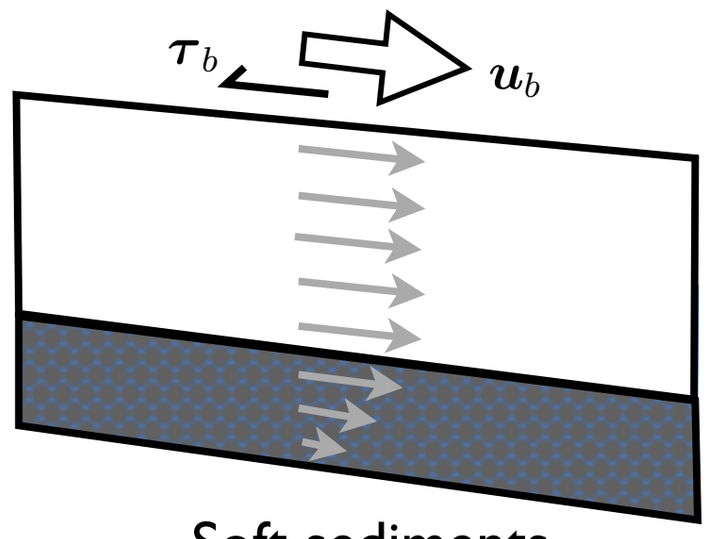
Cavities

$$\tau_b = C U_b^p N^q$$

Lliboutry 1979, Budd et al 1979, Fowler 1986

$$\tau_b = \mu N \left(\frac{u_b}{u_b + \lambda N^n} \right)^{1/n}$$

Schoof 2005, Gagliardini et al 2007

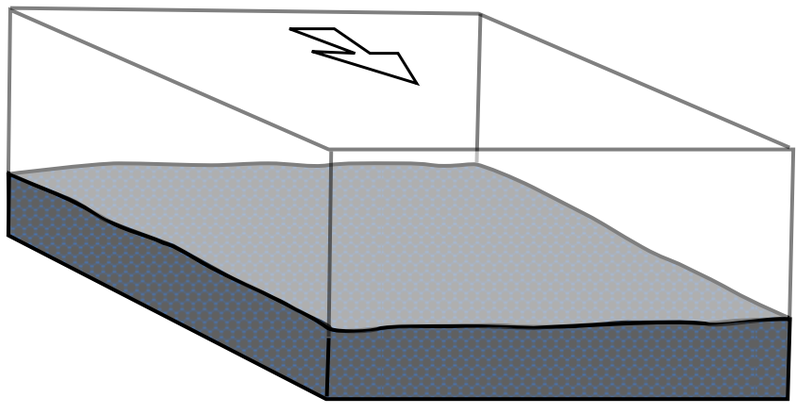


Soft sediments

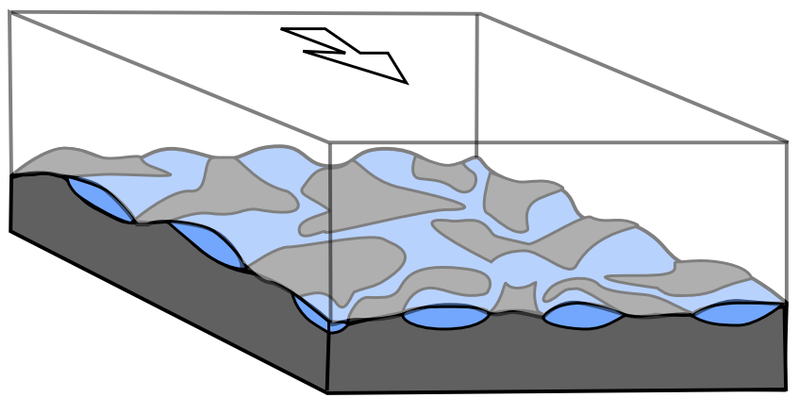
$$\tau_b = \mu N$$

Kamb 1991, Tulaczyk 2000

Subglacial drainage

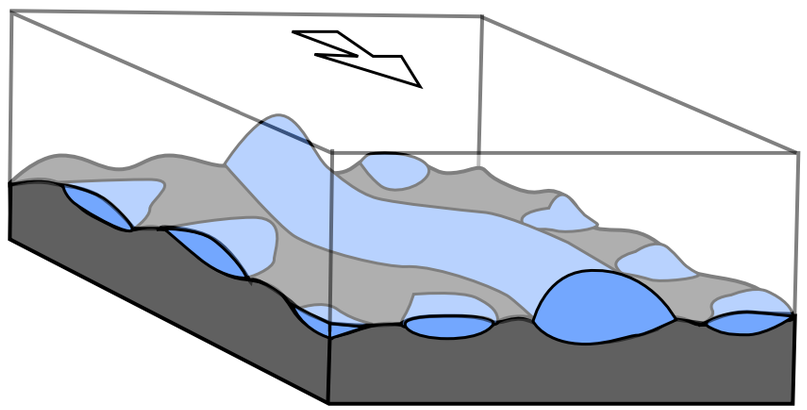


Saturated sediments



‘Distributed’ systems

- uneven water films
  Alley 1989, Creyts & Schoof 2009
- canals
  Walder & Fowler 1994
- linked cavities
  Walder 1986, Fowler 1986, Kamb 1987
- porous films
  Flowers & Clarke 2002, Hewitt 2011



‘Channel’ systems

Röthlisberger 1972, Nye 1976

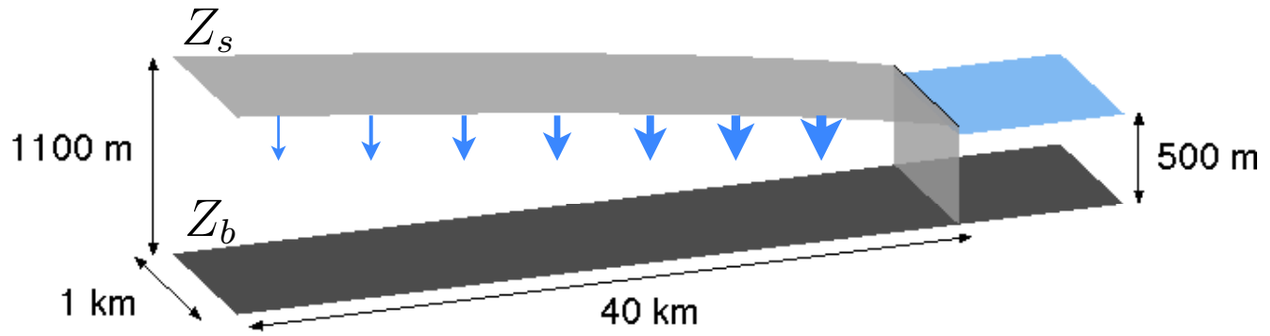
Steady state theory

Potential gradient

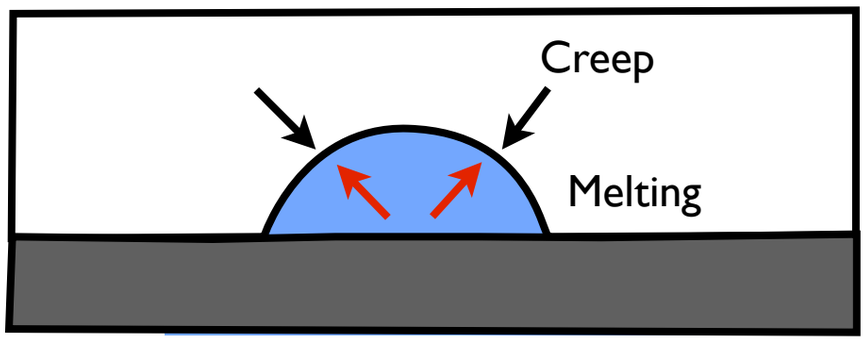
$$\mathcal{G} = -\nabla\phi$$

Hydraulic potential

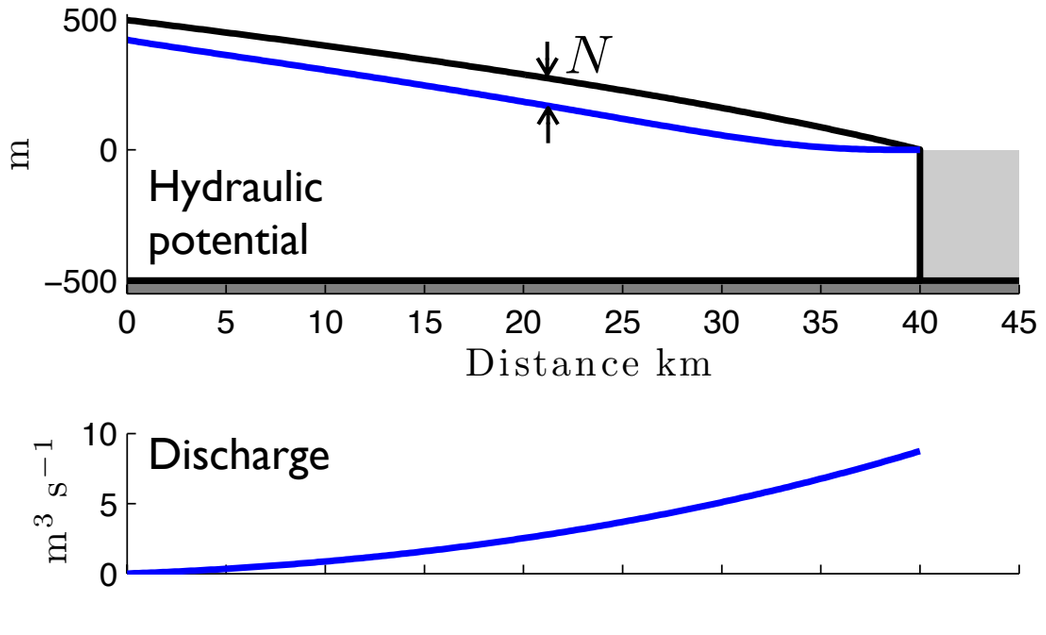
$$\phi = \rho_w g Z_b + p_w$$



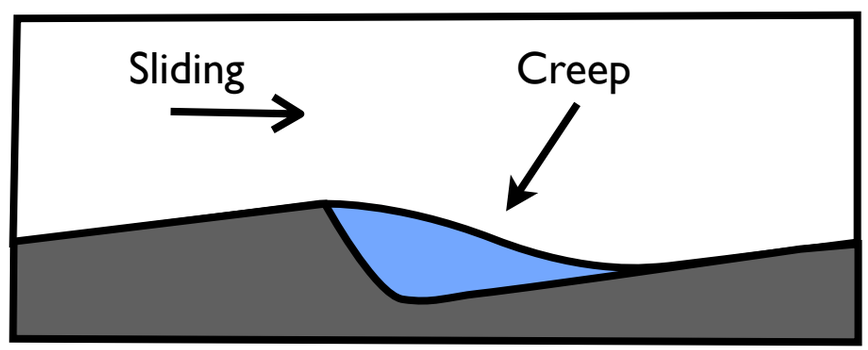
Channel theory



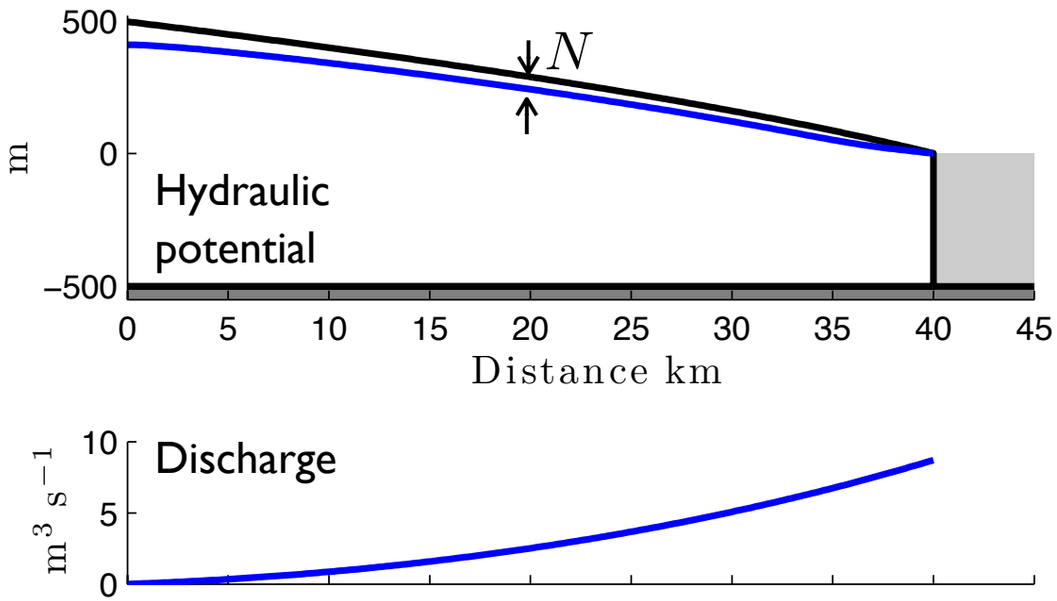
$$N \propto \mathcal{G}^{11/24} Q^{1/12}$$



Cavity theory



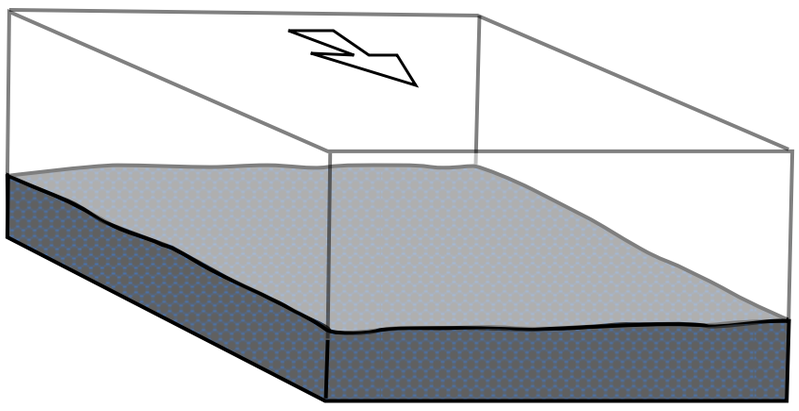
$$N \propto \frac{U_b^{1/3} \mathcal{G}^{1/9}}{Q^{1/9}}$$



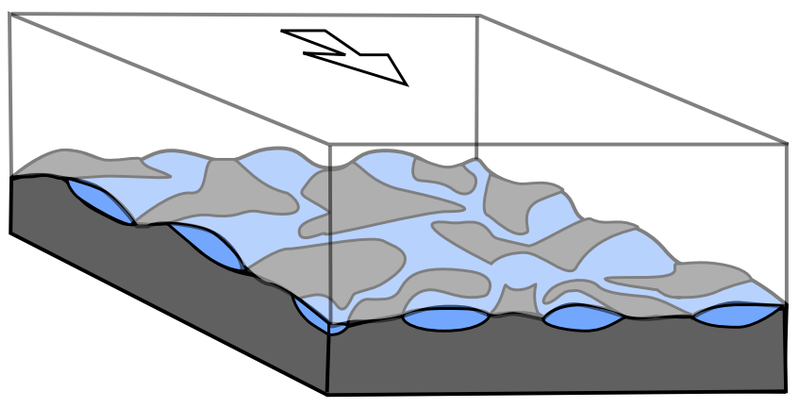
Observations suggest the system is rarely in a steady state

⇨ Evolution and transient behaviour is important

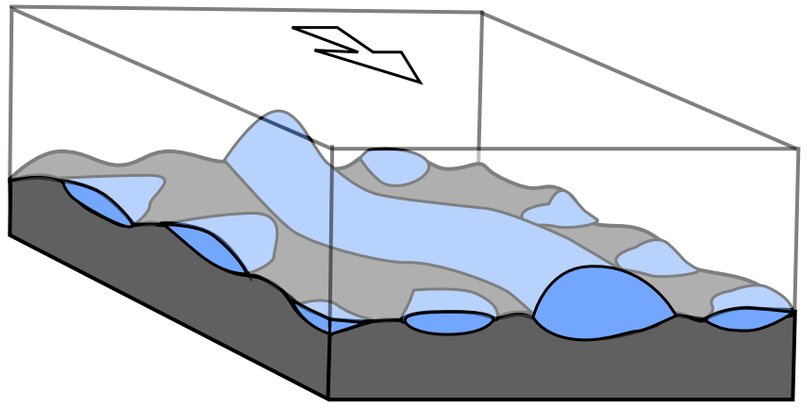
Large scale modelling



Saturated sediments



‘Distributed’ systems



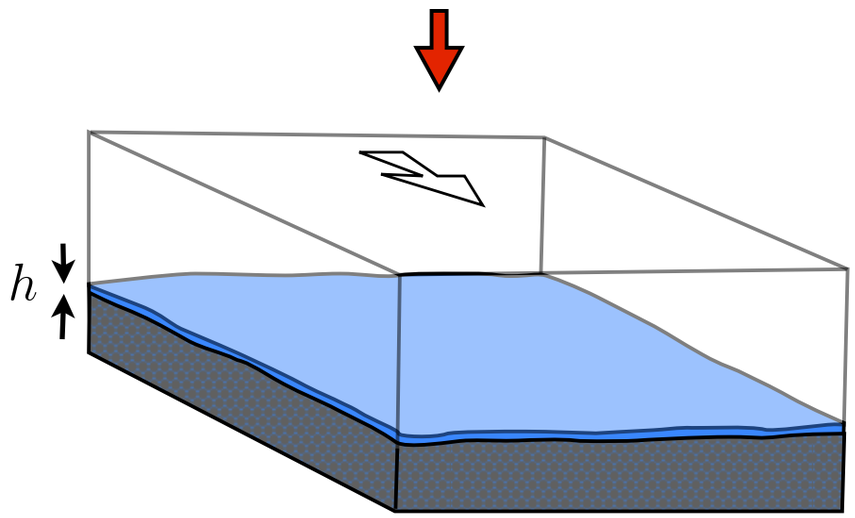
‘Channel’ systems

Sheet depth related directly to pressure

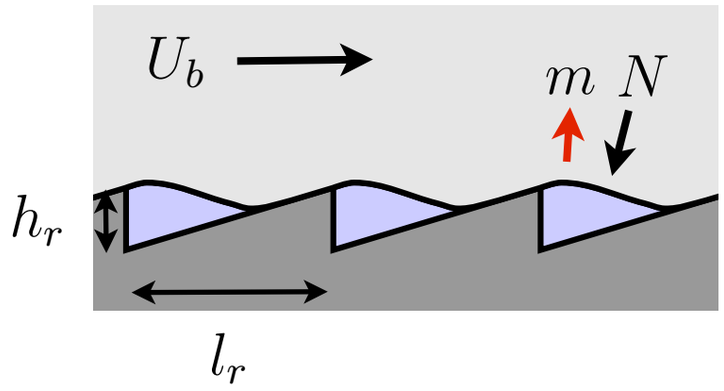
$$h(p_w, p_i)$$

or sheet depth evolves according to some law

$$\frac{\partial h}{\partial t} = \ldots$$



Distributed drainage model



h Porous sheet depth

Mass conservation

Sheet evolution

Runoff

$$\frac{\partial h}{\partial t} + \nabla \cdot \mathbf{q} = m + r \qquad \mathbf{q} = -K h^3 \nabla \phi$$

$$\frac{\partial h}{\partial t} = \frac{\rho_w}{\rho_i} m + U_b \frac{h_r - h}{l_r} - \frac{2A}{n^n} h N^n$$

Melting

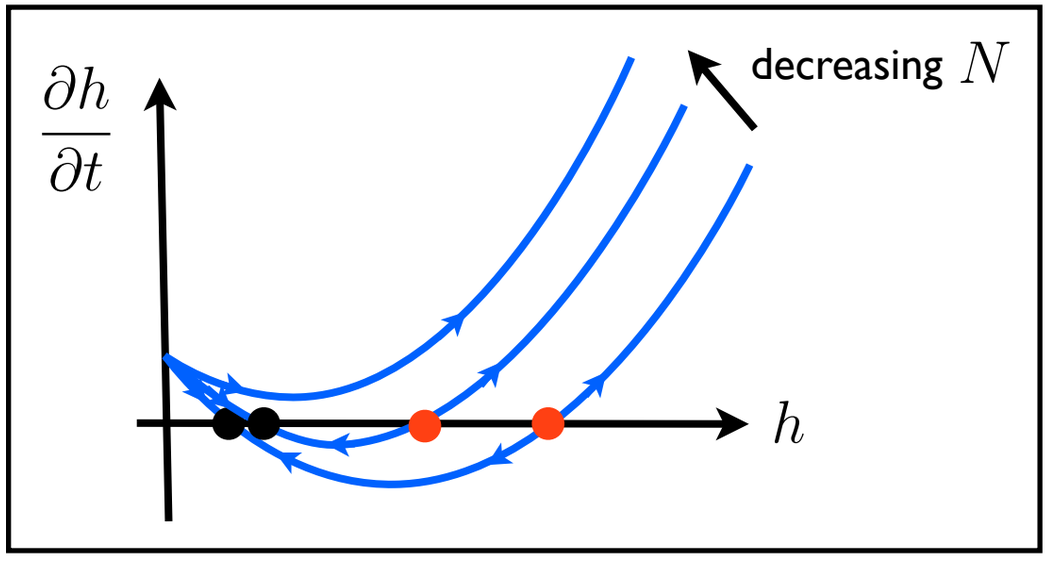
Sliding

Creep closure

$$m = \frac{G + \boldsymbol{\tau}_b \cdot \mathbf{u}_b + |\mathbf{q} \cdot \nabla \phi|}{\rho_w L}$$

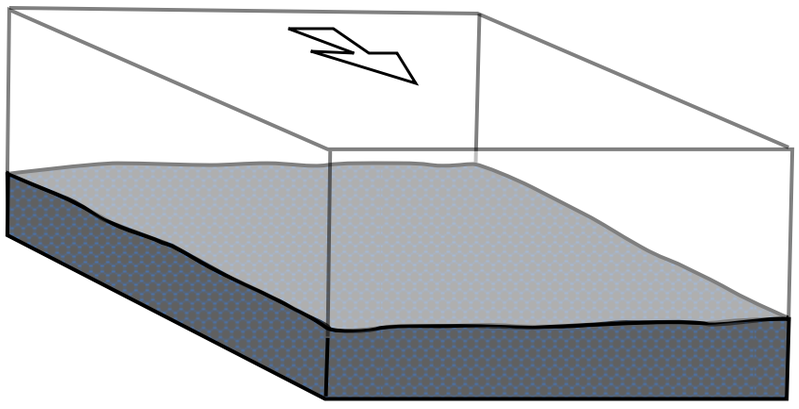
Dissipative heating

Walder 1986
Fowler 1986
Kamb 1987
Hewitt 2011
Schoof et al 2012

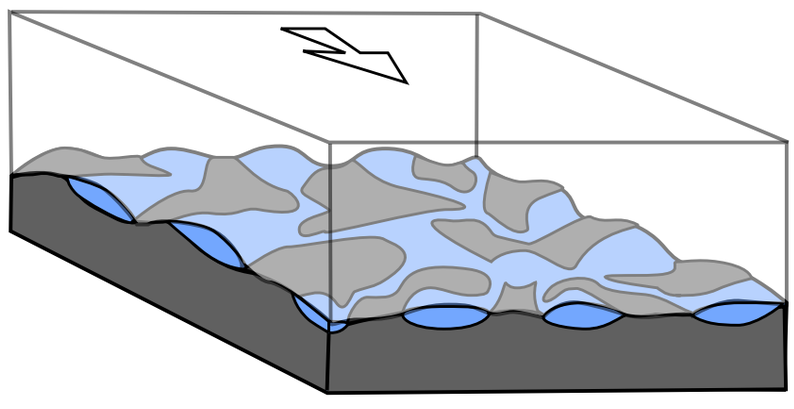


⇨ Unstable ‘channelization’ for large discharge

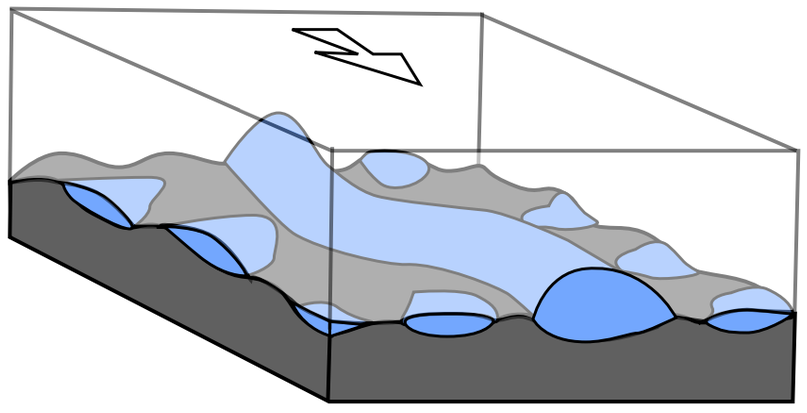
Large scale modelling



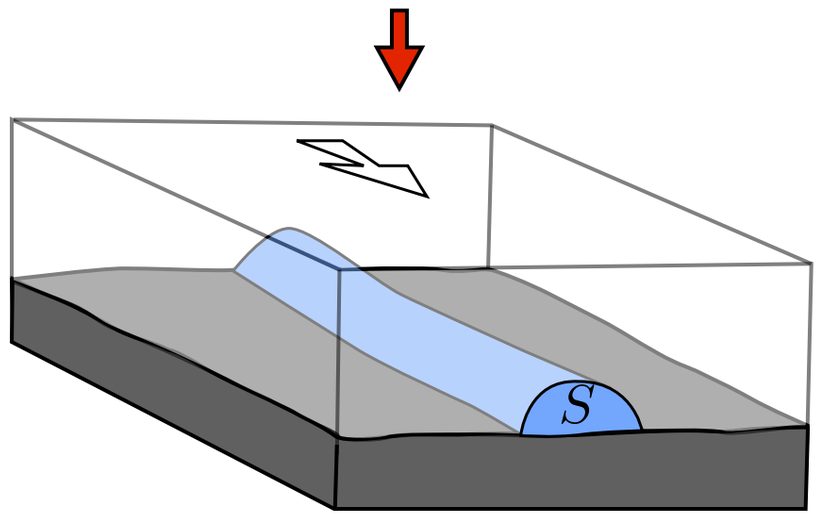
Saturated sediments



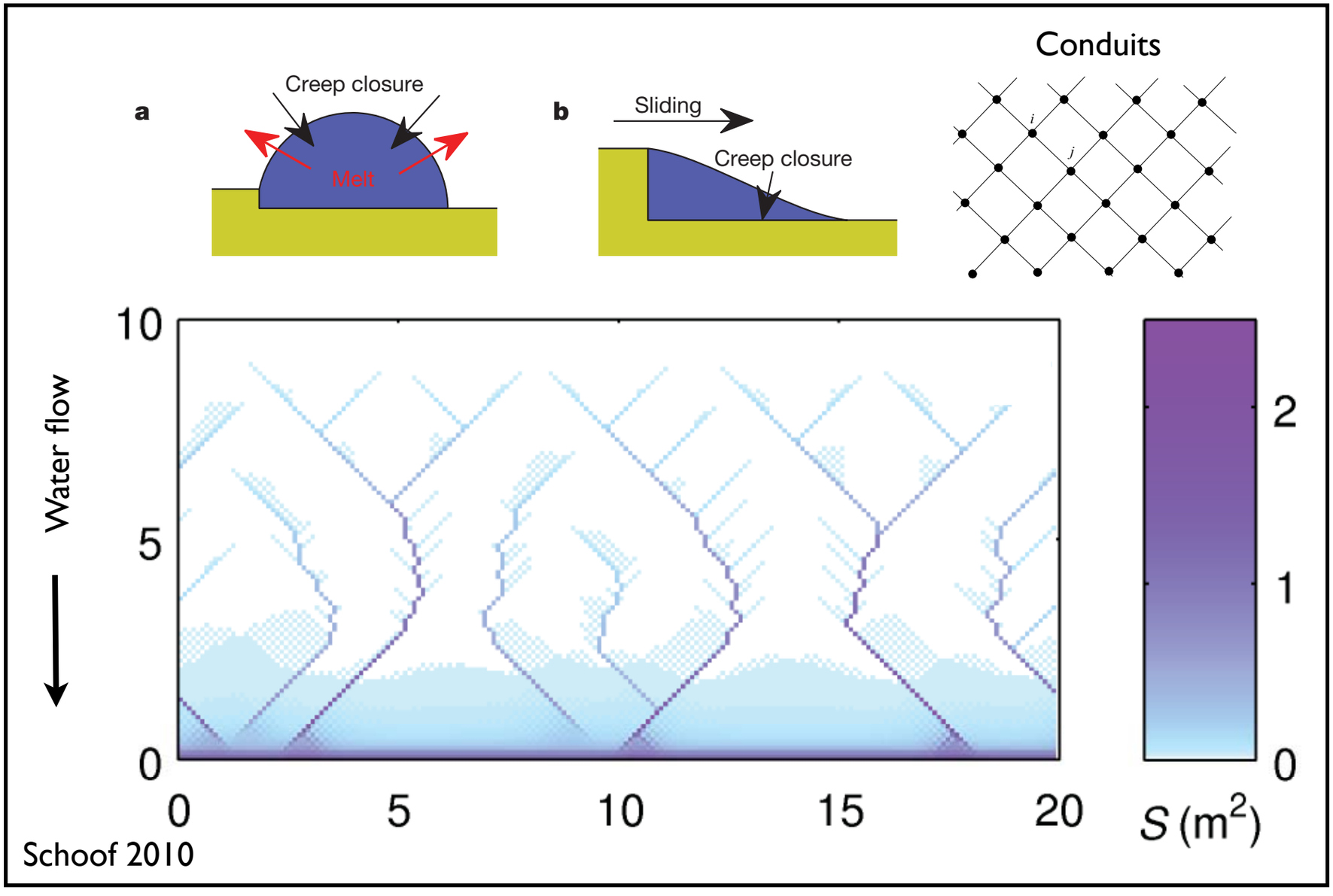
‘Distributed’ systems



‘Channel’ systems

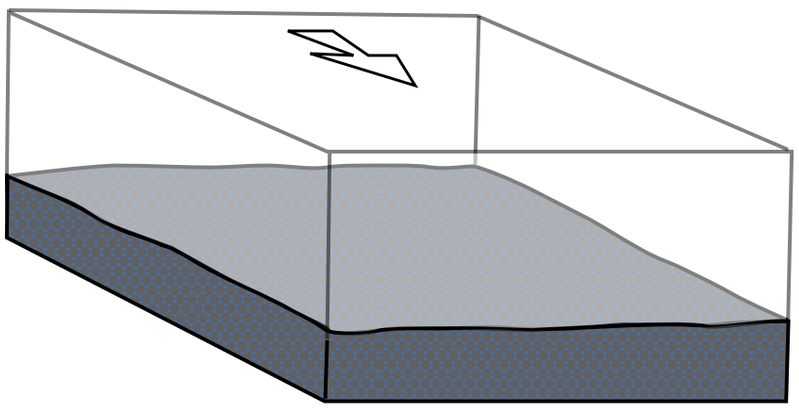


Conduit network

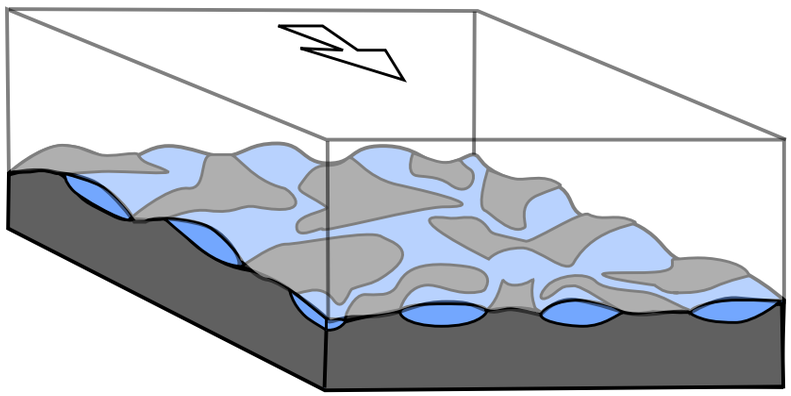


$$\frac{\partial S}{\partial t} = U_b \left(h_r - S/l_r\right) + \frac{\rho_w}{\rho_i} M - \frac{2A}{n^n} S N^n \qquad M = \frac{|Q \partial\phi/\partial s|}{\rho_w L} \qquad Q = -K_c S^{4/3} \frac{\partial \phi}{\partial s}$$

Large scale modelling



Saturated sediments



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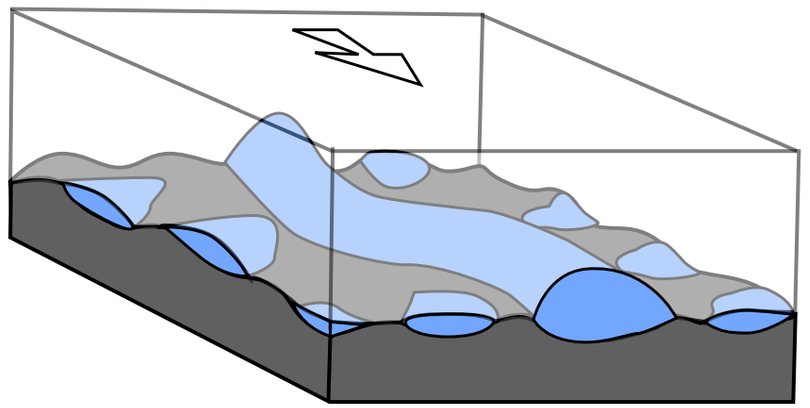

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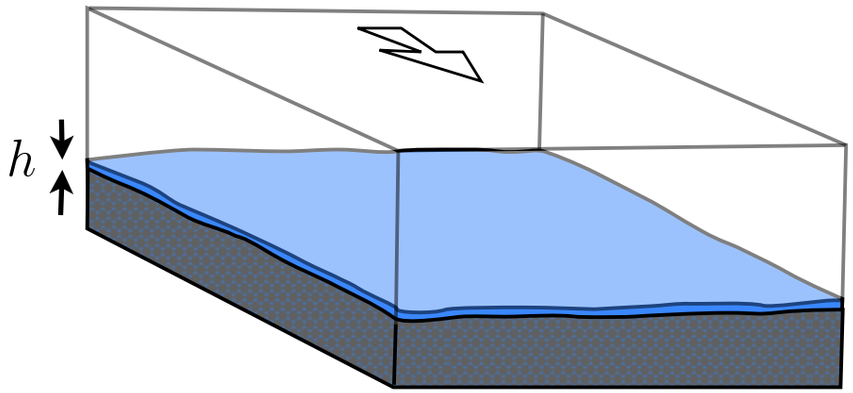
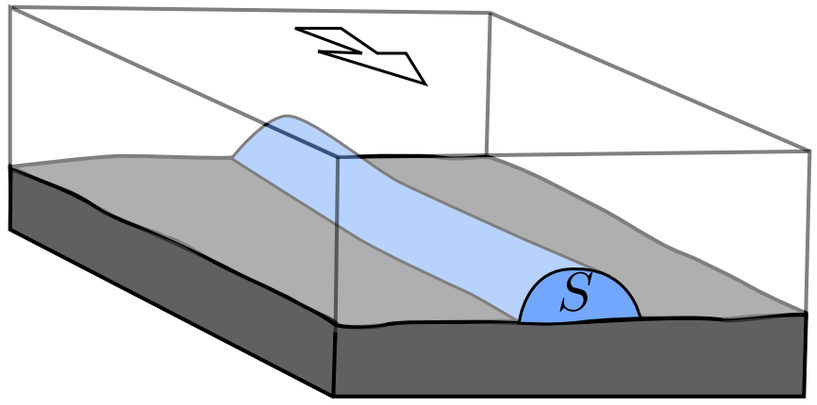
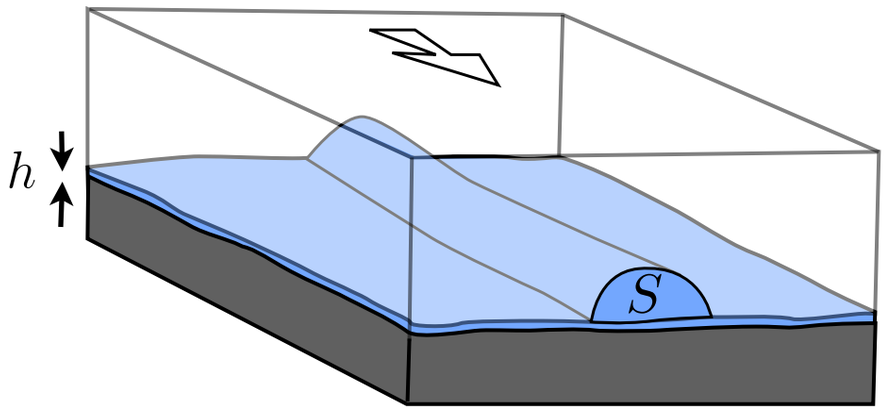
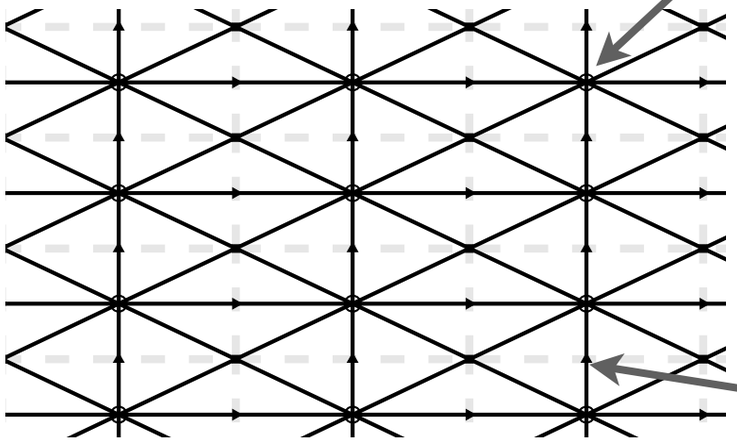


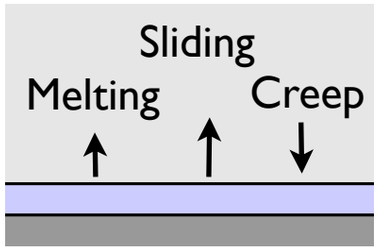




Combined drainage model



Sheet evolution

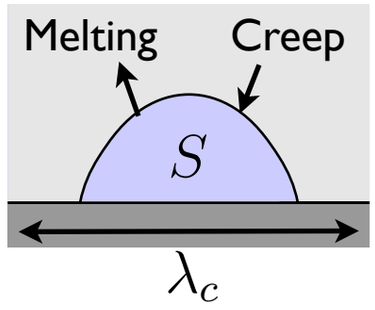


$$\frac{\partial h}{\partial t} = \frac{\rho_w}{\rho_i} m + U_b \frac{h_r - h}{l_r} - \frac{2A}{n^n} h N^n$$

Melting Sliding Creep closure

$$m = \frac{G + \boldsymbol{\tau}_b \cdot \mathbf{u}_b}{\rho_w L}$$

Channel evolution



$$\frac{\partial S}{\partial t} = \frac{\rho_w}{\rho_i} M - \frac{2A}{n^n} S N^n$$

Creep closure

Melting

$$M = \frac{|Q \partial \phi / \partial s| + \lambda_c |\mathbf{q} \cdot \nabla \phi|}{\rho_w L}$$

Mass conservation

Runoff

$$\frac{\partial h}{\partial t} + \nabla \cdot \mathbf{q} + \left[\frac{\partial S}{\partial t} + \frac{\partial Q}{\partial s} \right] \delta(\mathbf{x}_c) + \frac{\partial \Sigma}{\partial t} = m + M \delta(\mathbf{x}_c) + R \delta(\mathbf{x}_m)$$

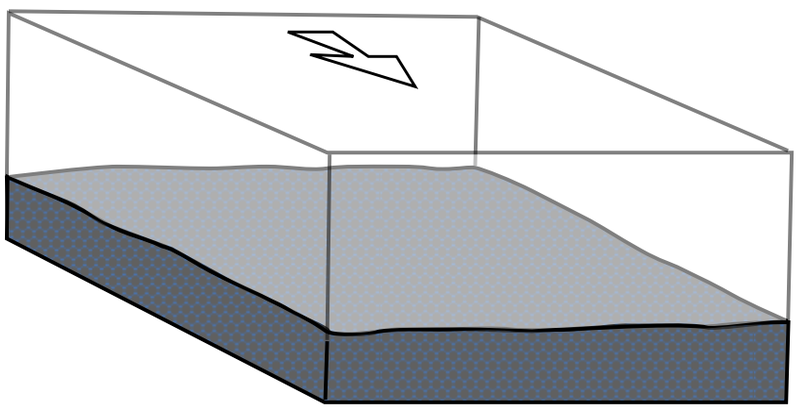
$$\mathbf{q} = -K h^3 \nabla \phi \qquad Q = -K_c S^{4/3} \frac{\partial \phi}{\partial s} \qquad \Sigma = \sigma p_w$$

⇨ Parabolic system for pressure

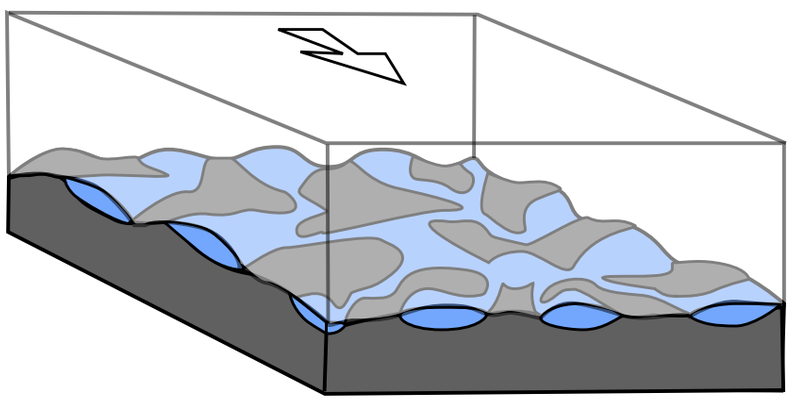
Solved with backward Euler ⇨ nonlinear discretized system for N h (nodes)

S (edges)

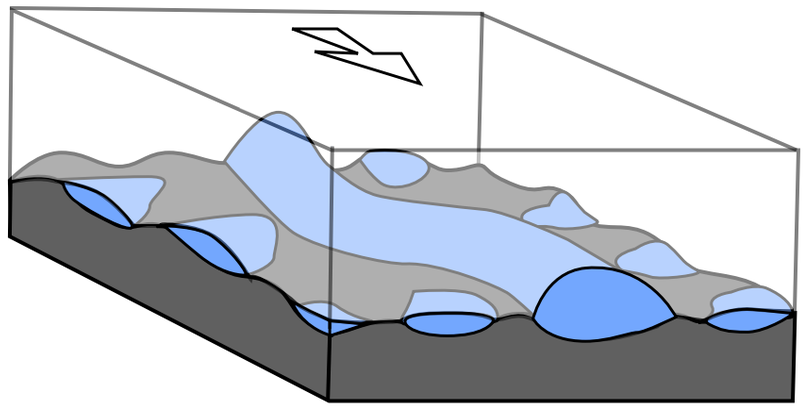
Large scale modelling



Saturated sediments



‘Distributed’ systems



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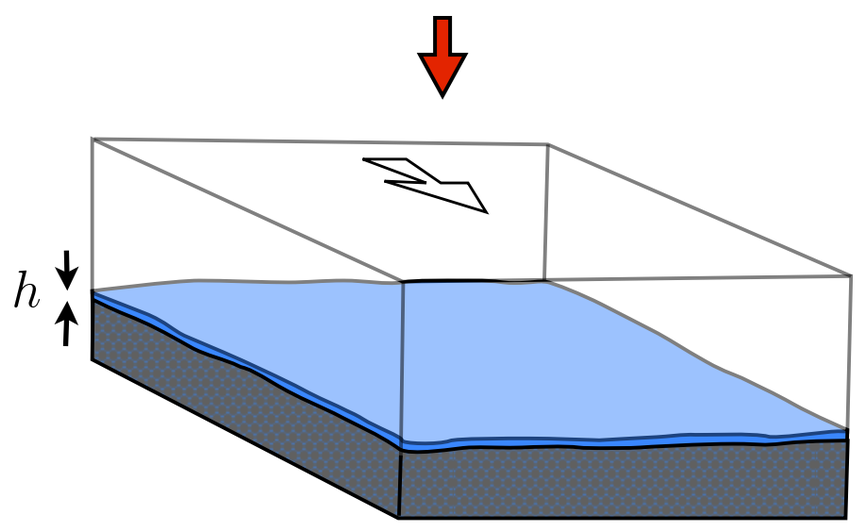
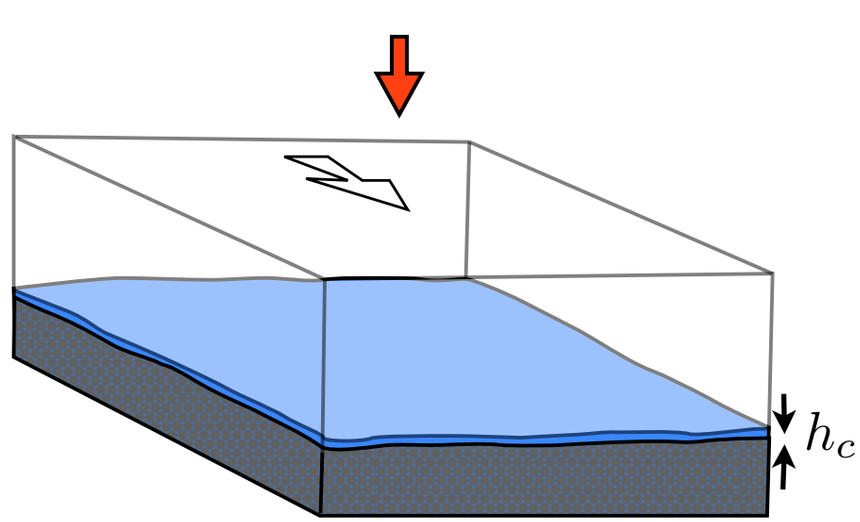
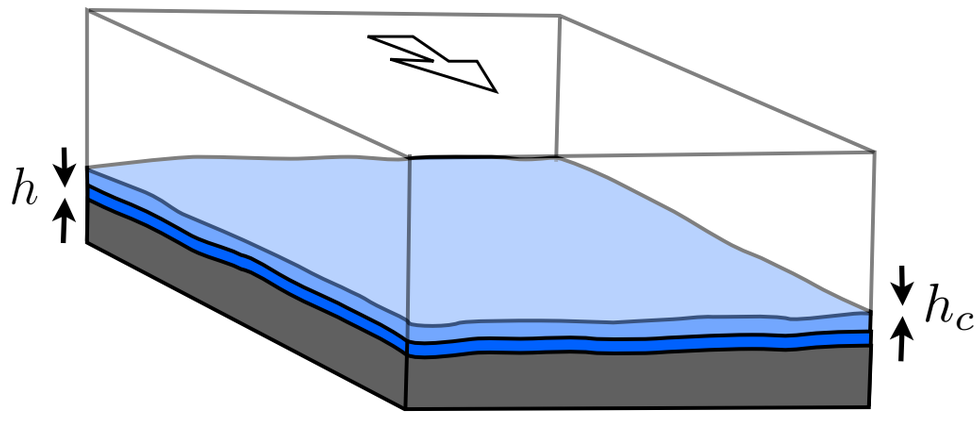
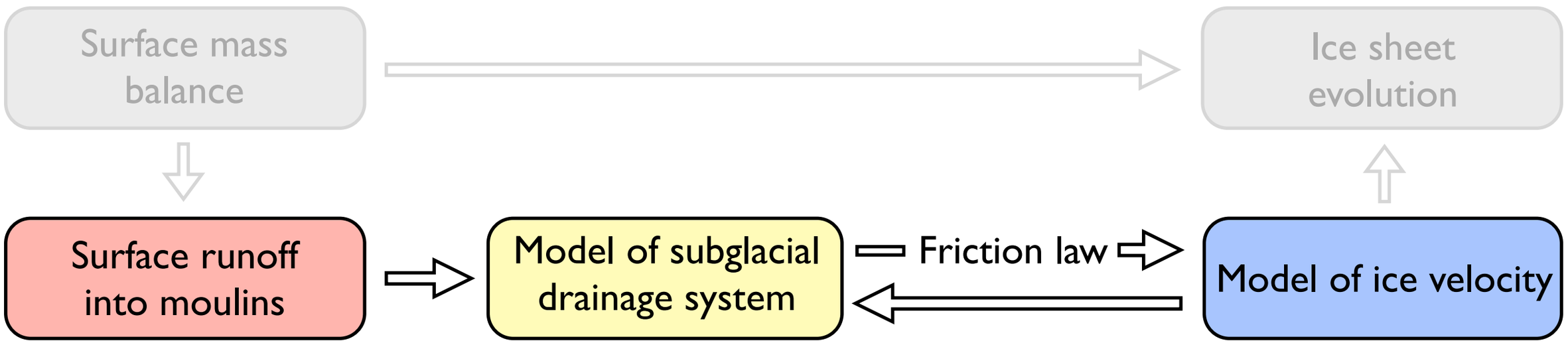






Modelling seasonal velocity changes



$$\tau_b = C U_b N$$

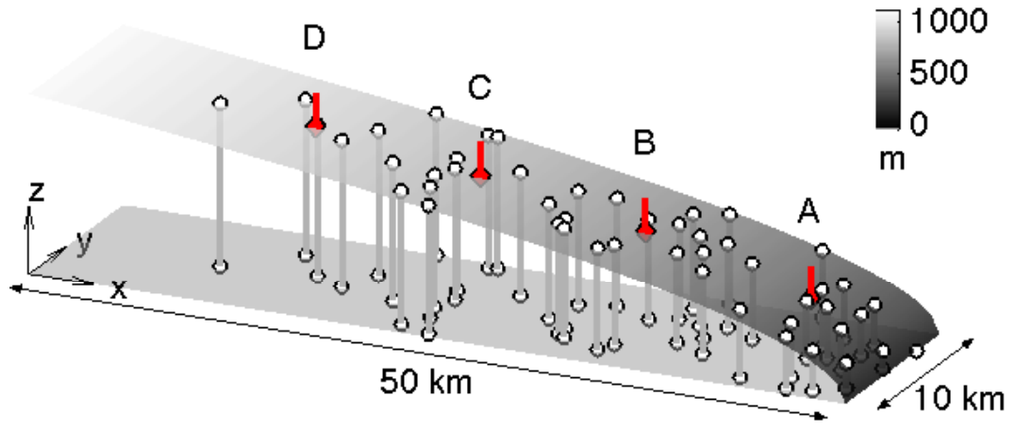
Vertically-integrated

Hindmarsh 2004,
Schoof & Hindmarsh 2010,
Goldberg 2011

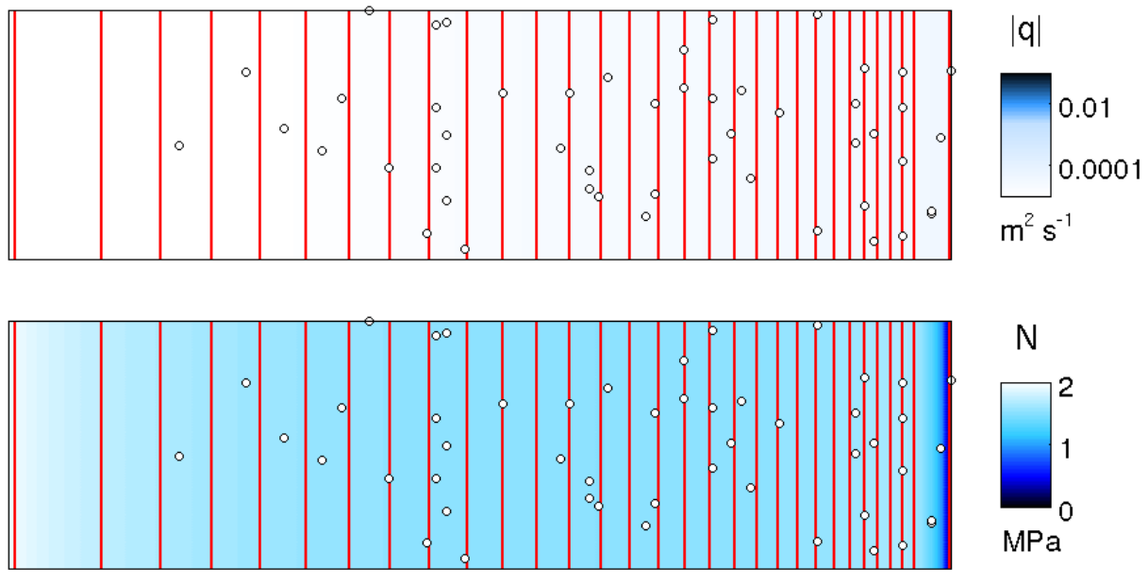
Hewitt 2013, *Earth Plan. Sci. Lett.*

Steady states

Basal melting only

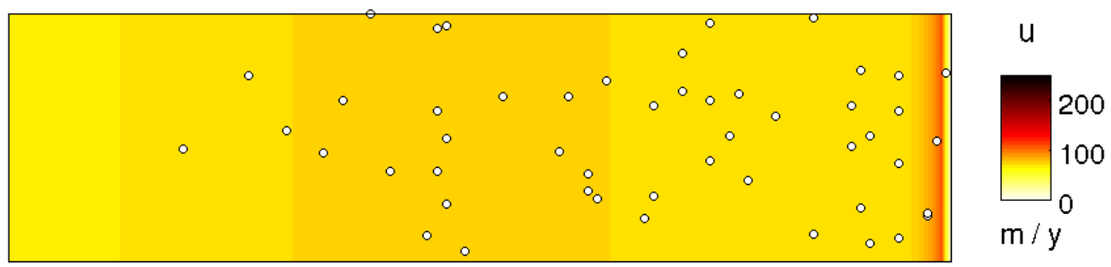


Water flux

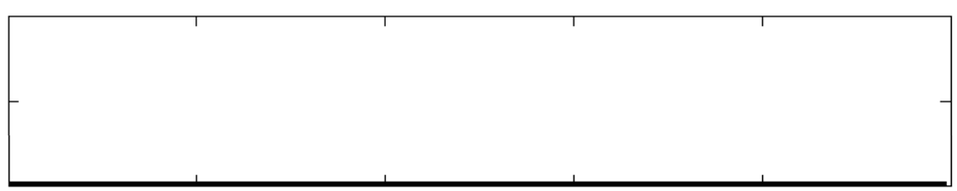


Effective pressure

Ice velocity



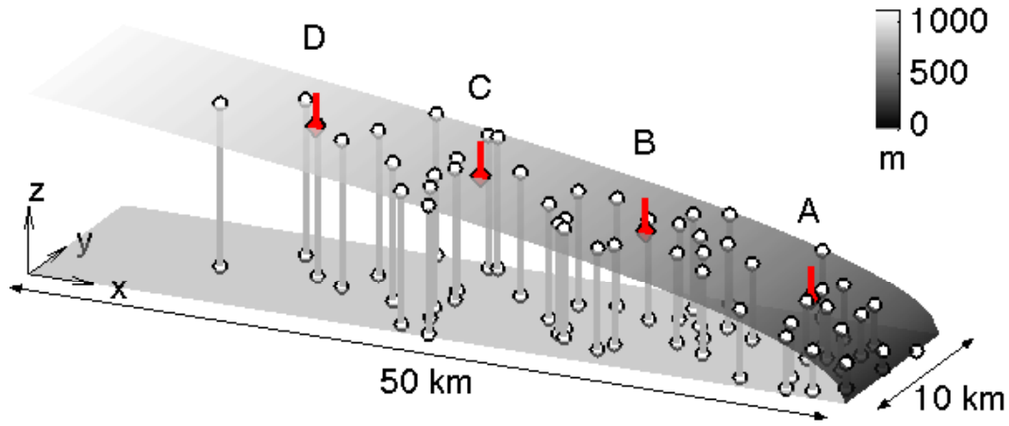
Water flux



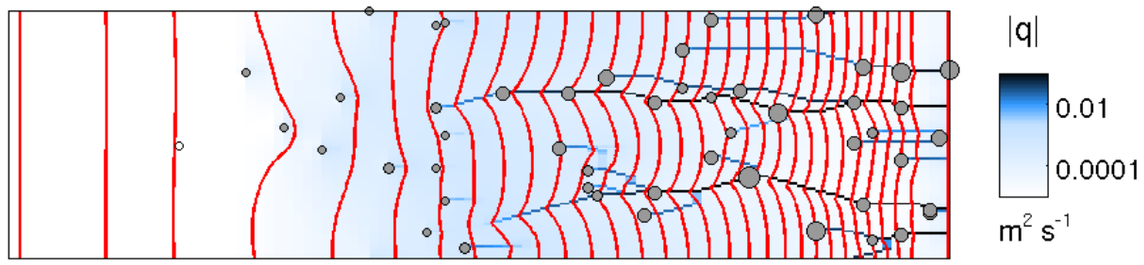
Ice flow →

Steady states

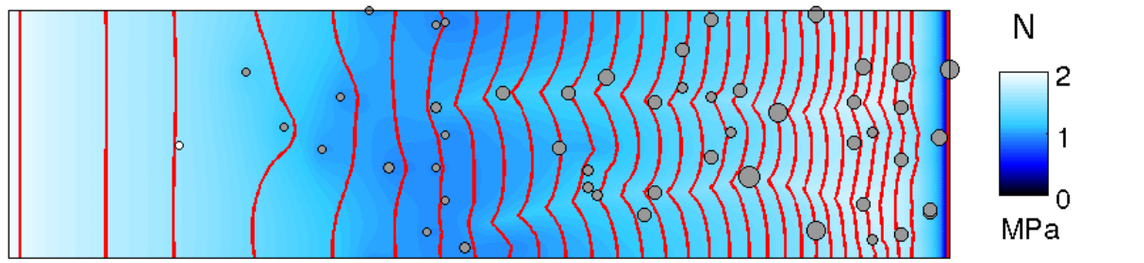
Surface + basal melting



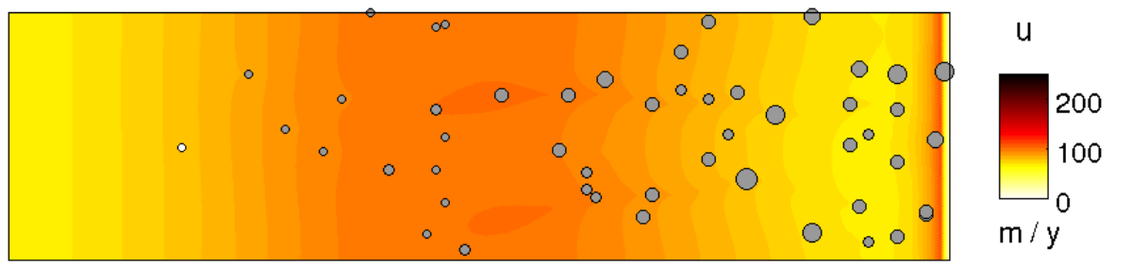
Water flux



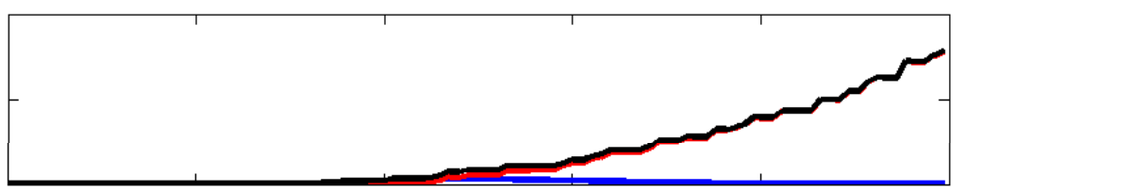
Effective pressure



Ice velocity

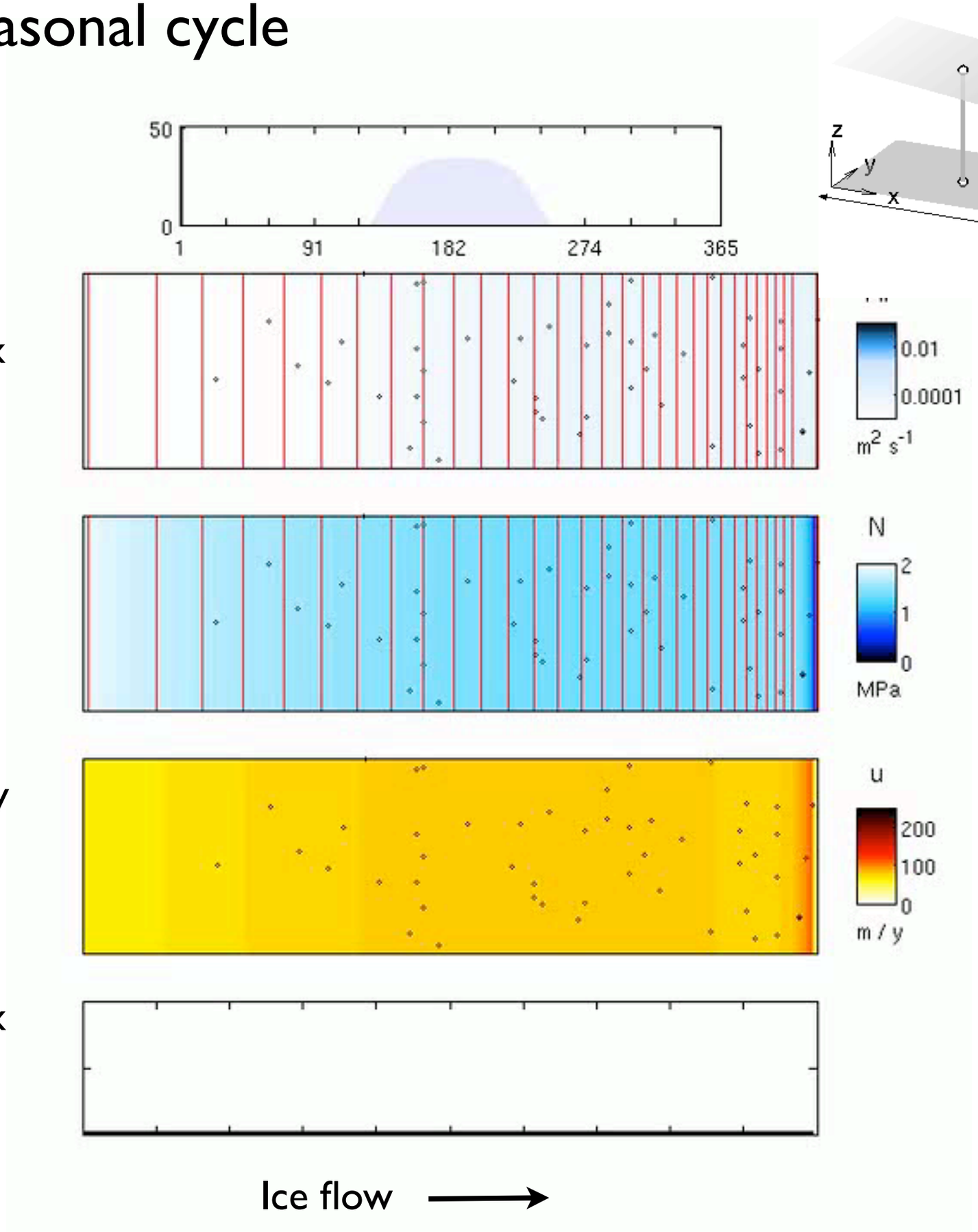


Water flux

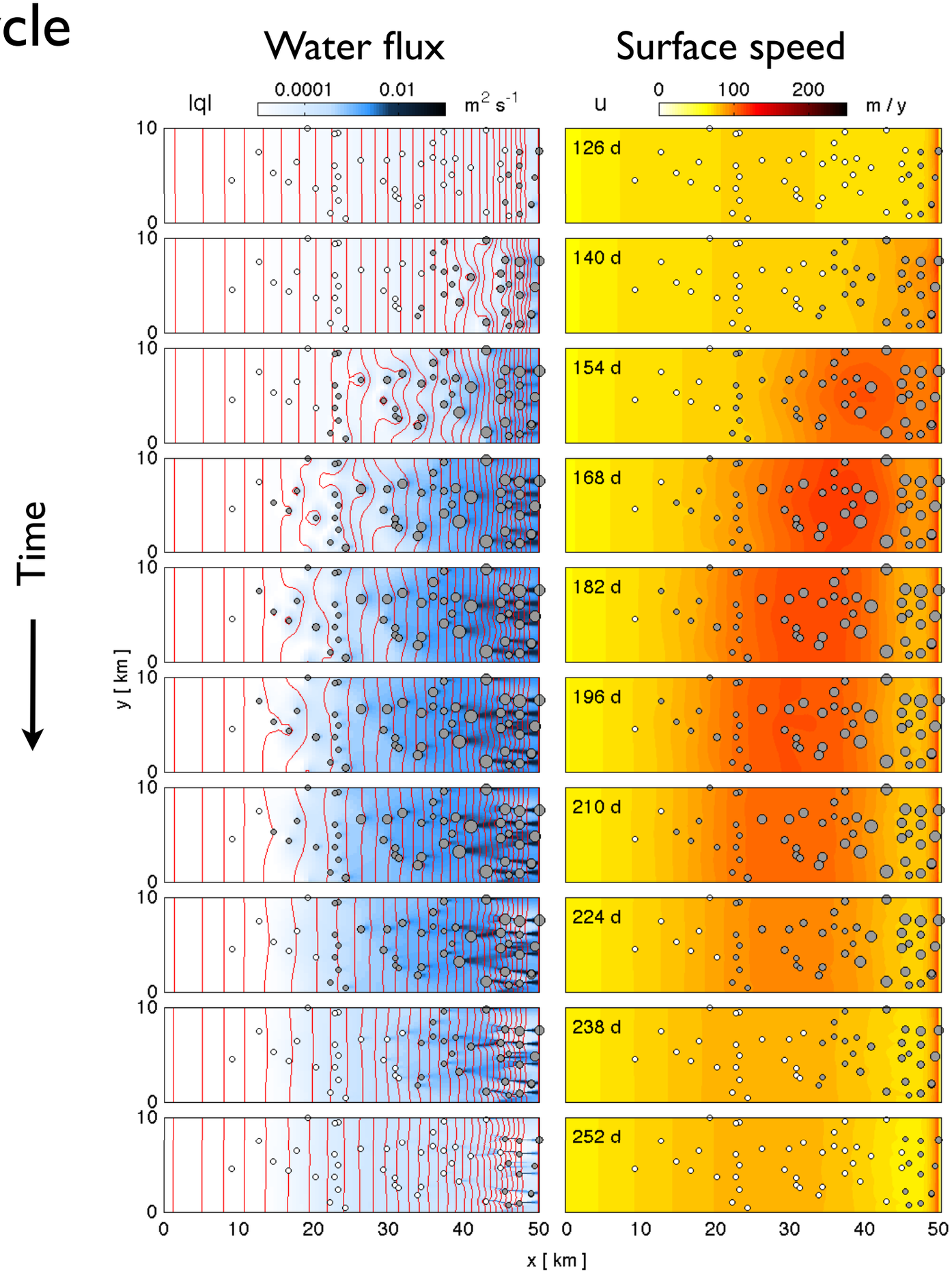


Ice flow →

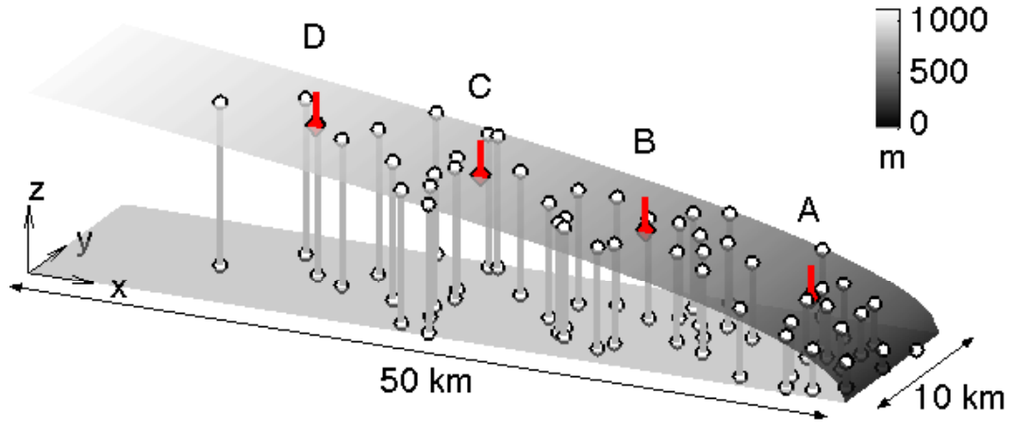
Movie: seasonal cycle

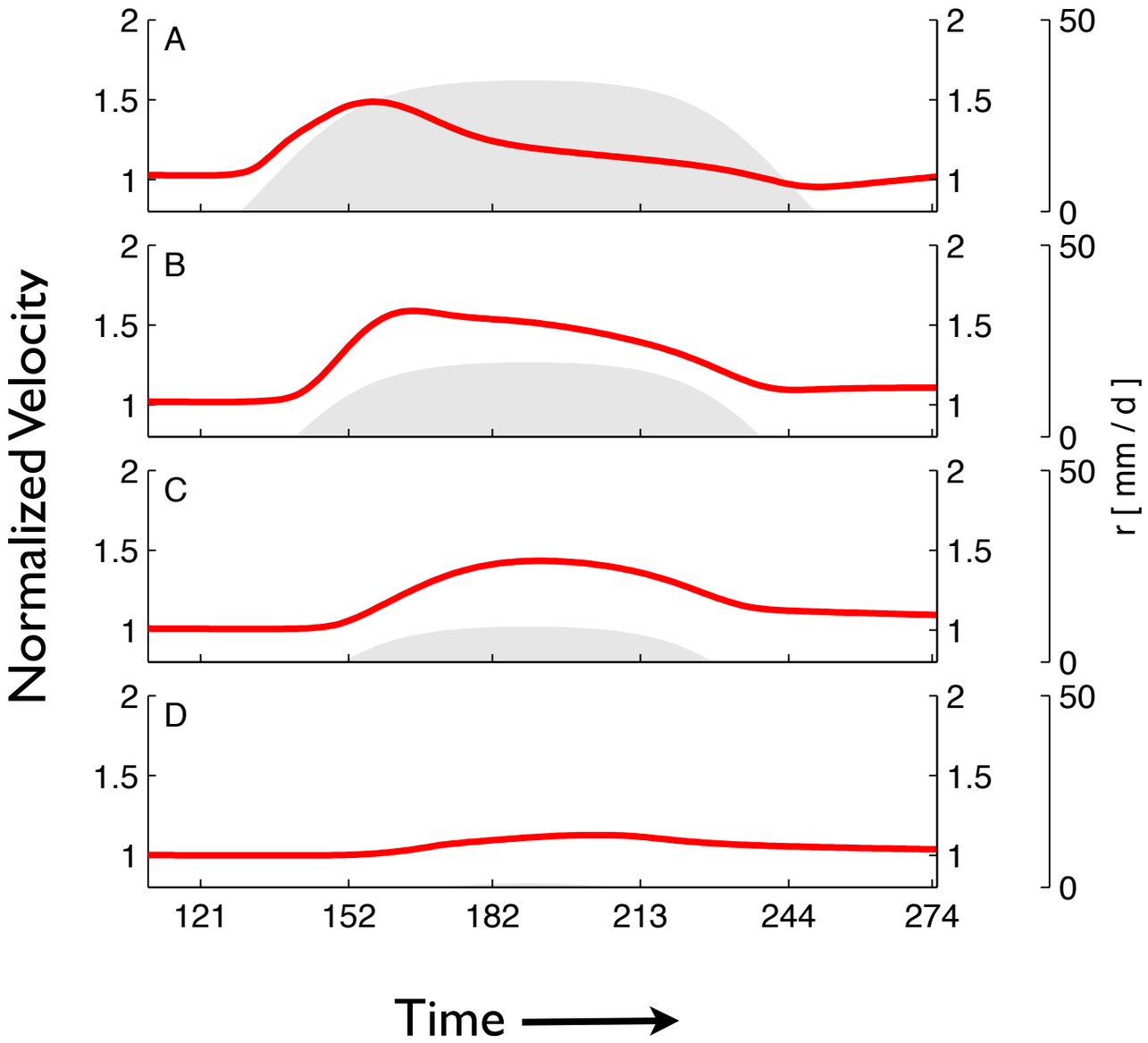


Seasonal cycle

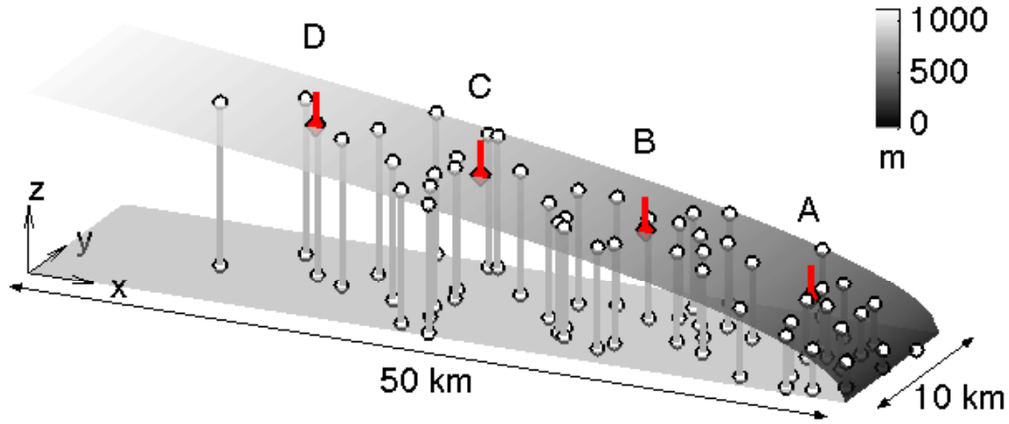


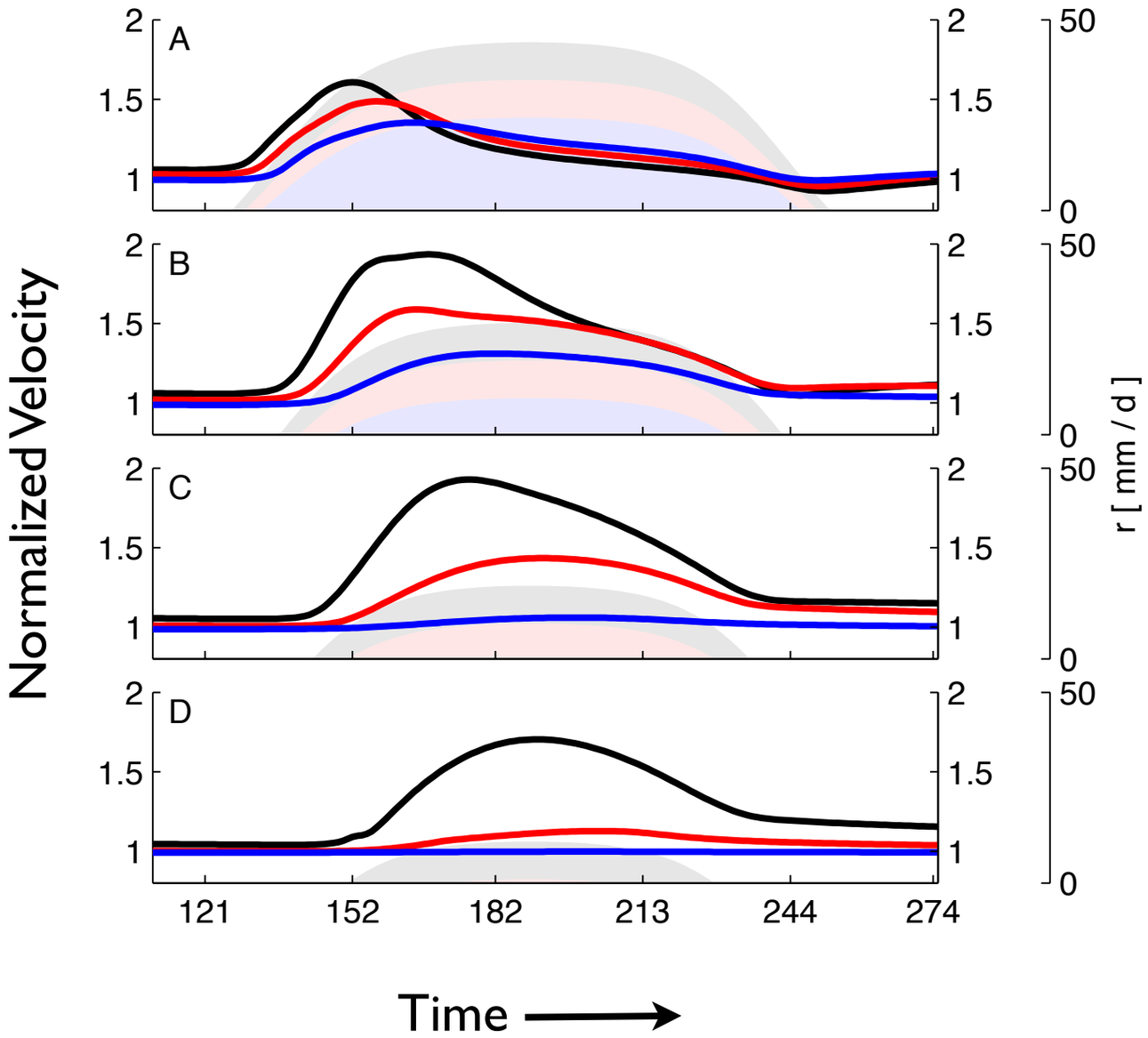
Seasonal cycle





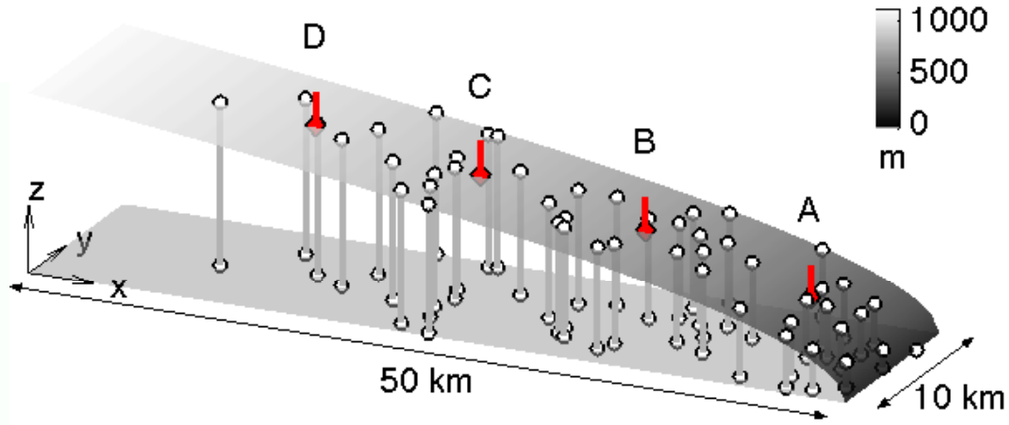
Seasonal cycle: surface melt dependence



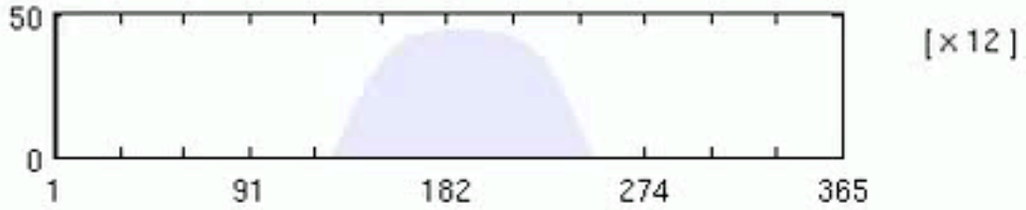


More melting generally leads to faster ice speed, but effect is limited by expansion of drainage system.

Movie: diurnally varying input

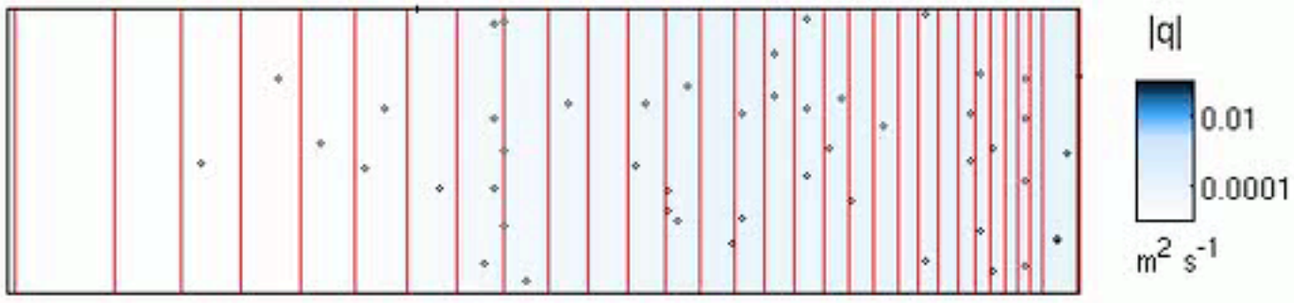


Inflow

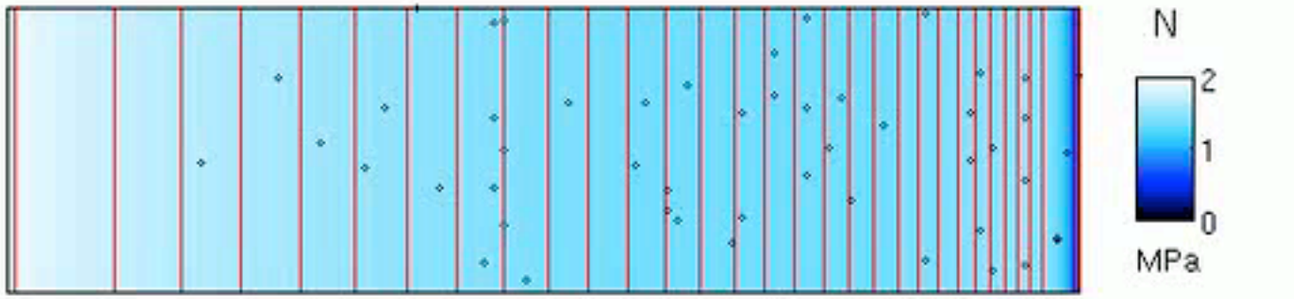


[x 12]

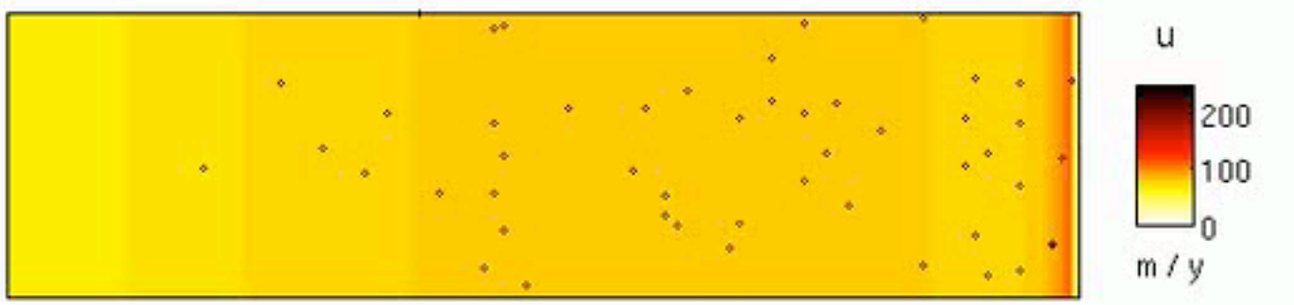
Water flux



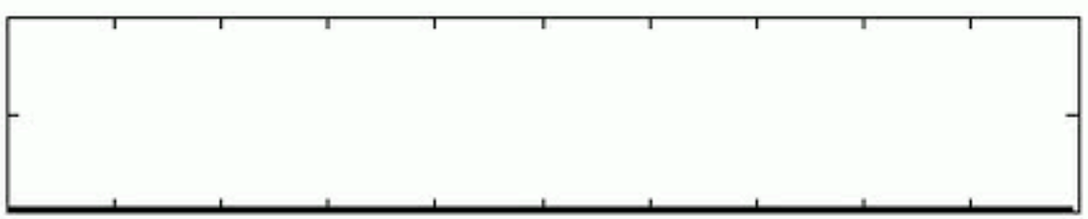
Effective pressure



Ice velocity

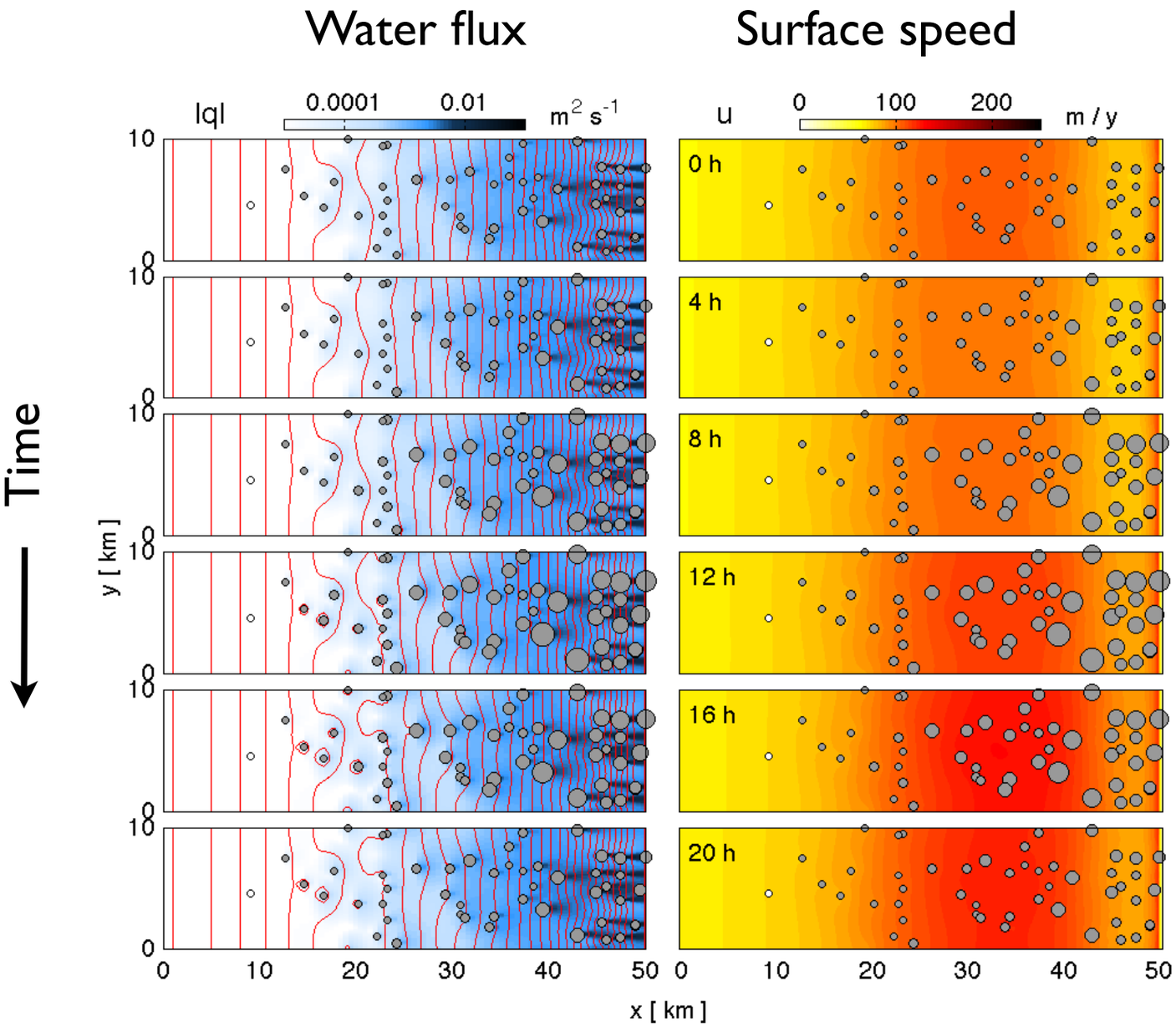


Water flux



Ice flow ⟶

Diurnal cycle

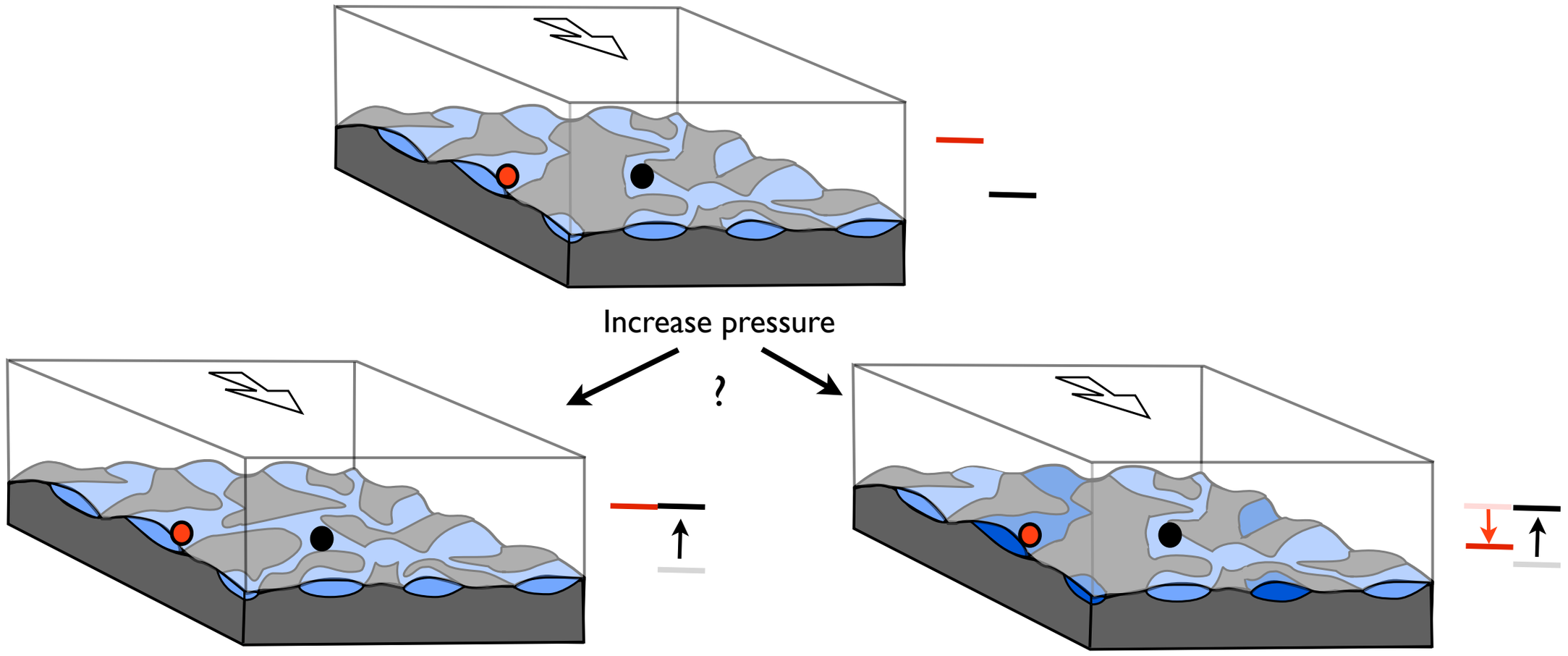


Difficulties

'Average' water pressure / effective pressure in distributed hydrology models is

an average over the connected drainage system

It cannot be expected to be the same as pressure in unconnected regions



Comparison with borehole measurements is very difficult!

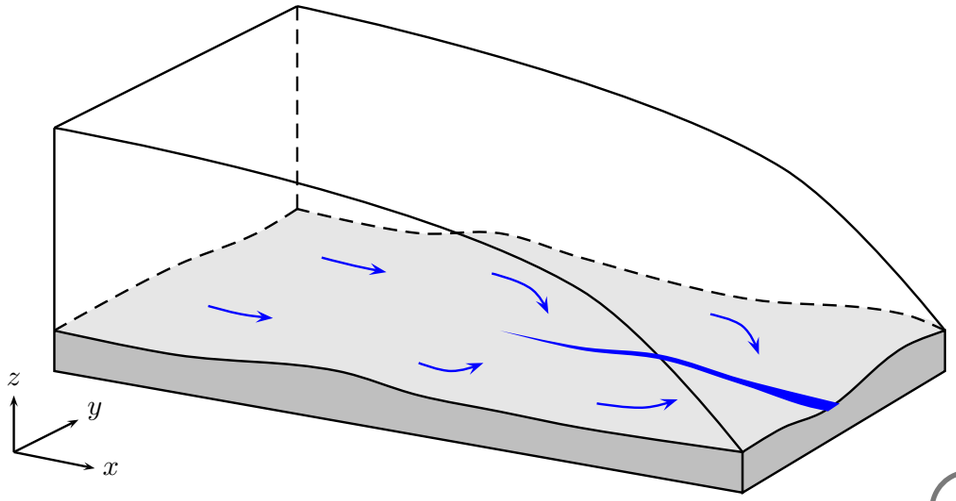
Summary

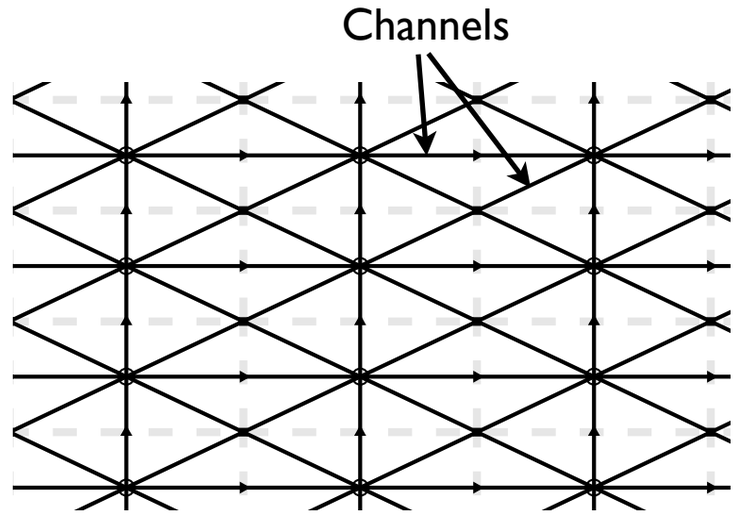
- Surface melt water can accelerate or decelerate ice flow with a complex spatial pattern.
- Models must include evolution of drainage space to show similar behaviour.

Challenges

- Rapid drainage events (lakes) and consequent ‘over’-pressure are an issue.
- Sediment transport and erosion may be crucial, giving ‘memory’ to drainage system.
- Subglacial discharge from marine terminating glaciers also influences melting/calving.
- Understanding hydrology also important for floods and erosion.

Drainage model





Mass conservation

$$\frac{\partial h}{\partial t} + \nabla \cdot \mathbf{q} + \left[\frac{\partial S}{\partial t} + \frac{\partial Q}{\partial s}\right] \delta(\mathbf{x}_c) + \frac{\partial \Sigma}{\partial t} = m + M\delta(\mathbf{x}_c) + R\delta(\mathbf{x}_m)$$

Water flow parameterizations

$$\mathbf{q} = -Kh^3 \nabla \phi$$

$$Q = -K_c S^{5/4} \left|\frac{\partial \phi}{\partial s}\right|^{-1/2} \frac{\partial \phi}{\partial s}$$

Evolution parameterizations

$$\frac{\partial h}{\partial t} = \frac{\rho_w}{\rho_i} m + U_b(h_r - h)/l_r - \frac{2A}{n^n} h |N|^{n-1} N$$

$$\frac{\partial S}{\partial t} = \frac{\rho_w}{\rho_i} M - \frac{2A}{n^n} S |N|^{n-1} N$$

Energy conservation

$$m = \frac{G + \boldsymbol{\tau}_b \cdot \mathbf{u}_b}{\rho_w L} \qquad M = \frac{|Q \partial \phi / \partial s| + \lambda_c |\mathbf{q} \cdot \nabla \phi|}{\rho_w L}$$

Englacial storage

$$\Sigma = \sigma \rho_w g p_w$$

Hewitt 2013, EPSL

Ice model

Basal force balance

$$\frac{f(U_b,N)}{U_b}u_b = -\rho_i g H\frac{\partial s}{\partial x} + \frac{\partial}{\partial x}\left[H\left(2\overline{\tau}_{xx}+\overline{\tau}_{yy}\right)\right] + \frac{\partial}{\partial y}\left[H\overline{\tau}_{xy}\right]$$

$$\frac{f(U_b,N)}{U_b}v_b = -\rho_i g H\frac{\partial s}{\partial y} + \frac{\partial}{\partial y}\left[H\left(\overline{\tau}_{xx}+2\overline{\tau}_{yy}\right)\right] + \frac{\partial}{\partial x}\left[H\overline{\tau}_{xy}\right]$$

In-plane force balance

$$\tau_{xz} = -\rho_i g(s-z)\frac{\partial s}{\partial x} + \frac{\partial}{\partial x}\left[\int_z^s \left(2\tau_{xx}+\tau_{yy}\right)\,\mathrm{d}z\right] + \frac{\partial}{\partial y}\left[\int_z^s \tau_{xy}\,\mathrm{d}z\right]$$

$$\tau_{yz} = -\rho_i g(s-z)\frac{\partial s}{\partial y} + \frac{\partial}{\partial y}\left[\int_z^s \left(\tau_{xx}+2\tau_{yy}\right)\,\mathrm{d}z\right] + \frac{\partial}{\partial x}\left[\int_z^s \tau_{xy}\,\mathrm{d}z\right]$$

Approximate constitutive law

$$\tau_{xz} = \eta\frac{\partial u}{\partial z}, \quad \tau_{yz} = \eta\frac{\partial v}{\partial z}$$

$$\eta = \tfrac{1}{2}\overline{A}^{-1}\left[\tau_{xz}^2+\tau_{yz}^2+\tau_{xy}^2+\tau_{xx}^2+\tau_{xx}\tau_{yy}+\tau_{yy}^2\right]^{(1-n)/2}$$

$$\tau_{xx} = 2\tilde{\eta}\frac{\partial u_b}{\partial x}, \quad \tau_{yy} = 2\tilde{\eta}\frac{\partial v_b}{\partial y}, \quad \tau_{xy} = \tilde{\eta}\left(\frac{\partial u_b}{\partial y}+\frac{\partial v_b}{\partial x}\right)$$

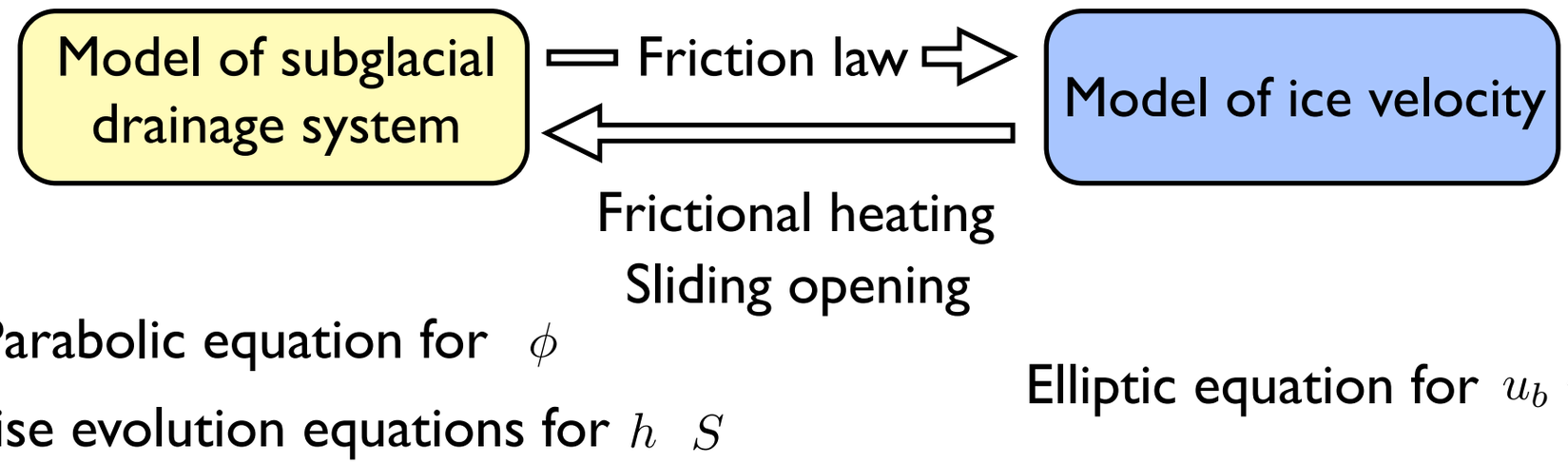
$$\tilde{\eta} = \tfrac{1}{2}\overline{A}^{-1}\left[\tilde{\tau}_{xz}^2+\tilde{\tau}_{yz}^2+\tau_{xy}^2+\tau_{xx}^2+\tau_{xx}\tau_{yy}+\tau_{yy}^2\right]^{(1-n)/2}$$

$$\tilde{\tau}_{xz} = -\rho_i g(s-z)\frac{\partial s}{\partial x} \qquad \tilde{\tau}_{yz} = -\rho_i g(s-z)\frac{\partial s}{\partial y}$$

Implementation

Two-way coupling

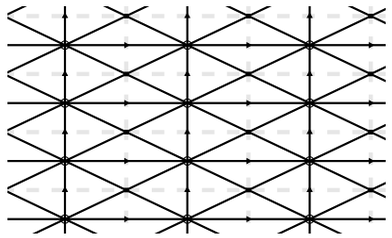
Short time-steps required (<1h for diurnally varying discharge)



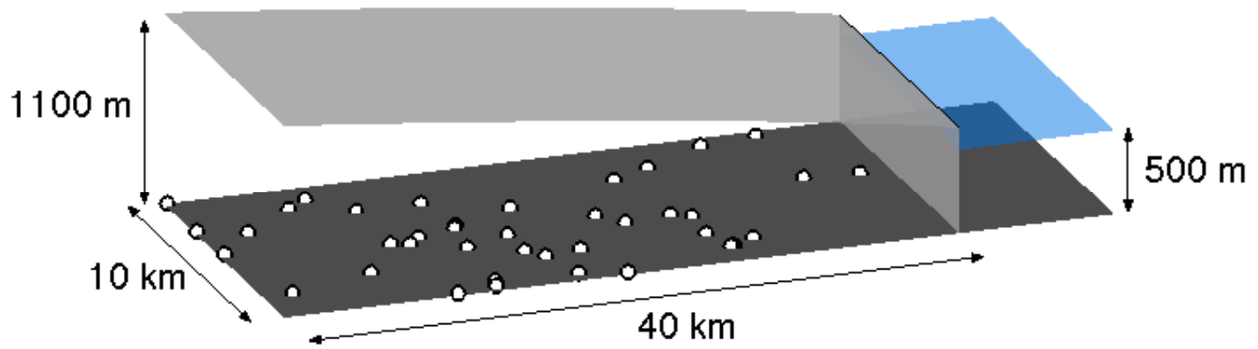
Slight grid-size dependence due to channel network, but mostly unimportant.

Widespread reduction of effective pressure to zero causes very large accelerations.

Channel spacing



Conduits + cavities



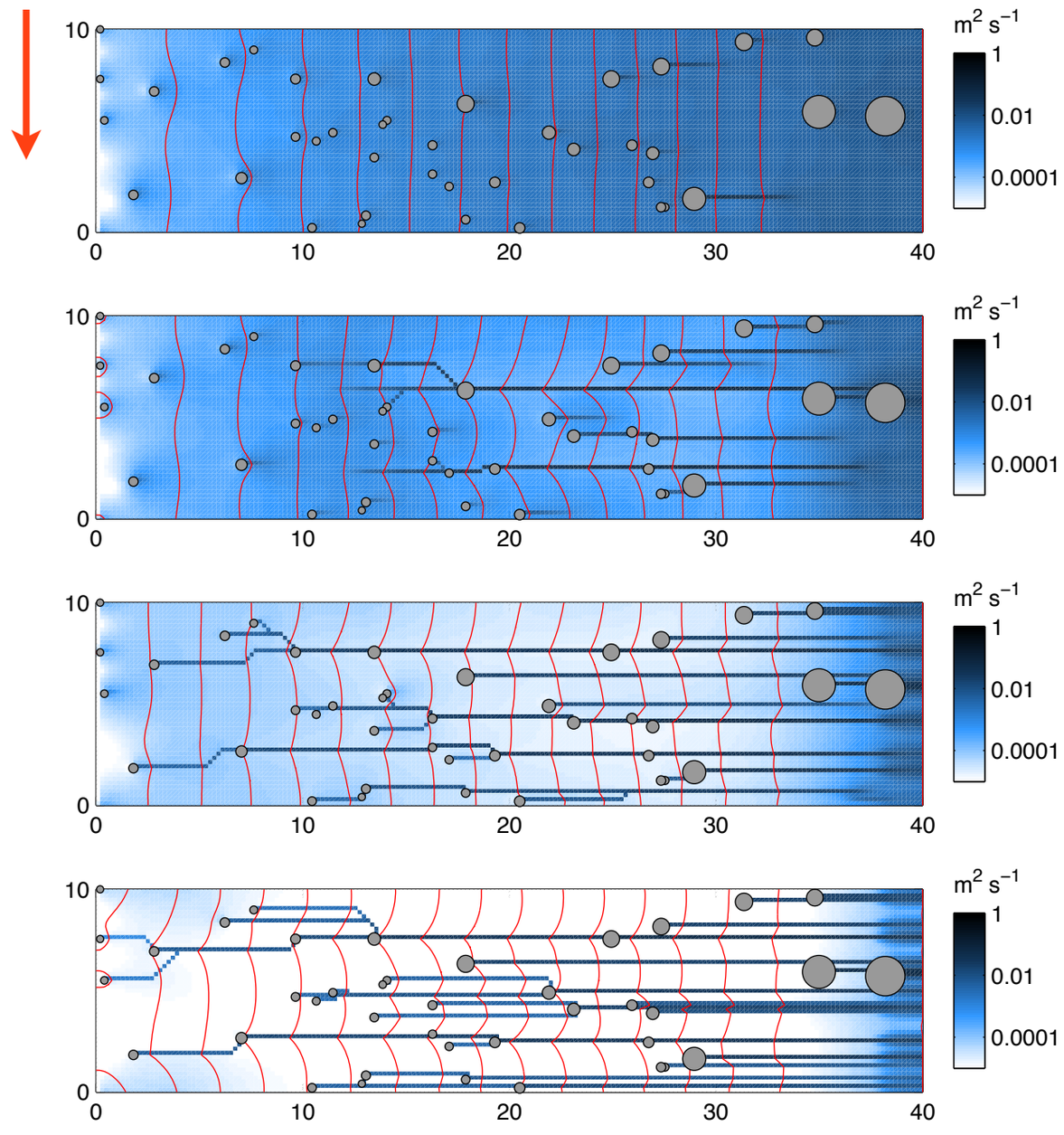
- More densely spaced smaller channels likely if
  - slope is large
  - distributed system is poorly connected

- Steady state may never occur in practice.

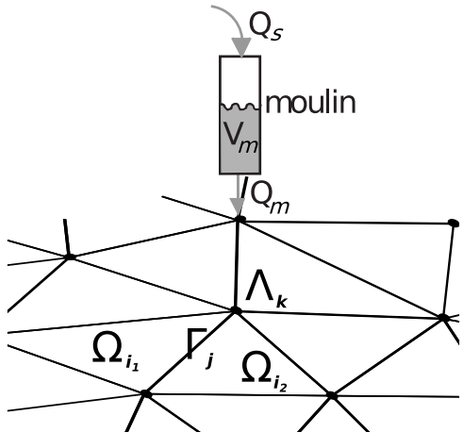
- Continuum of scales, from orifices to channels.

- Modelled channels tend to coarsen over time.

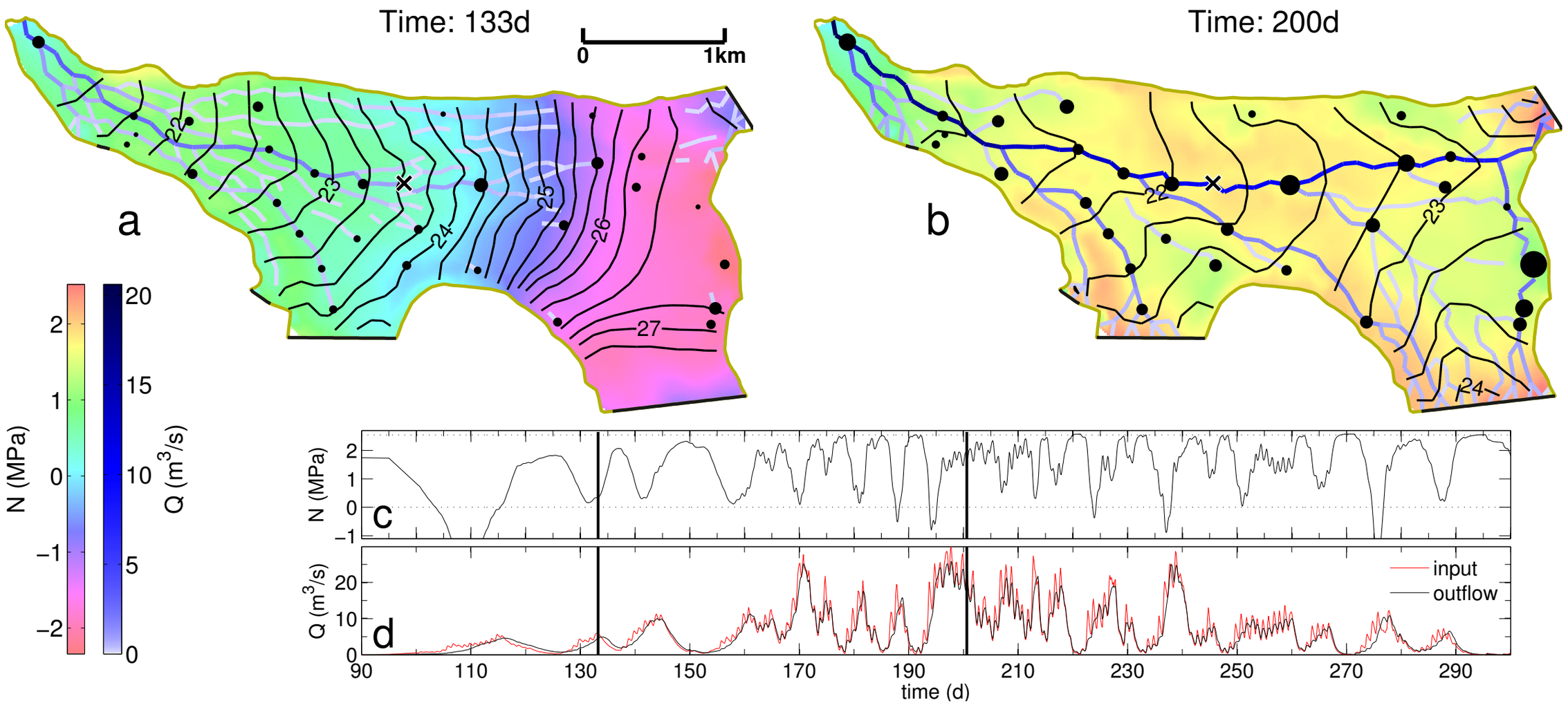
More poorly connected cavities



Unstructured meshes



Gornergletscher - 2007 melt forcing

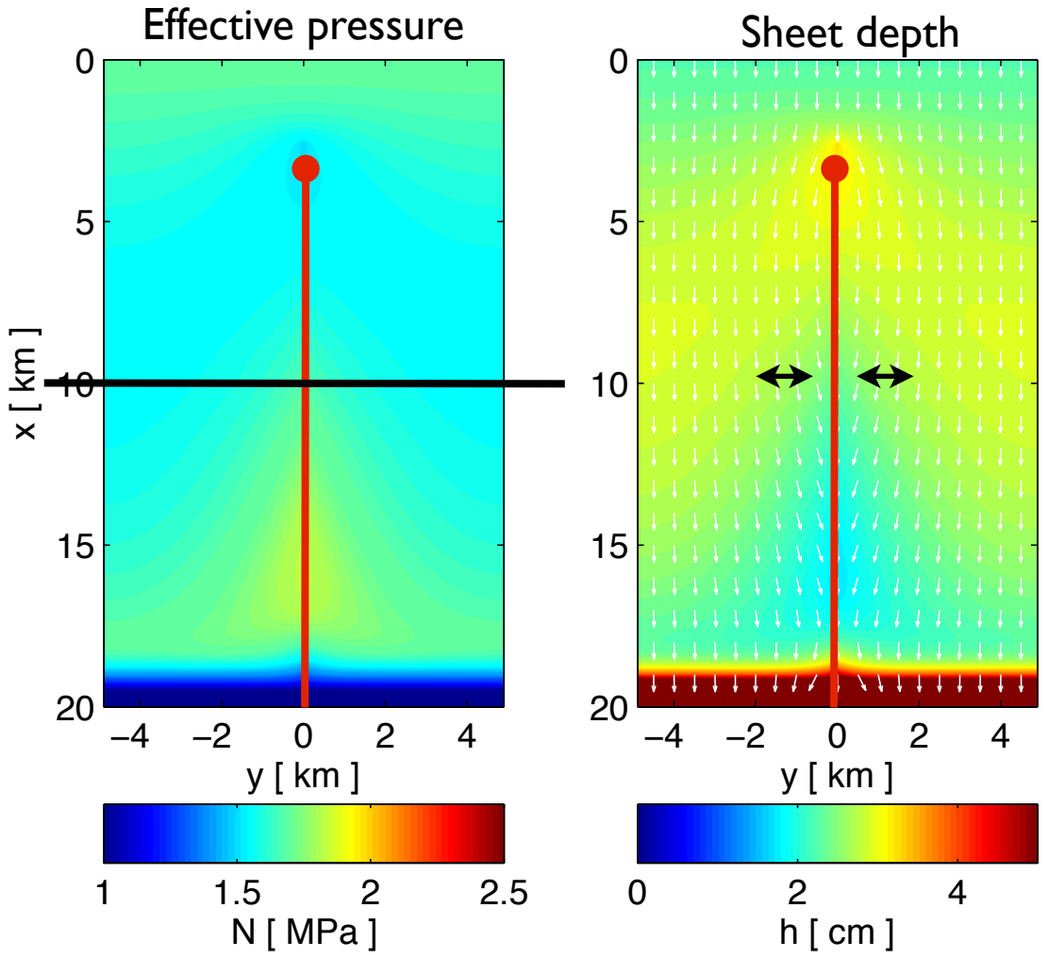


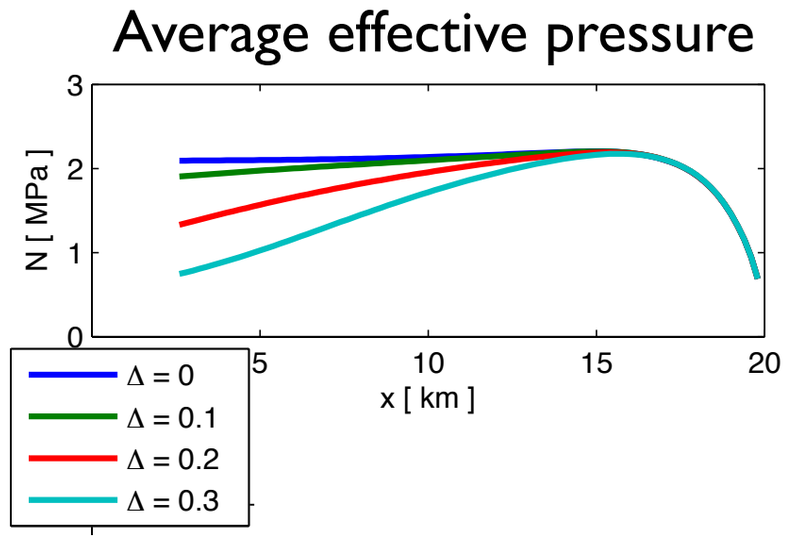
Werder et al *in press*

Single channel model

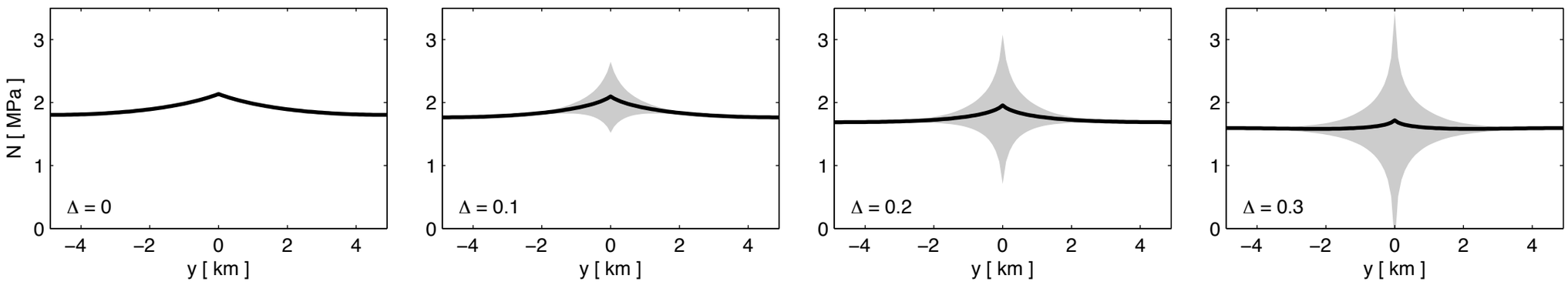
Variable input to isolated moulin

Surface gradient





Effective pressure transects



Increasing amplitude of diurnal input →