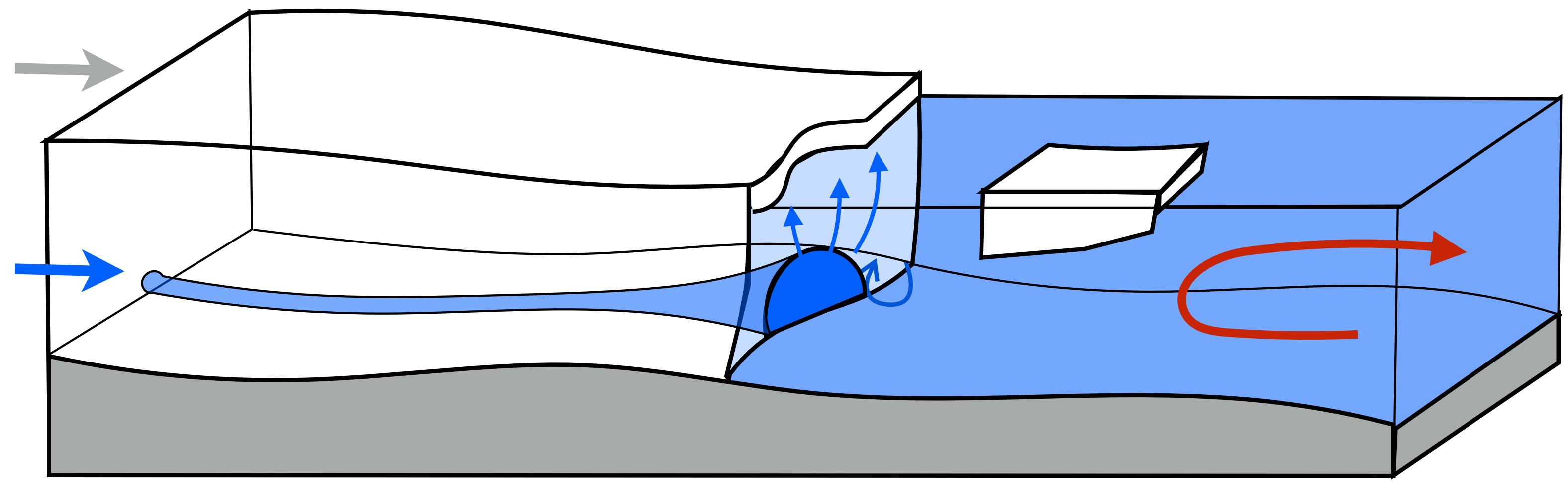
A new model for polythermal ice

incorporating gravity-driven meltwater drainage

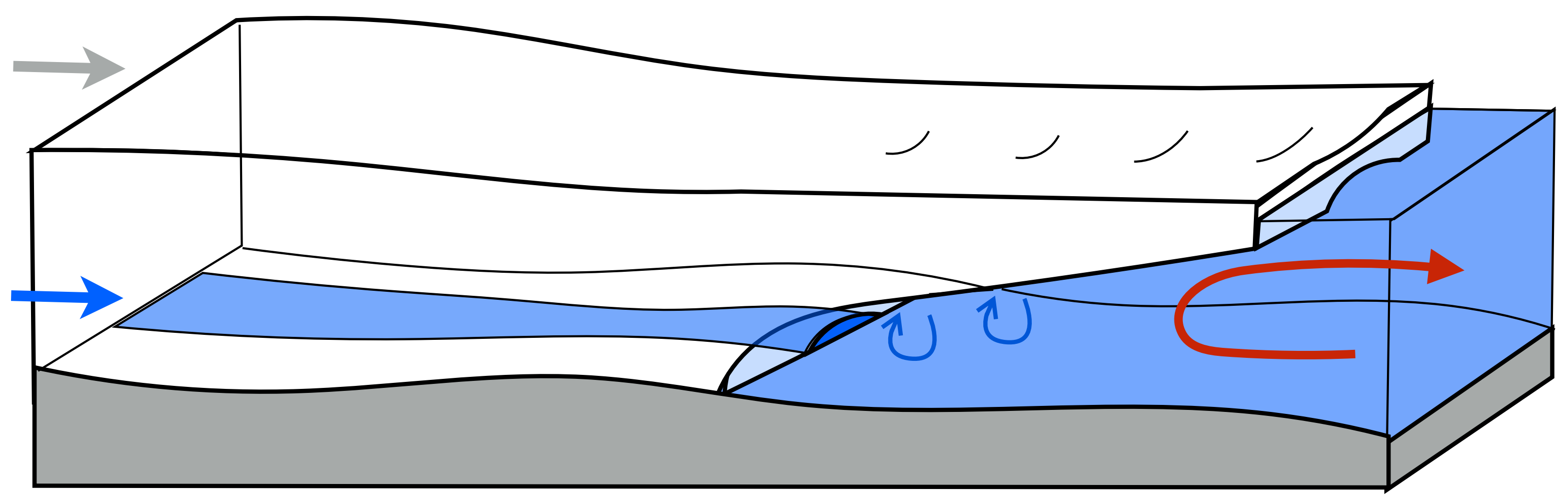
Ian Hewitt, University of Oxford
Christian Schoof, University of British Columbia

From EGU:

I. What do models tell us about how subglacial discharge is delivered at grounding lines?



II. How does the spatial distribution of subglacial discharge affect the shape of ice shelves?



Subglacial channels have a ‘trumpet-like’ shape near the margin

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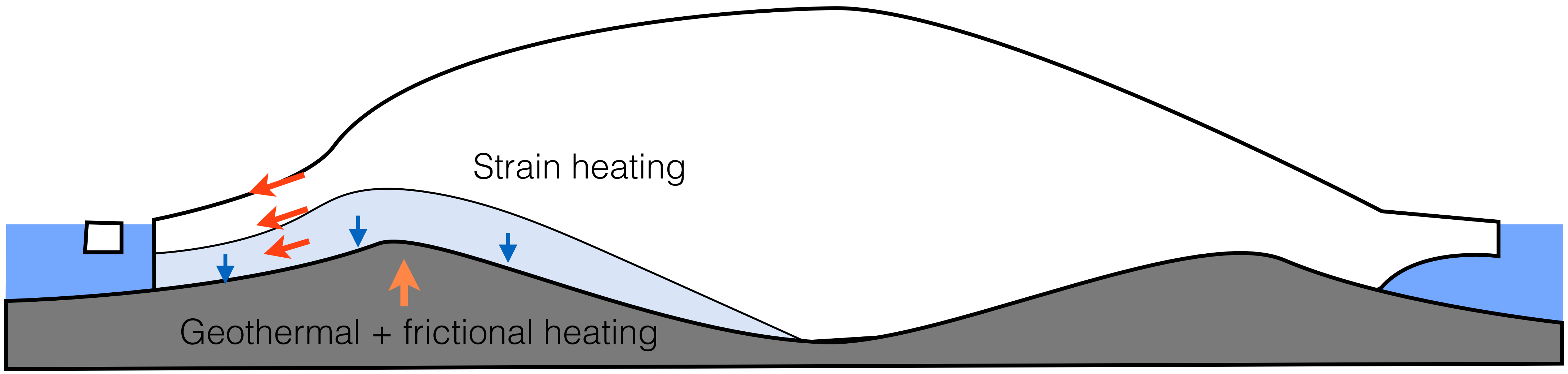
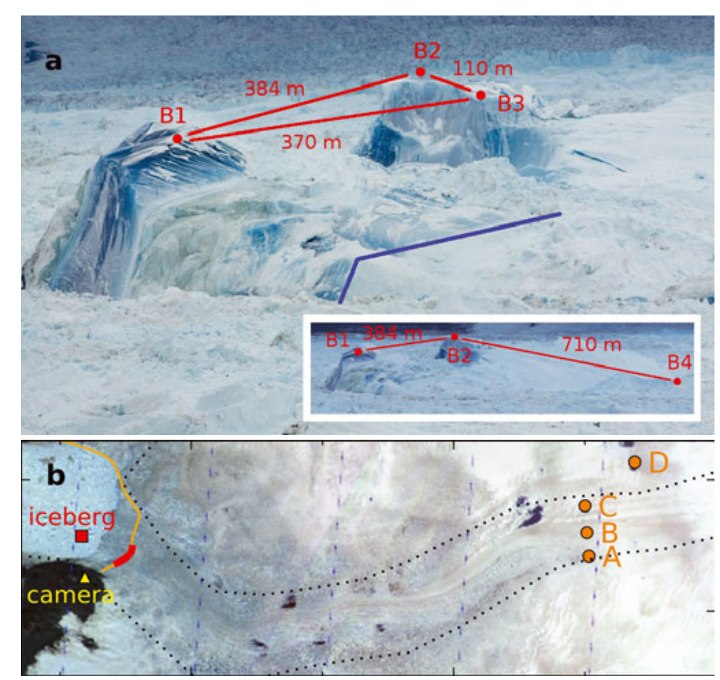
Motivation

Goal - provide a simple model that

- predicts temperature and water content of polythermal ice.
- allows water to drain from the ice by porous flow.

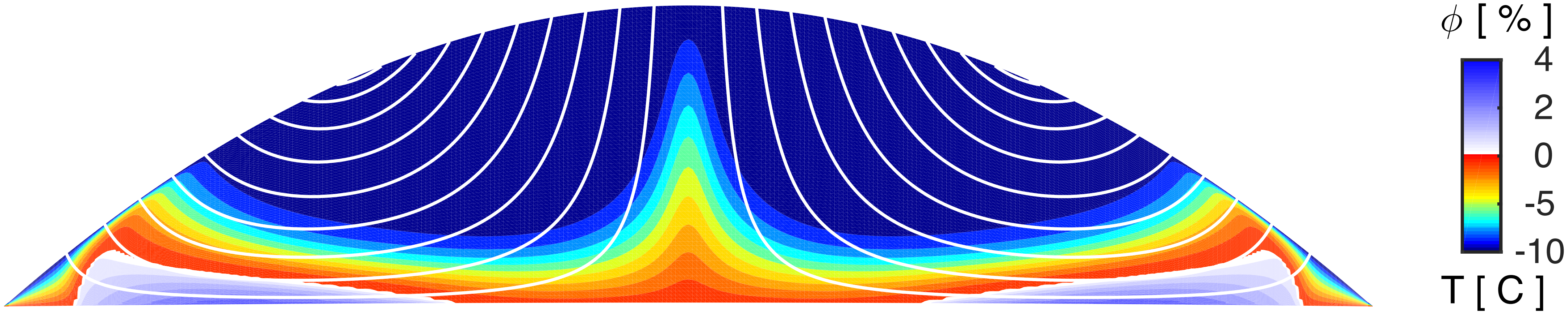


- Ice flow depends on temperature and water content.
- May be fast dynamical feedbacks between water content and ice flow.



Lüthi et al (2009)

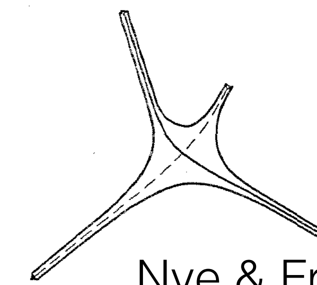
Motivation



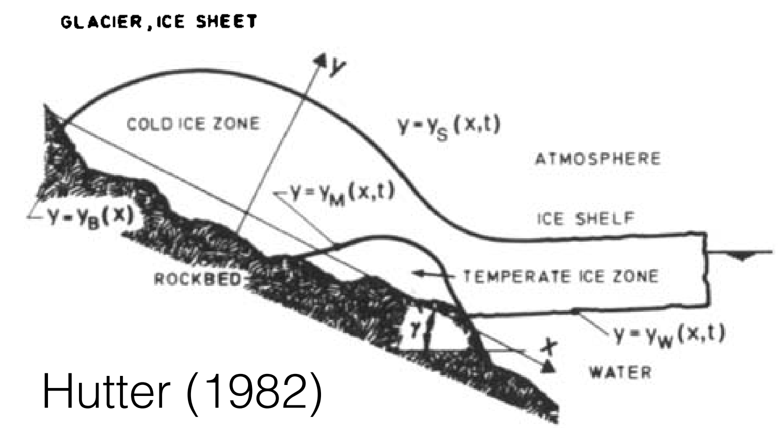
Previous work on polythermal ice

Theory

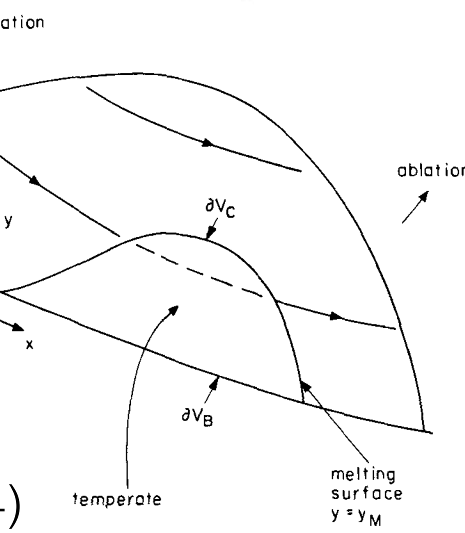
- Lliboutry (1971,1976), Nye & Frank (1973) - permeability
- Fowler & Larson (1978) - continuum formulation, no moisture movement
- Hutter (1982) - mixture theory, diffusive moisture transport
- Fowler (1984) - two-phase theory, Darcy's law for moisture transport



Nye & Frank (1973)



Hutter (1982)

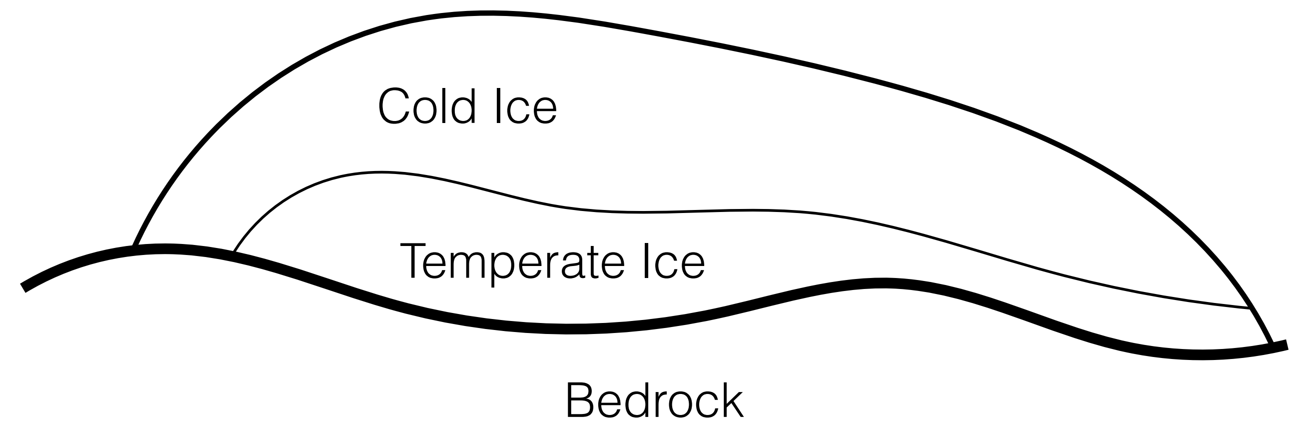


Fowler (1984)

Computational models

- Greve (1997) - two layer, explicit determination of 'CTS', switch-like drainage function
- Aschwanden et al (2012) - enthalpy gradient method

Problem formulation



Stokes flow (or approximation)

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \tau_{ij}}{\partial x_j} - \frac{\partial p}{\partial x_i} = -\rho g_i$$

$$\tau_{ij} = A^{-1/n} \dot{\varepsilon}^{1/n-1} \dot{\varepsilon}_{ij}$$

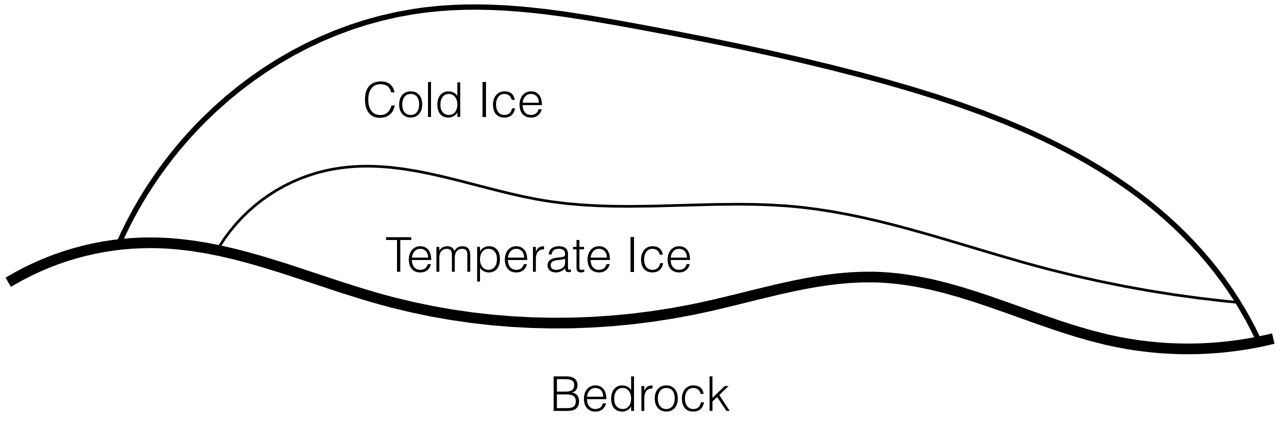
$$A = A(T, \phi)$$

ϕ = water content (porosity)

This talk - ice velocity prescribed (decoupled from thermodynamics)

Schoof & Hewitt 2015 *in review*

Problem formulation



Energy conservation

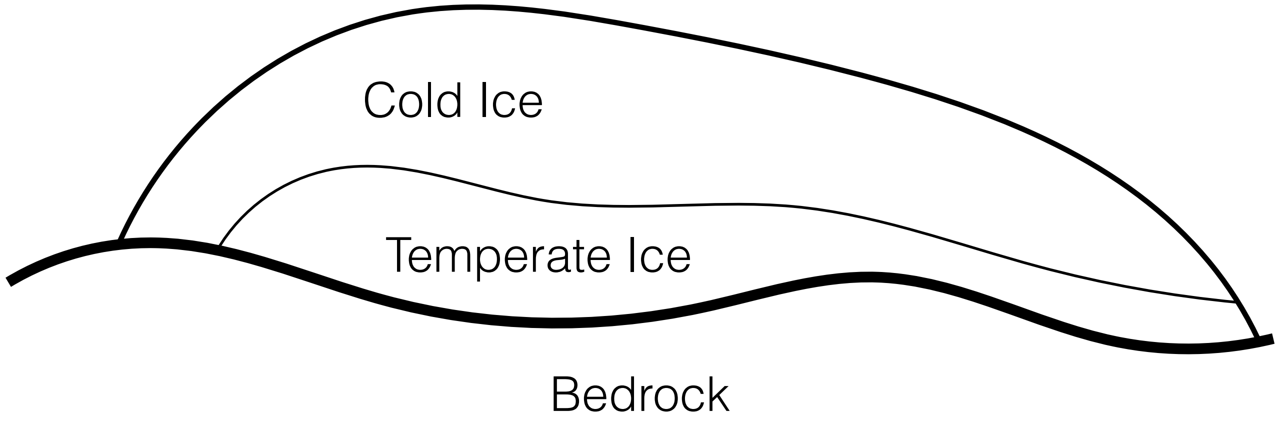
Cold ice $$\rho c \left(\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) = \nabla \cdot (k \nabla T) + \tau_{ij} \dot{\varepsilon}_{ij}, \qquad \phi = 0, \qquad T \leq T_m$$

Temperate ice $$\rho_w L \left(\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi \right) + \rho_w L \nabla \cdot \mathbf{j} = \tau_{ij} \dot{\varepsilon}_{ij}, \qquad T = T_m, \qquad \phi > 0$$

Relative water flux $\mathbf{j} = -\nu \nabla \phi$ - enthalpy gradient method

Schoof & Hewitt 2015 *in review*

Problem formulation



Energy conservation

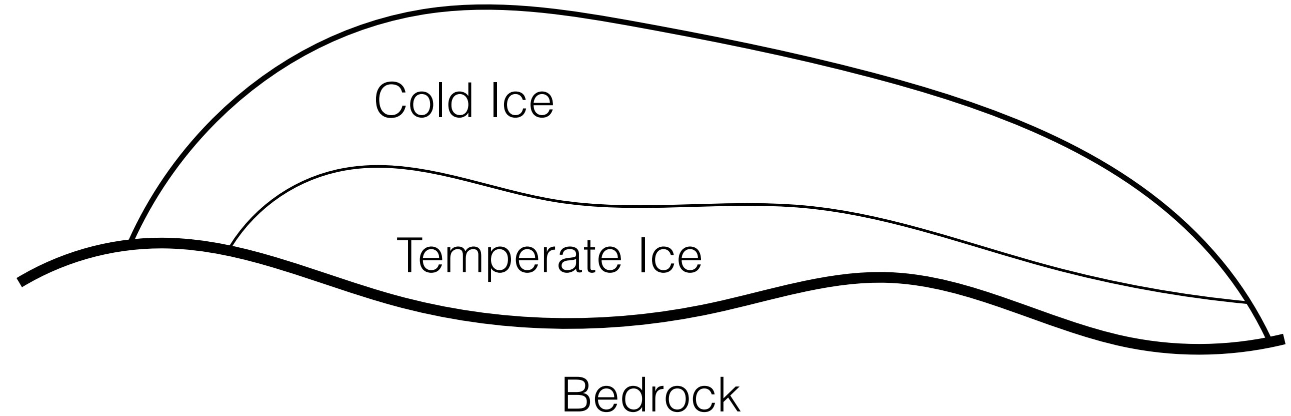
In terms of 'enthalpy',

$h = \rho c T + \rho_w L \phi$

$$\left(\frac{\partial h}{\partial t} + \mathbf{u} \cdot \nabla h\right) + \nabla \cdot \mathbf{Q} = \tau_{ij}\dot{\varepsilon}_{ij}, \qquad \mathbf{Q} = \begin{cases} -k\nabla T & h < \rho c T_m, \\ \rho_w L\,\mathbf{j} & h \geq \rho c T_m. \end{cases}$$

Schoof & Hewitt 2015 *in review*

Problem formulation



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Permeability (?)

Relative moisture flux
(Darcy's law)

$$\mathbf{j} = \frac{k_0 \phi^2}{\eta_w}\left(\rho_w \mathbf{g} - \nabla p_w\right)$$

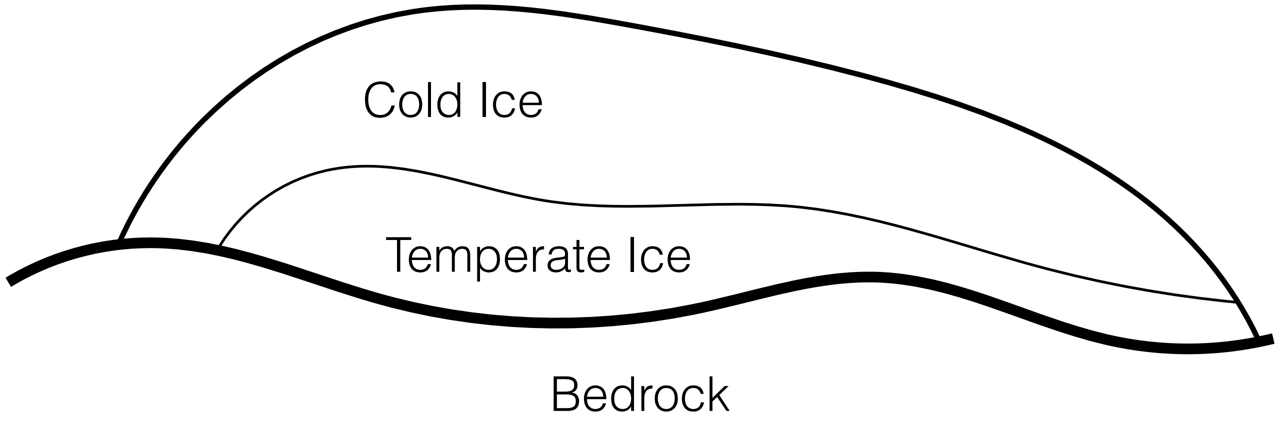
Effective pressure

Pore pressure $p_w = p - p_e$

$\nabla p \approx \rho \mathbf{g}$

Schoof & Hewitt 2015 *in review*

Problem formulation



Energy conservation

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$\nabla p \approx \rho \mathbf{g}$

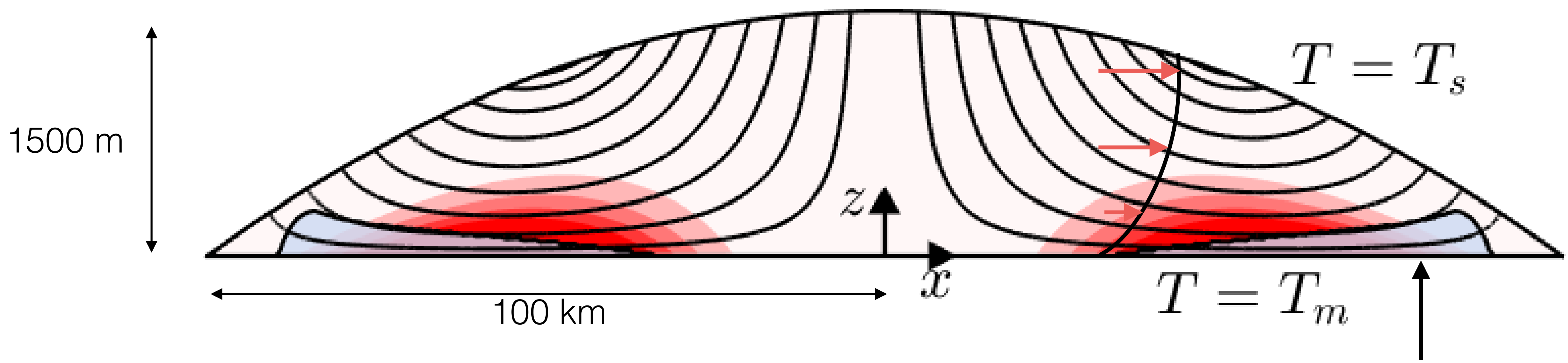
Viscous compaction
e.g. Hewitt & Fowler 2008

$$\nabla\cdot\mathbf{j} = \frac{\phi p_e}{\eta}$$

$$\mathbf{j} = \frac{k_0 \phi^\alpha}{\eta_w}((\rho_w - \rho)\mathbf{g} + \nabla p_e)$$

Schoof & Hewitt 2015 *in review*

Ice divide example

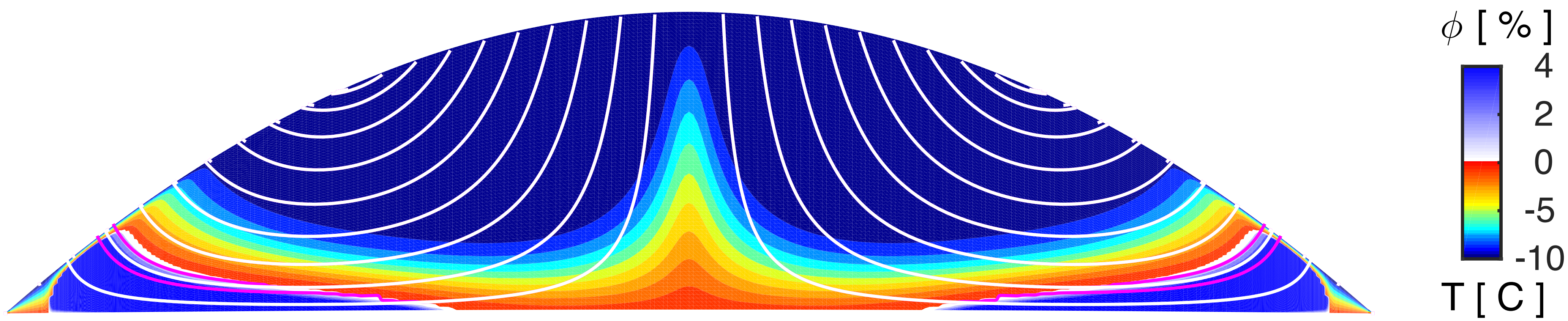


Pore pressure = subglacial drainage pressure $p_e = N_0$

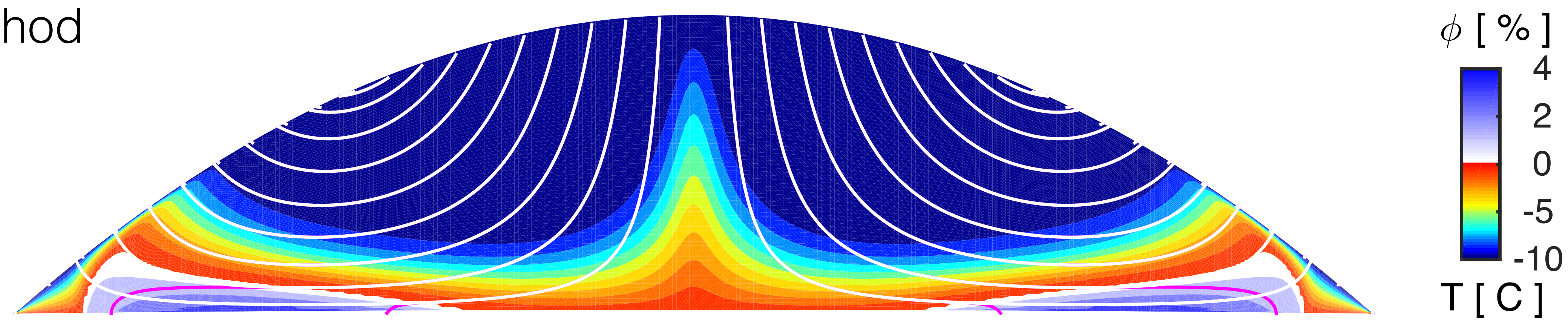
- Velocity from shallow ice approximation (thermodynamically decoupled).

Ice divide example

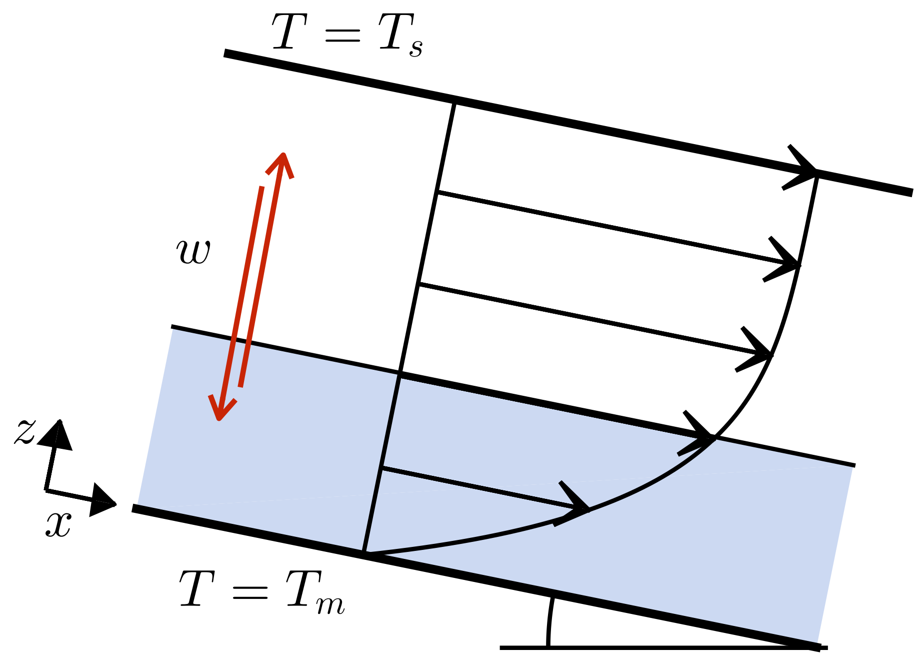
‘Standard’ enthalpy
gradient method



Compaction pressure
method



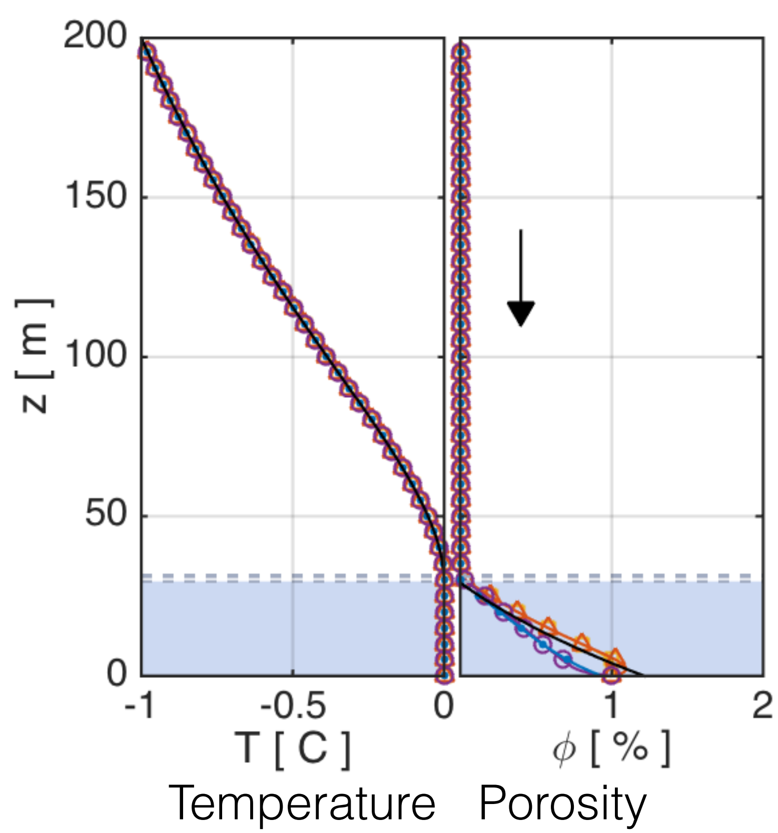
Slab glacier test case

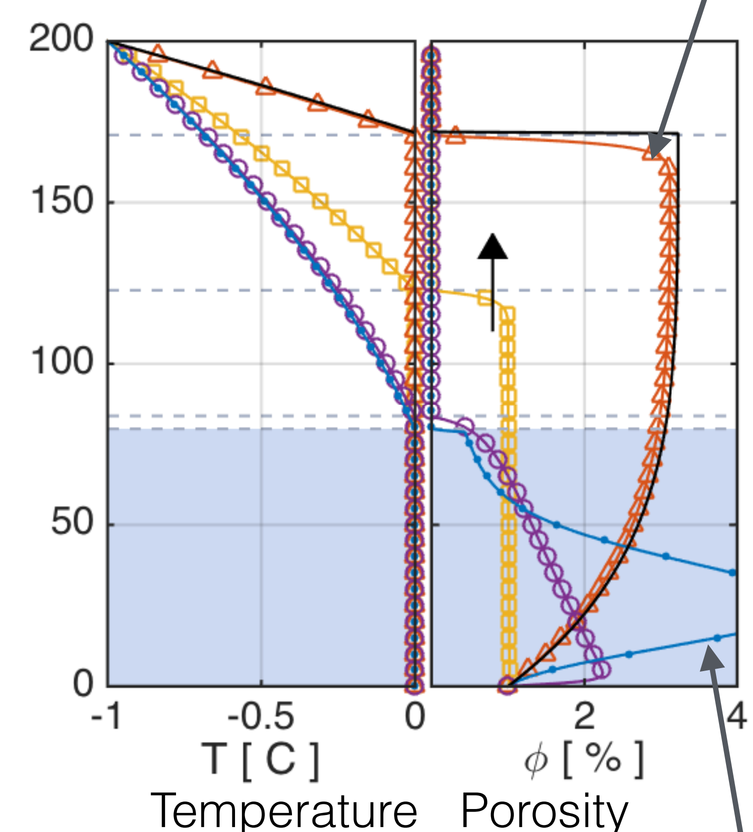


- Greve & Blatter (2009), Kleiner et al (2015), Blatter & Greve (2015)

Slab glacier test case

- Comparison of different models



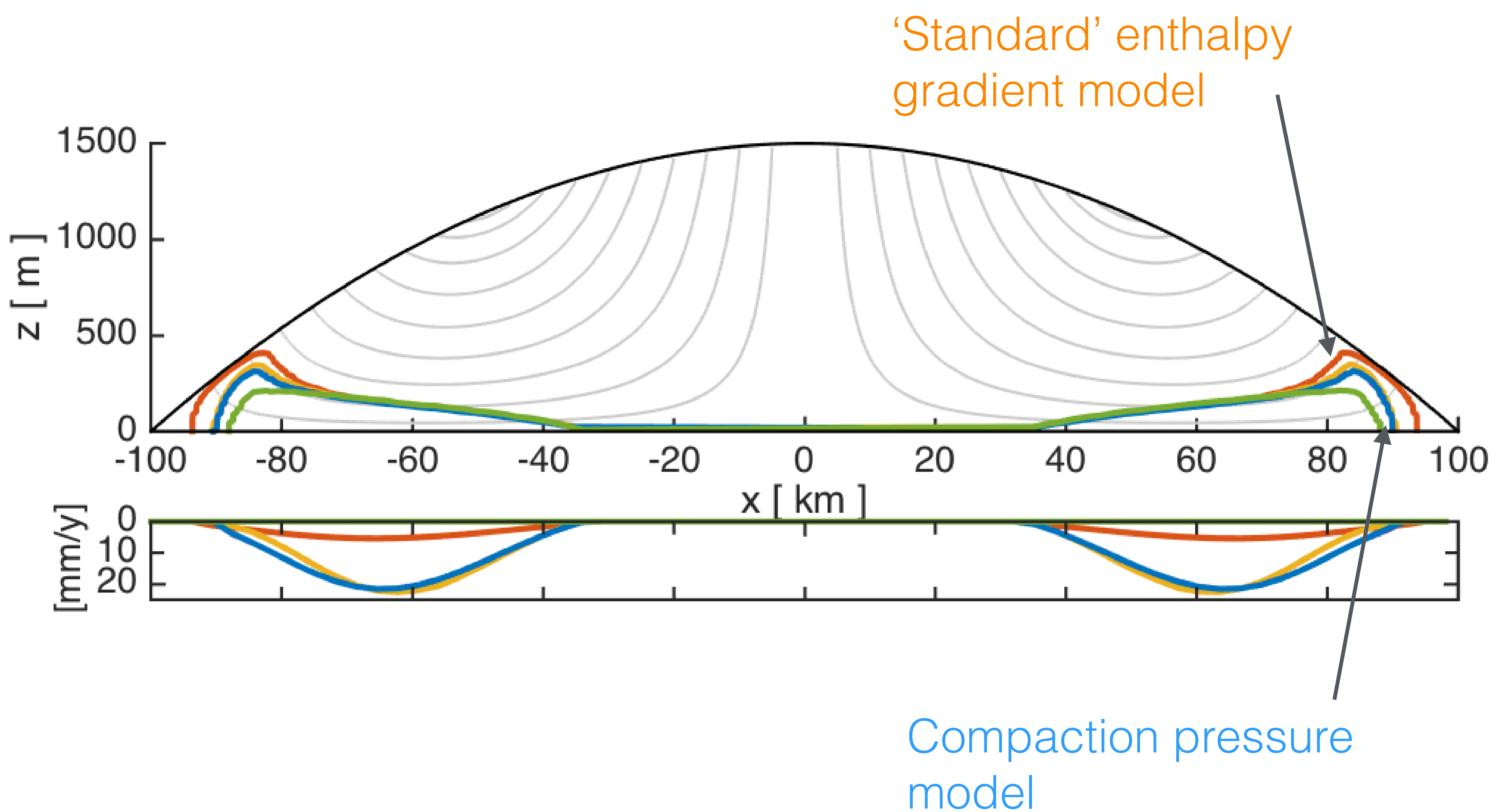


- Standard enthalpy gradient well approximates no-water-transport solution.
- Gravity-driven drainage results in less temperate ice - and some interesting behaviour…

Ice divide example

Cold-temperate transition

Meltwater flux to bed



Summary

- Suggested a simple model to incorporate polythermal ice in existing ice-sheet models - alternative to enthalpy gradient method.

- Model allows water transport through the ice, and connection with subglacial drainage - but more knowledge of permeability needed.

- Worth exploring dynamics of temperate ice + subglacial water + sediments further.