# Glacier and Ice-Sheet Hydrology

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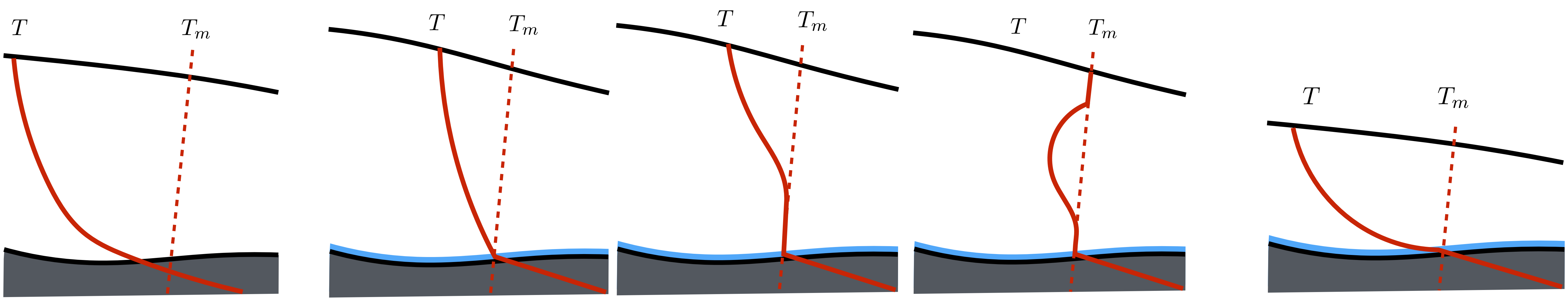
Why is glacier or ice sheet hydrology important?

Where is water produced on a glacier or ice sheet?  How much?

What happens to that water?

How does water move at the base of a glacier or ice sheet?

# Thermal setting




Frozen bed

Basal melting

Basal freezing

Geothermal heating $G \sim 0.06\ \mathrm{W\ m^{-2}}$

Frictional heating $\tau_b \sim 100\ \mathrm{kPa}$

$u_b \sim 30\ \mathrm{m\ y^{-1}}$

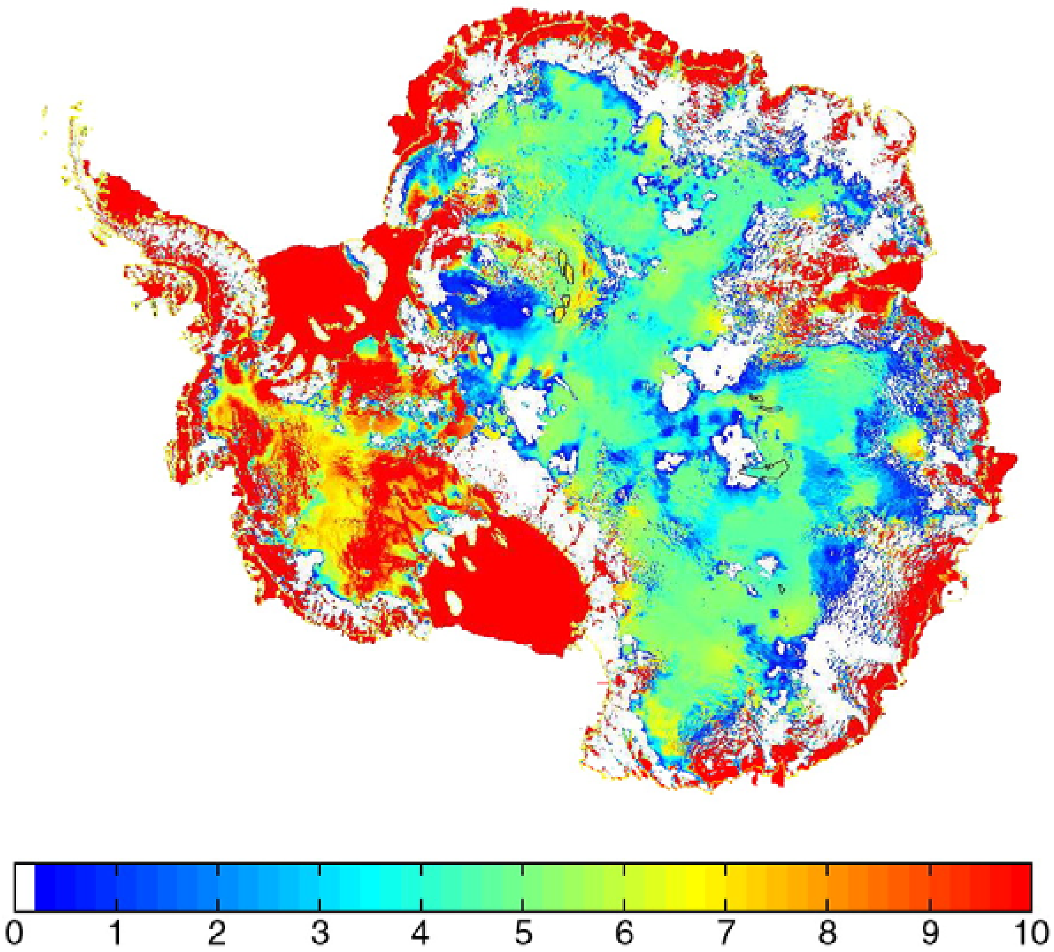
$\tau_b u_b \sim 0.1\ \mathrm{W\ m^{-2}}$

Basal melting

$$m \sim \frac{Q}{\rho L} \sim 10\ \mathrm{mm\ y^{-1}}$$

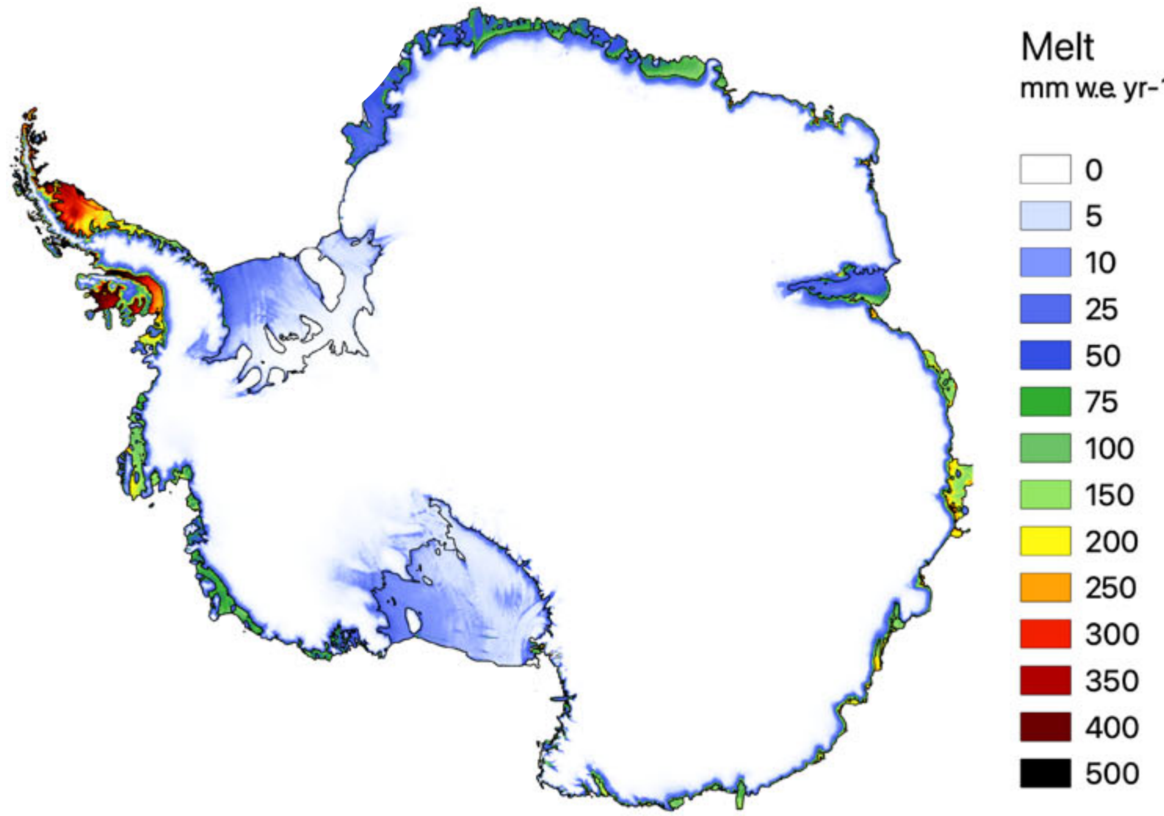
# Water sources in Antarctica

Basal melting ~ 10 mm/y




Pattyn 2010

Surface melting ~ 10 cm/y



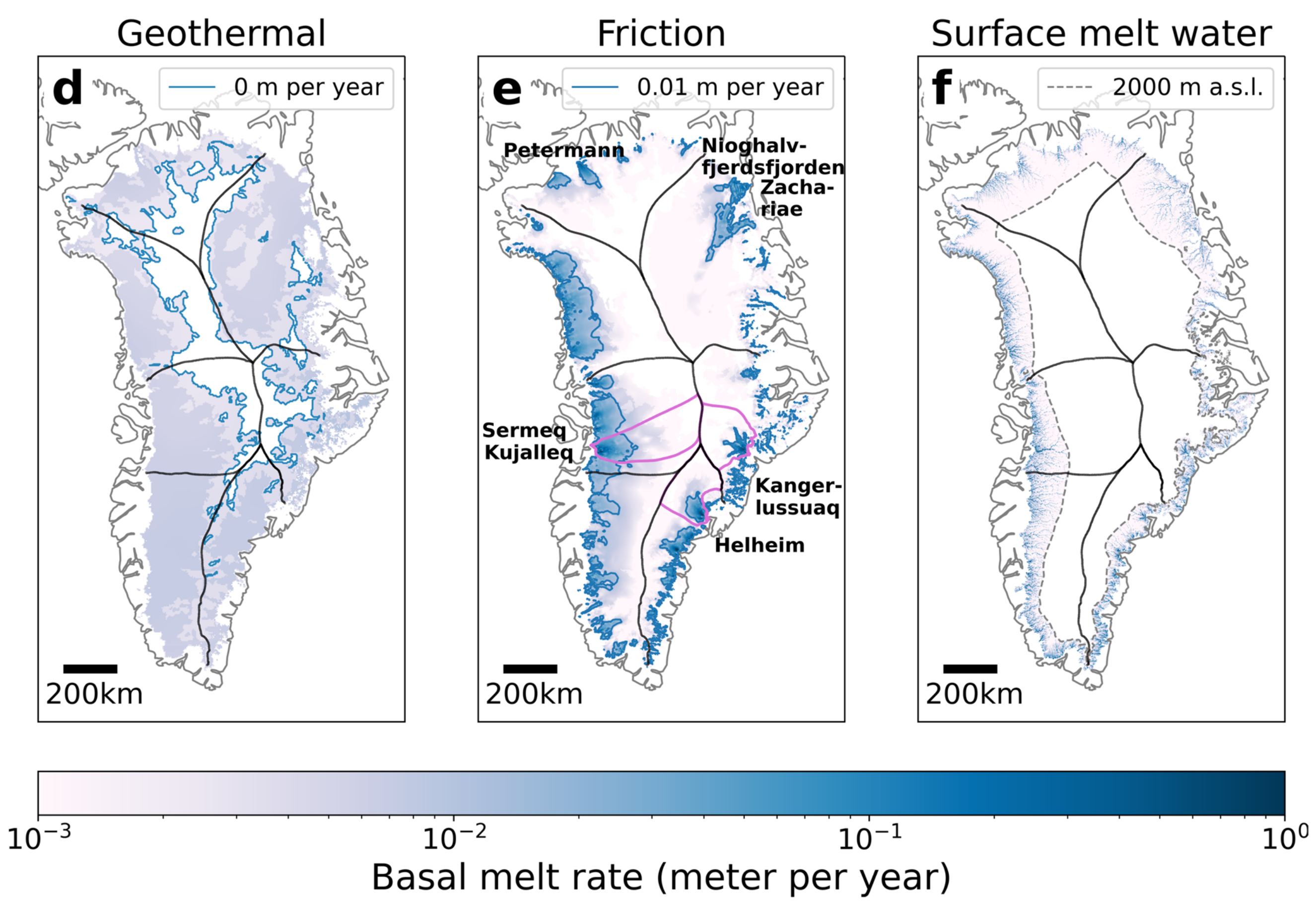

Noël et al 2023

⇨ Basal melting (grounded ice) ~ 65 Gt/y Surface melting ~ 150 Gt/y

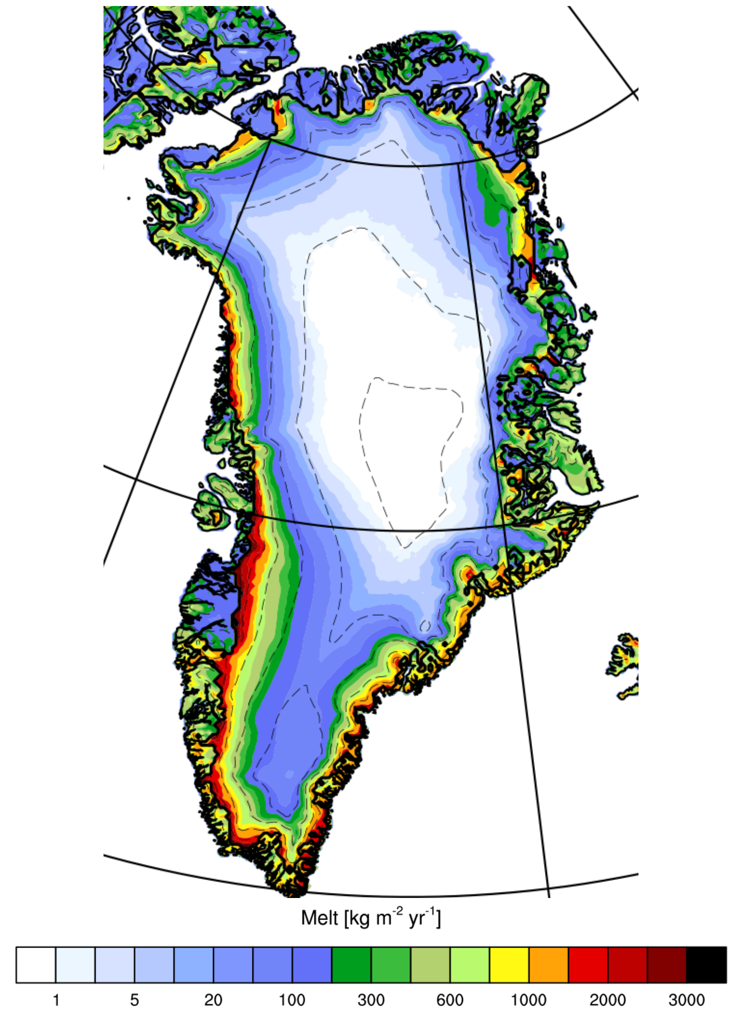
# Water sources in Greenland

Basal melting ~ 10 mm/y

Surface melting ~ 1 m/y



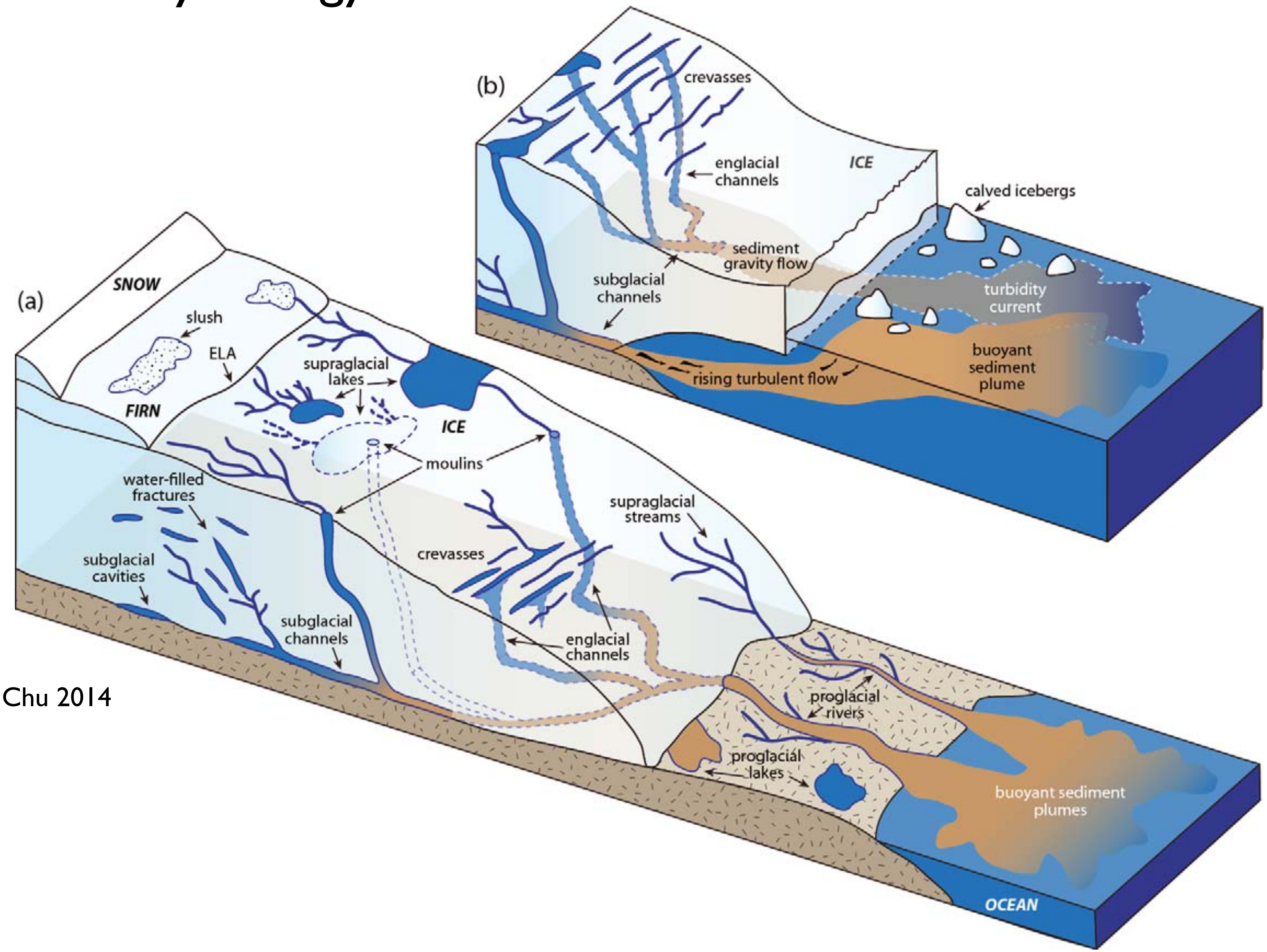

Karlsson et al 2021




Van den Broeke et al 2016

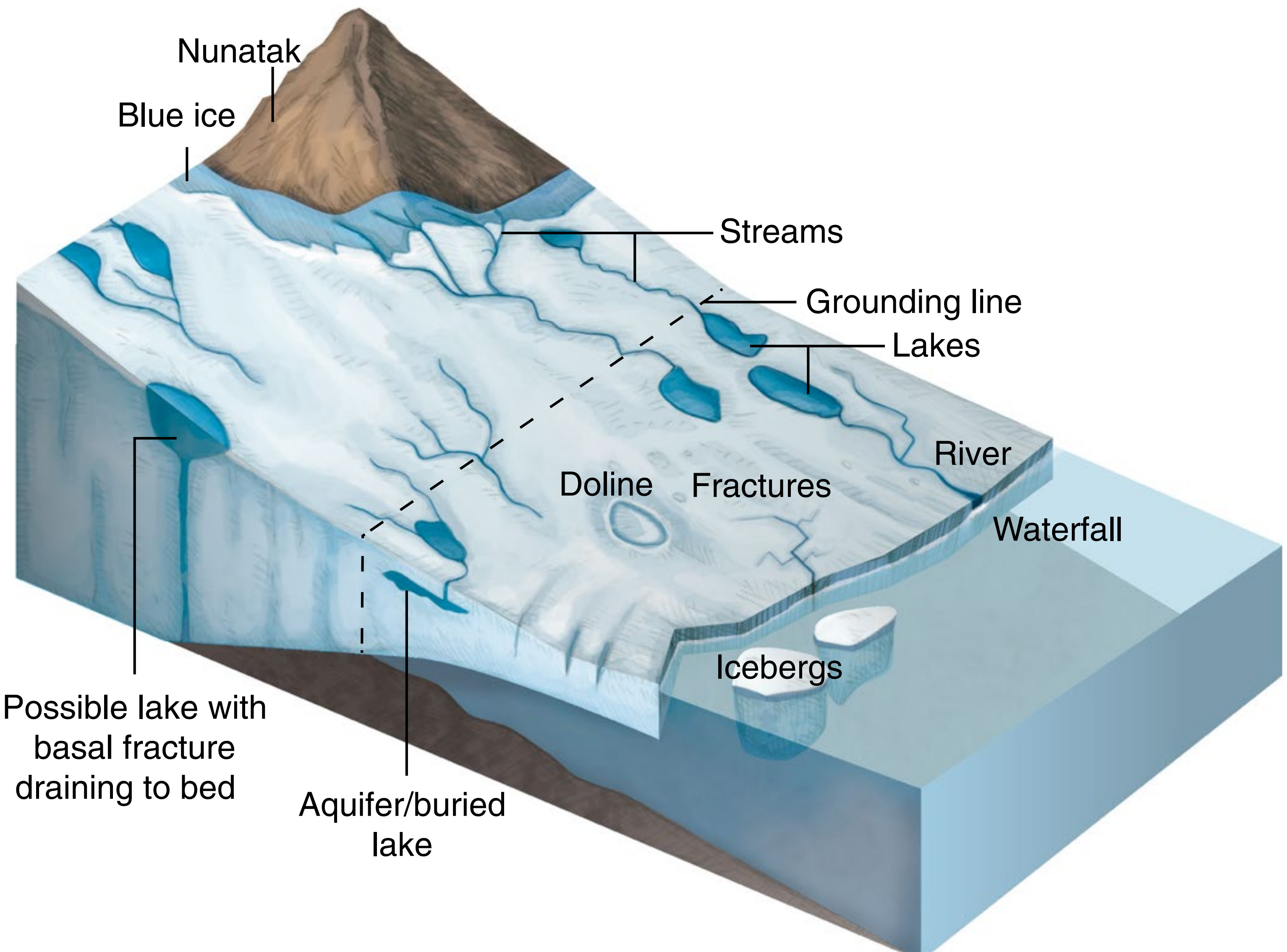
⇨ Basal melting ~ 20 Gt/y Surface melting ~ 400 Gt/y

# Greenland hydrology




Chu 2014

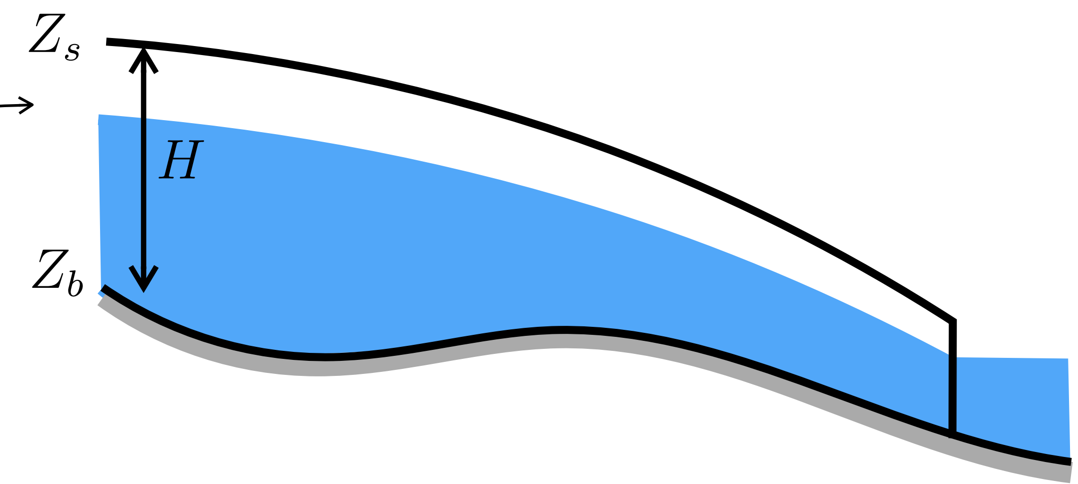
# Antarctic hydrology




Bell et al 2018

# Pressurised subglacial water

Hydraulic head $\dfrac{\phi}{\rho_w g}$




**Hydraulic potential**

$$\phi = \rho_w g Z_b + p_w$$

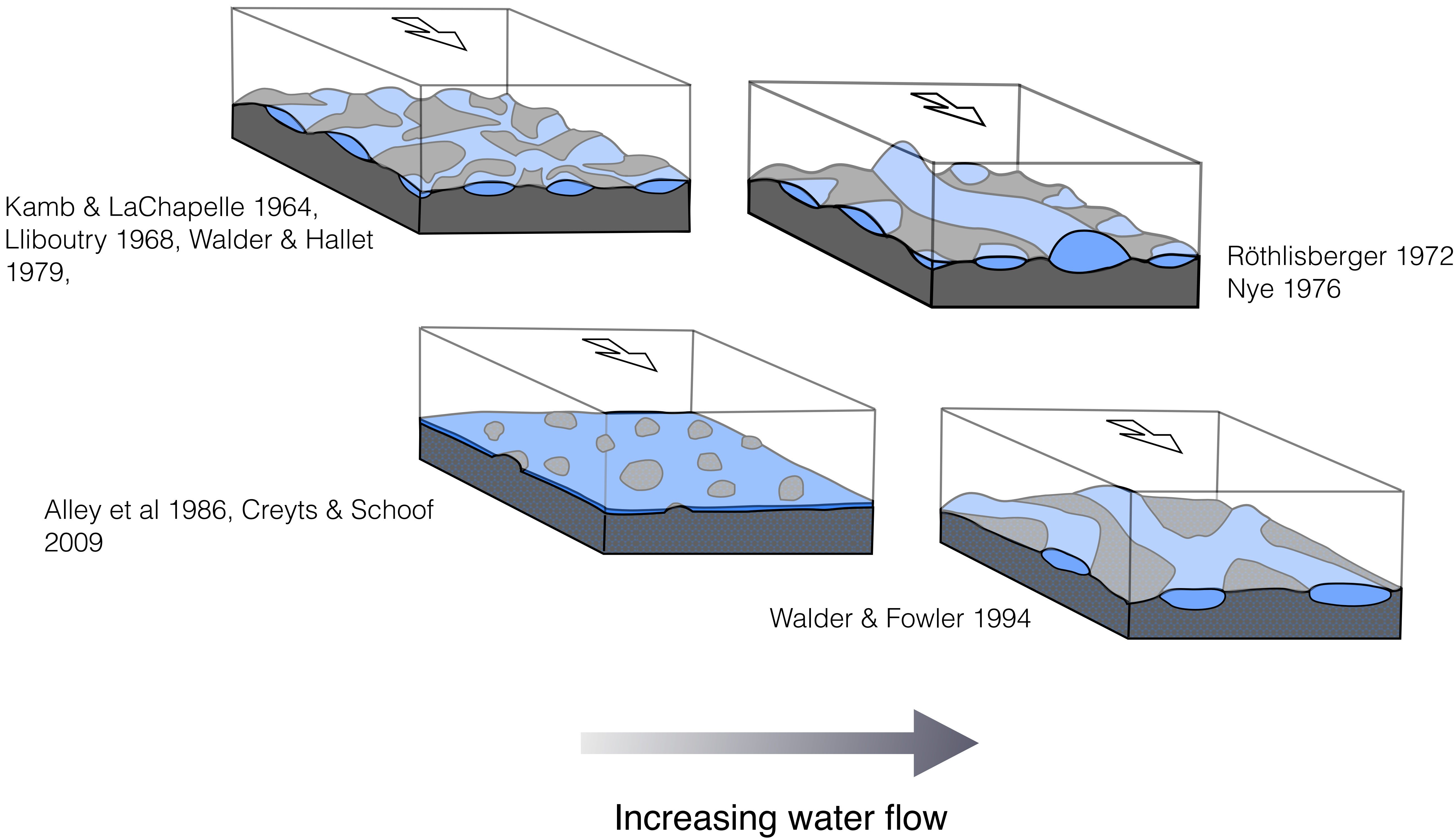
$$= \rho_w g Z_b + \rho_i g (Z_s - Z_b) - N$$ in terms of **effective pressure** $N = p_i - p_w$

**Potential gradient** $-\nabla \phi = \Psi + \nabla N \approx \Psi$ $\qquad \Psi = -\rho_i g \nabla Z_s - (\rho_w - \rho_i) g \nabla Z_b$

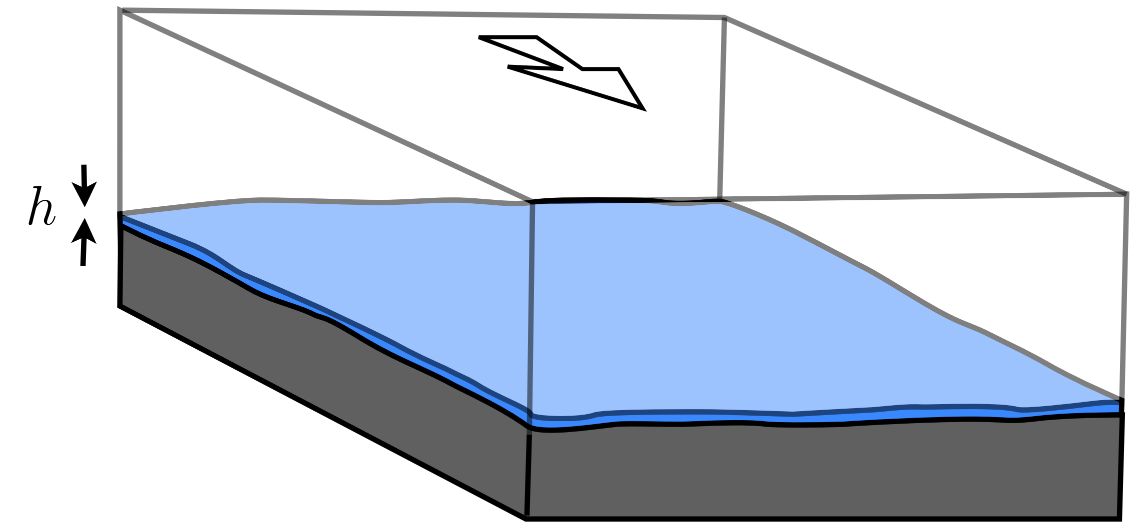
A common assumption is $p_w \approx k p_i$ $\Rightarrow$ $-\nabla \phi \approx -\rho_i g \nabla Z_s - k(\rho_w - \rho_i) g \nabla Z_b$ 'Shreve potential'

$\Rightarrow$ Subglacial water predominantly flows **down ice surface slope**

# Subglacial drainage systems



# Water film Weertman 1972, Walder 1982




Poiseuille flux $Q = \frac{h^3}{12\eta}(\Psi + \nabla N)$

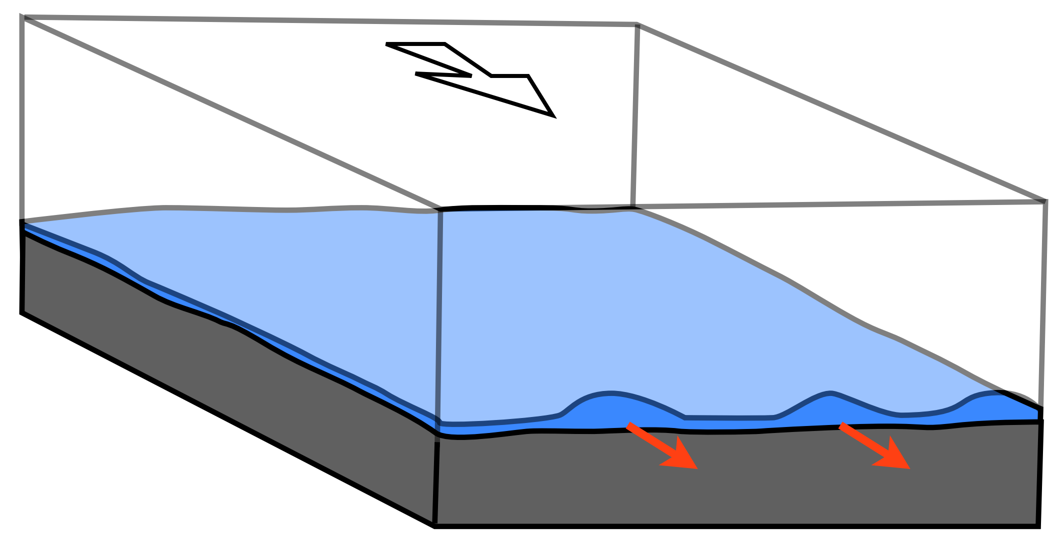
Water flow dissipates energy through heating

⇨ Leads to an instability

Larger sheet thickness

→ Larger flux

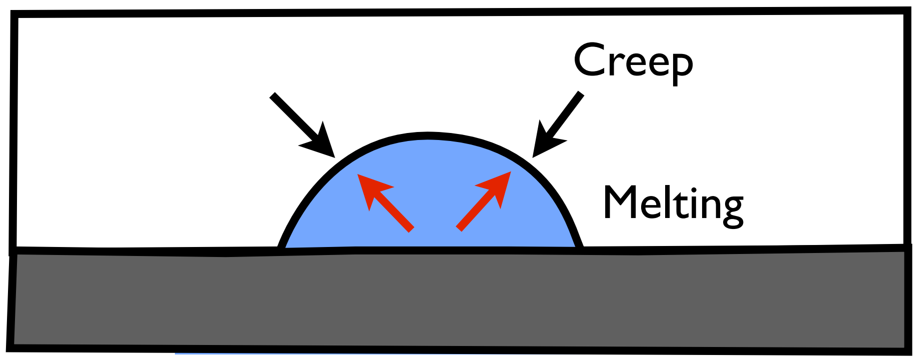
→ Melting of ice roof

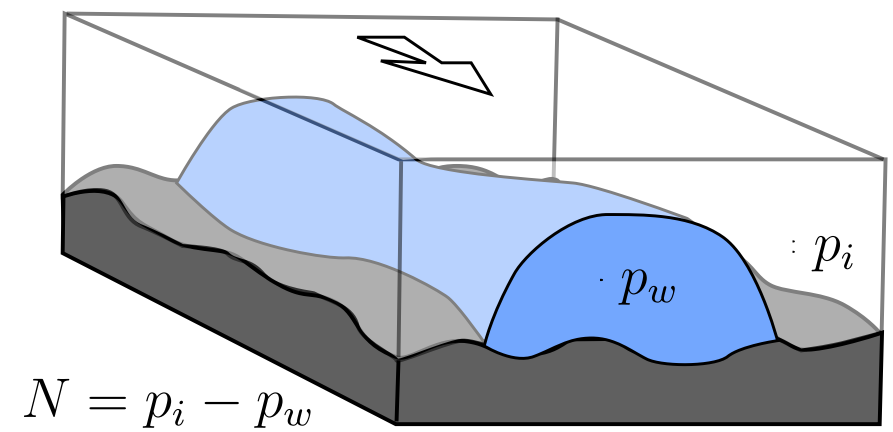


⇨ Flow wants to concentrate in **localized channels / tunnels**

# Röthlisberger channels   Röthlisberger 1972, Nye 1976

Ice wall **melting** is counteracted by **viscous creep**




Model (ignoring pressure dependence of melting temperature)

$$\frac{\partial S}{\partial t} + \frac{\partial Q}{\partial x} = \frac{m}{\rho_w} + M$$ water mass conservation

$$\frac{\partial S}{\partial t} = \frac{m}{\rho_i} - \tilde{A} S N^n$$ wall evolution

$$mL = Q\left(\Psi + \frac{\partial N}{\partial x}\right)$$ local energy conservation

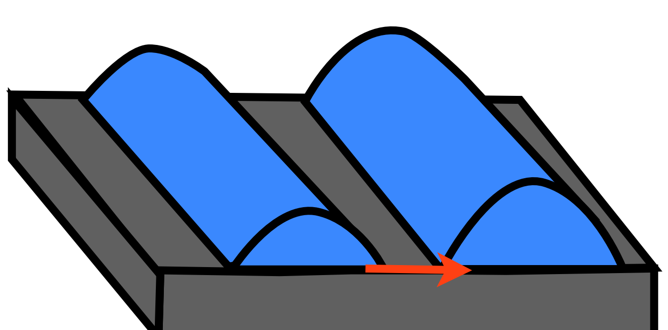
$$Q = K_c S^{4/3}\left(\Psi + \frac{\partial N}{\partial x}\right)^{1/2}$$ momentum conservation (turbulent flow parameterization)

Steady state ⇨

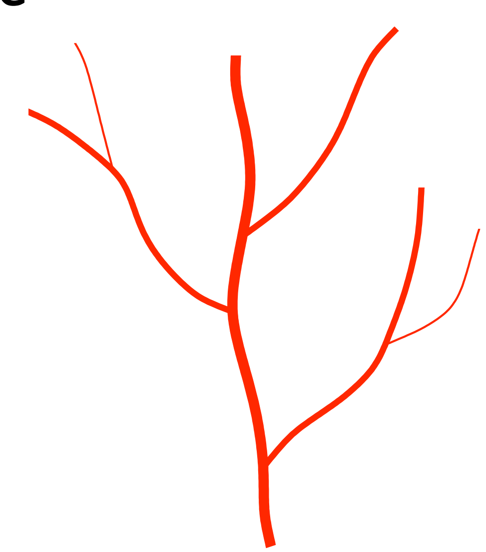
$$\boxed{N \approx \left(\frac{K_c^{3/4}}{\rho_i L \tilde{A}}\right)^{1/n} \Psi^{11/8n}\, Q^{1/4n}}$$

Effective pressure INCREASES with discharge

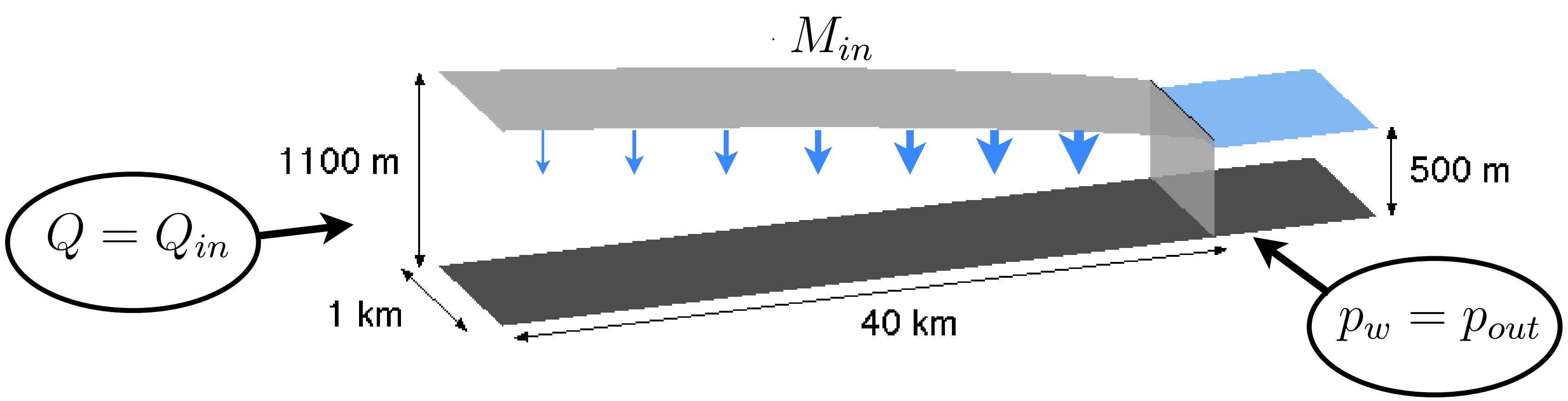
Neighbouring channels compete with one another

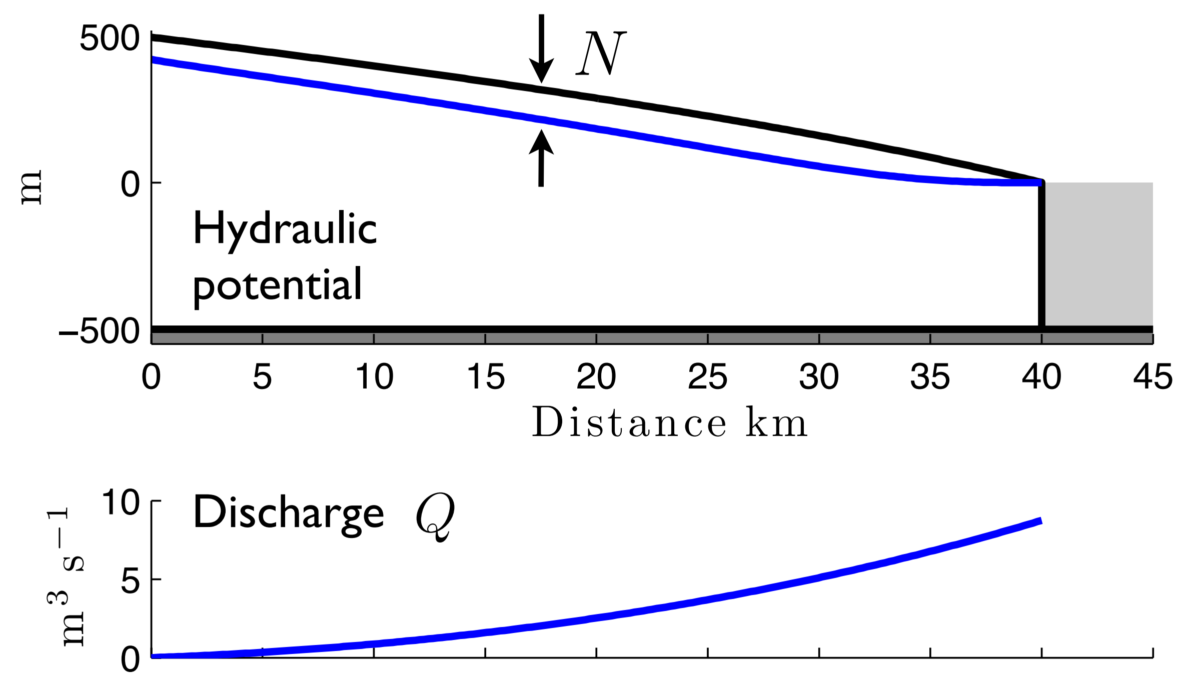


⇨ leads to an arterial network



# Röthlisberger channels



# Jökulhlaups (Glacial Lake Outburst Floods)

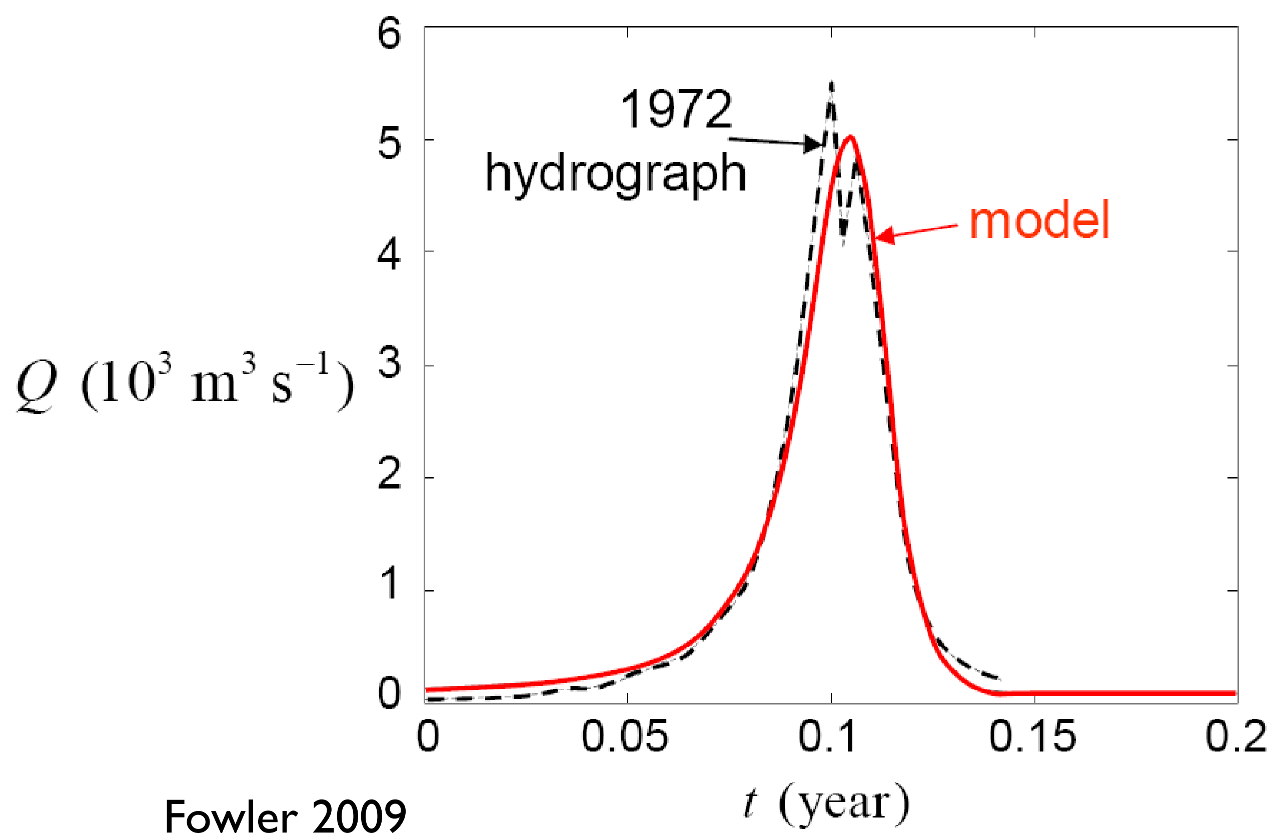
Skeidarársandur, Iceland 1996

# Jökulhlaups Nye 1976, Spring & Hutter 1981, Clarke 2003

A significant success of the channel theory is the application to **floods from ice-dammed lakes**

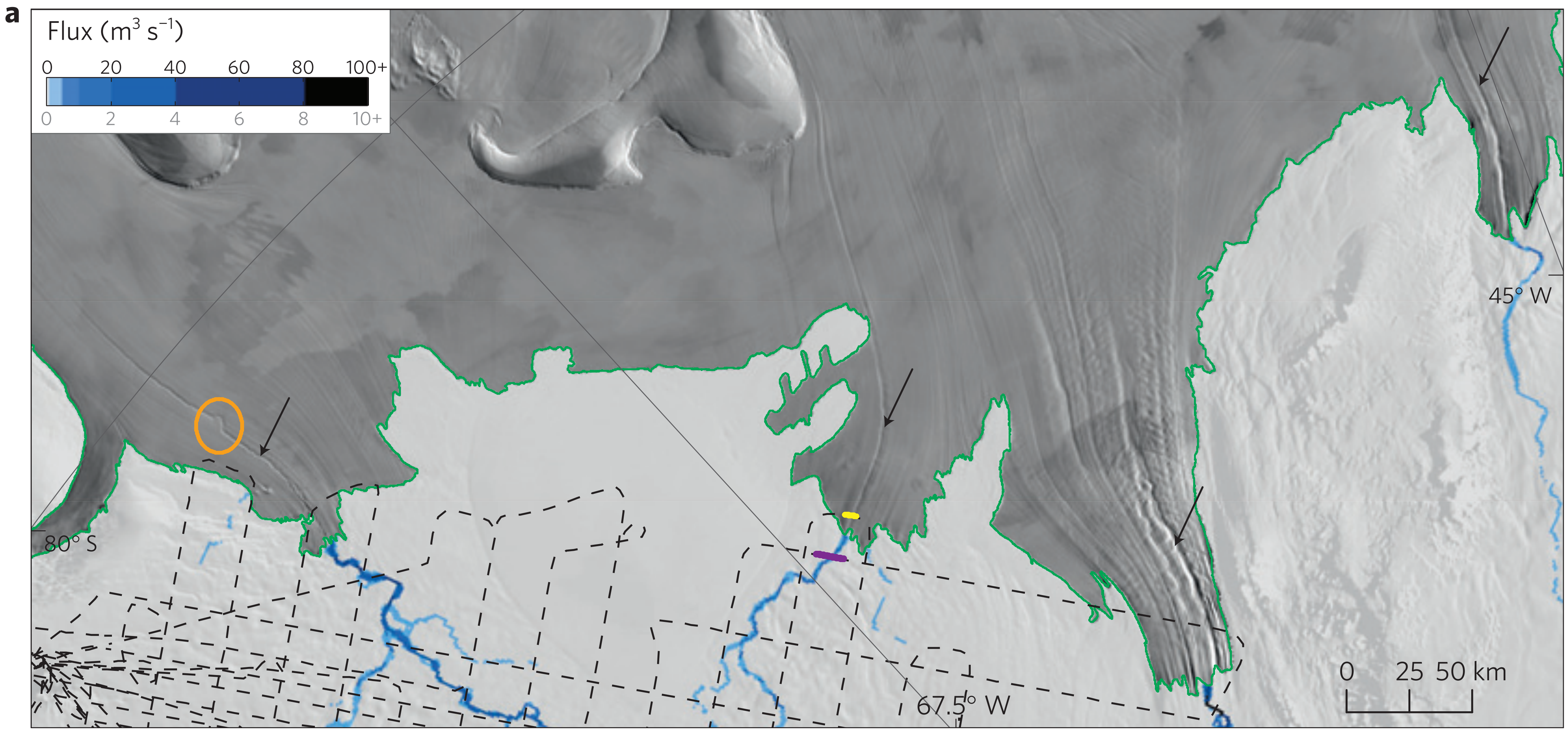
Combine **channel evolution** equation
$$\frac{\partial S}{\partial t} = \frac{S^{4/3}\Psi^{3/2}}{\rho_i L} - \tilde{A} S N^n$$

with a **lake filling** equation
$$-\frac{A_L}{\rho_w g}\frac{\partial N}{\partial t} = m_L - Q$$




Fowler 2009

# Evidence for channelised water flow beneath grounding lines

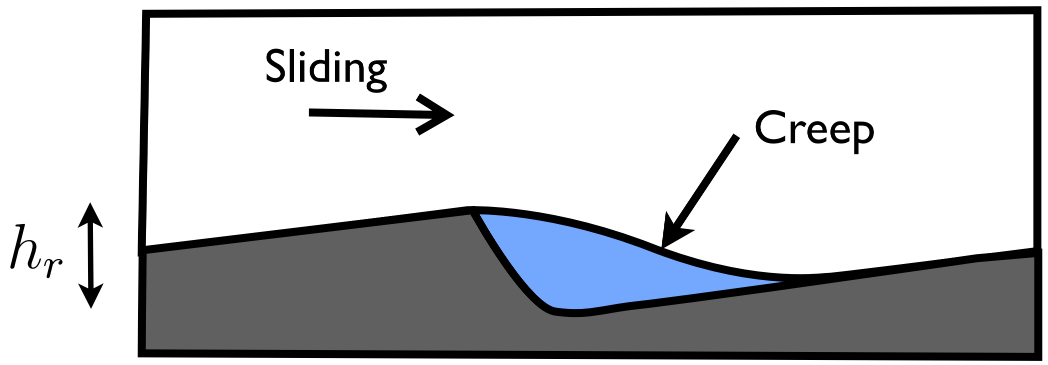


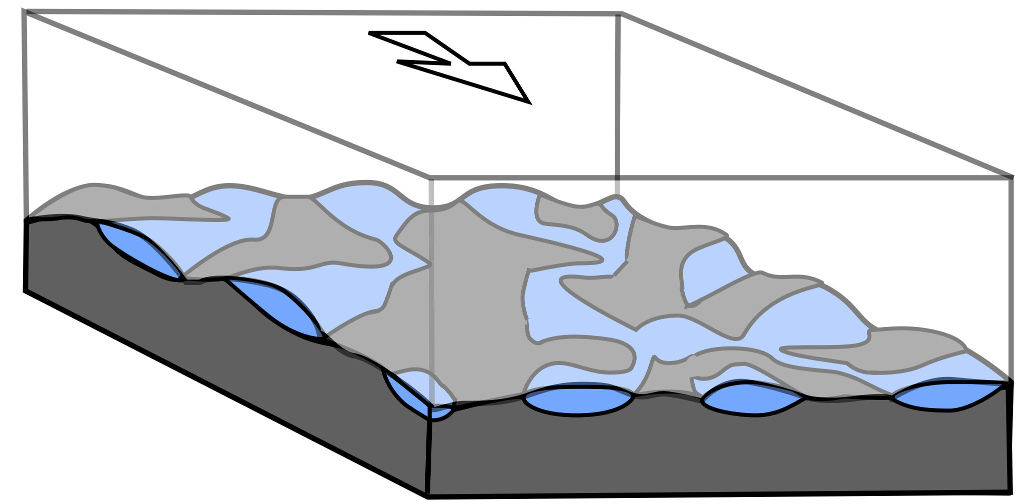

Le Brocq et al 2013

Localised subglacial discharge initiates sub-shelf plumes and ice-shelf channels

# Linked cavities Walder 1986, Kamb 1987

Cavities grow through sliding over bedrock



**Model**

$$\frac{\partial \hat{S}}{\partial t} = U_b h_r - \tilde{A}\hat{S}N^n$$

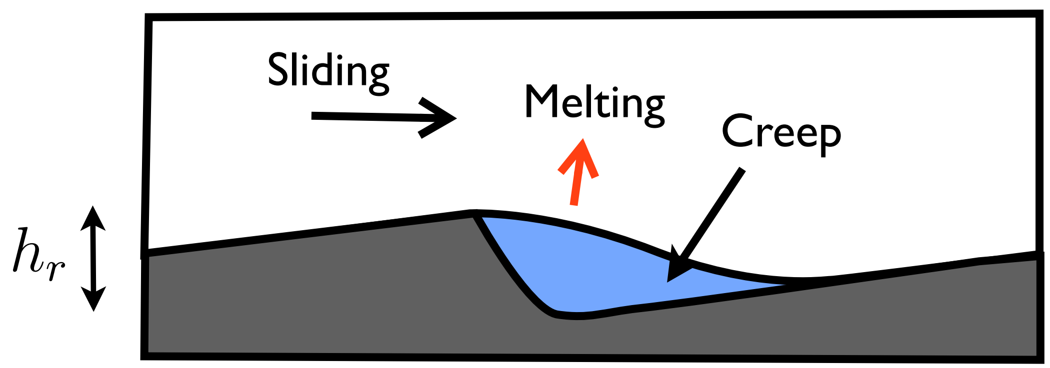
Approximate steady-state ⇨ $\boxed{N(Q) \qquad \frac{\partial N}{\partial Q} < 0}$

Effective pressure DECREASES with discharge ⇨ Flow is distributed


2 m

# Drainage system stability

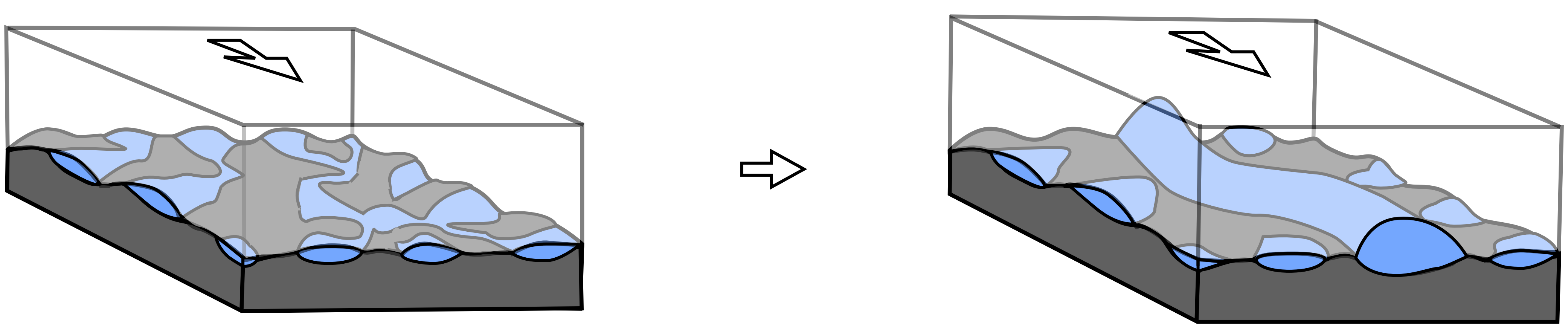
Energy is still dissipated by distributed water flow




$$\frac{\partial S}{\partial t} = \underline{\frac{m}{\rho_i}} + U_b h_r - \tilde{A} S N^n$$

A linked cavity system can become unstable to produce **channels**

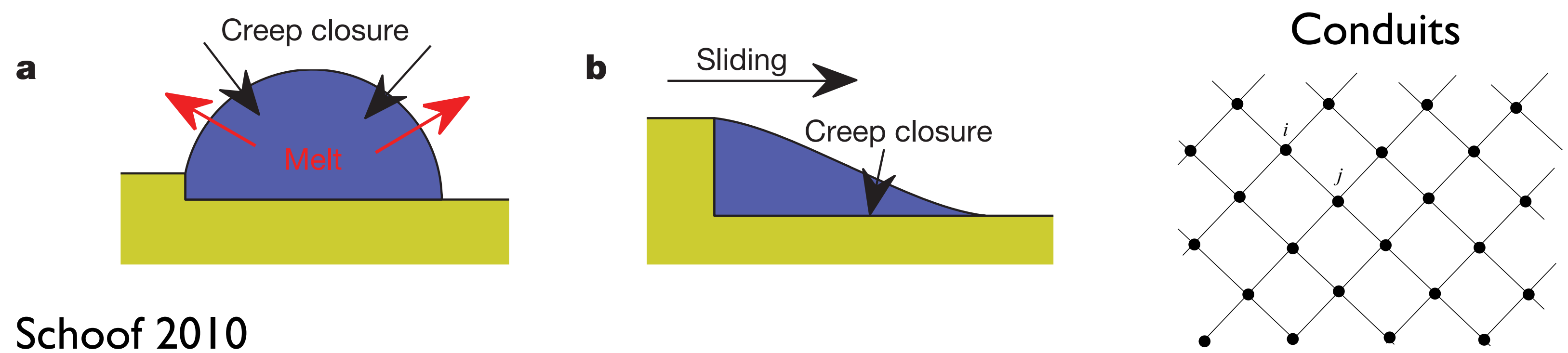
eg. if discharge becomes sufficiently large, or sliding speed sufficiently low



Conversely, a channel can become unstable to **cavities**

eg. if discharge low, or sliding speed sufficiently high

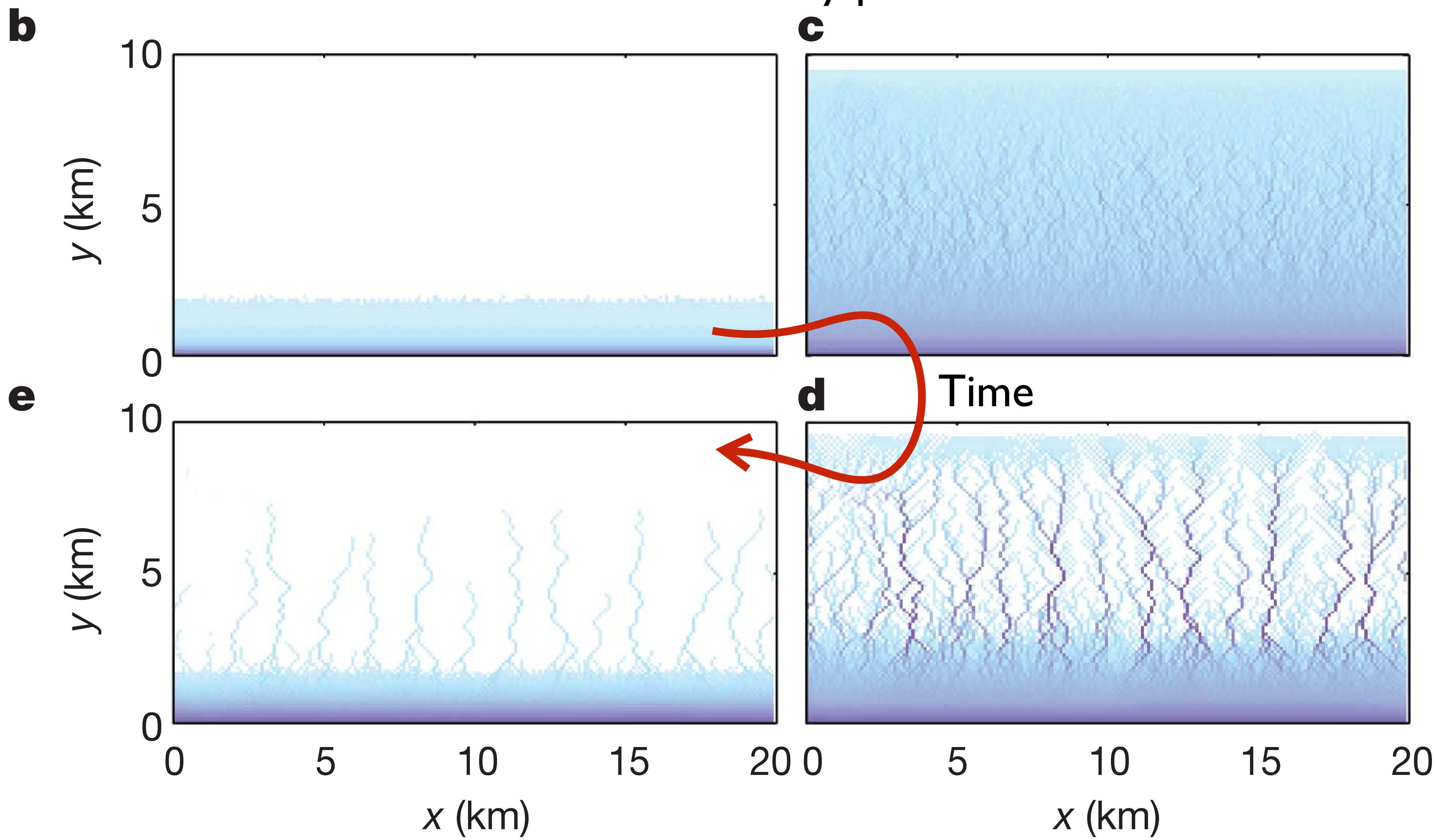
# Seasonal evolution of drainage system



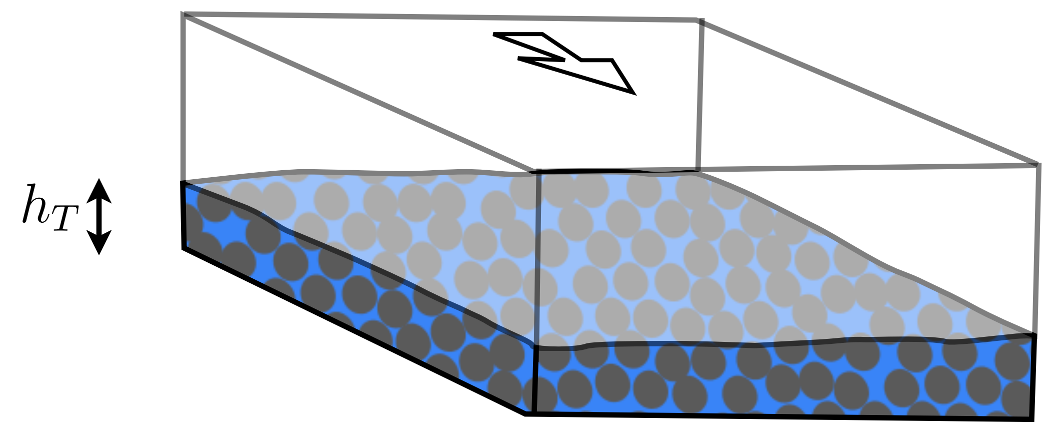

Schoof 2010

Network of 'conduits' forced by prescribed surface runoff

Ice flow

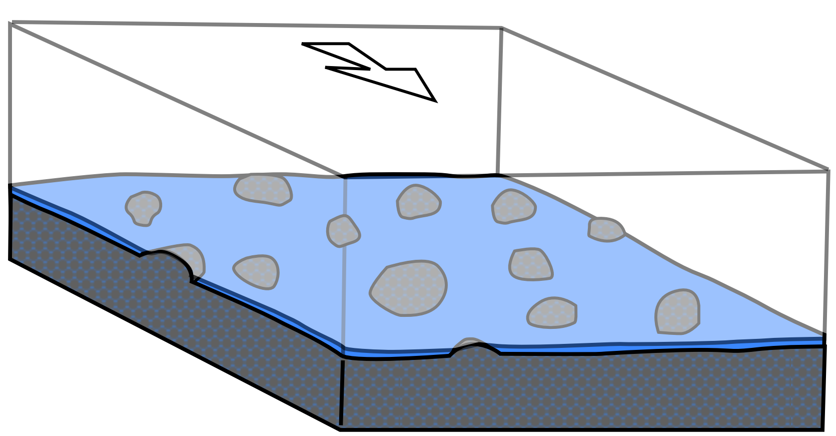


# Drainage through sediments




**Hydraulic conductivity** of till is generally too small to allow significant horizontal flow.

⇨ Water flows in a **patchy film** at the ice-till interface, or in some form of **channels or canals**.

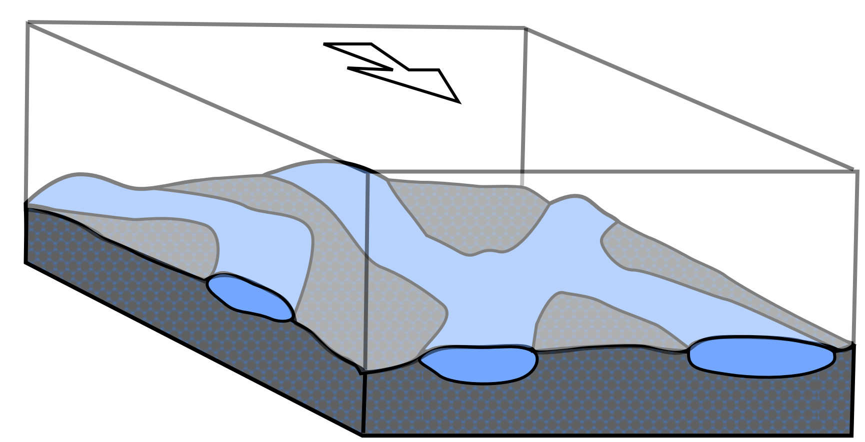


Patchy sheet

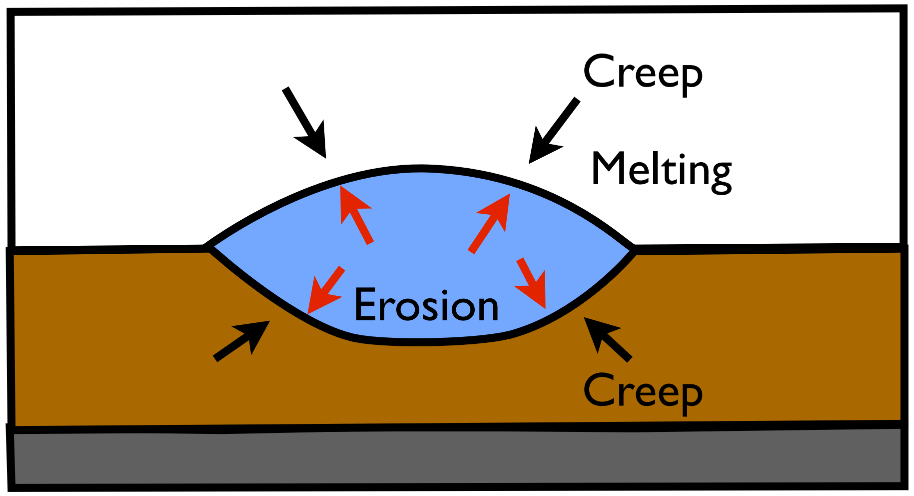
Alley 1989, Creyts & Schoof 2009

Canals

Walder & Fowler 1994



# Canals Walder & Fowler 1994, Ng 2000




Gravitational potential gradient

$$\Psi = -\rho_i g \nabla Z_s - (\rho_w - \rho_i) g \nabla Z_b$$

Walder & Fowler suggested two possibilities for steady states:

Channels - mostly melted into ice $\boxed{N \propto \Psi^{7/15} Q^{1/15}}$ $N > \tilde{N}$

$\tilde{N} \approx 0.8 \text{ MPa}$

Canals - mostly eroded into sediment $\boxed{N \propto \Psi^{-1/3} Q^{-1/3}}$ $N < \tilde{N}$

Effective pressure in canals DECREASES with increasing discharge $\Rightarrow$ Flow is distributed

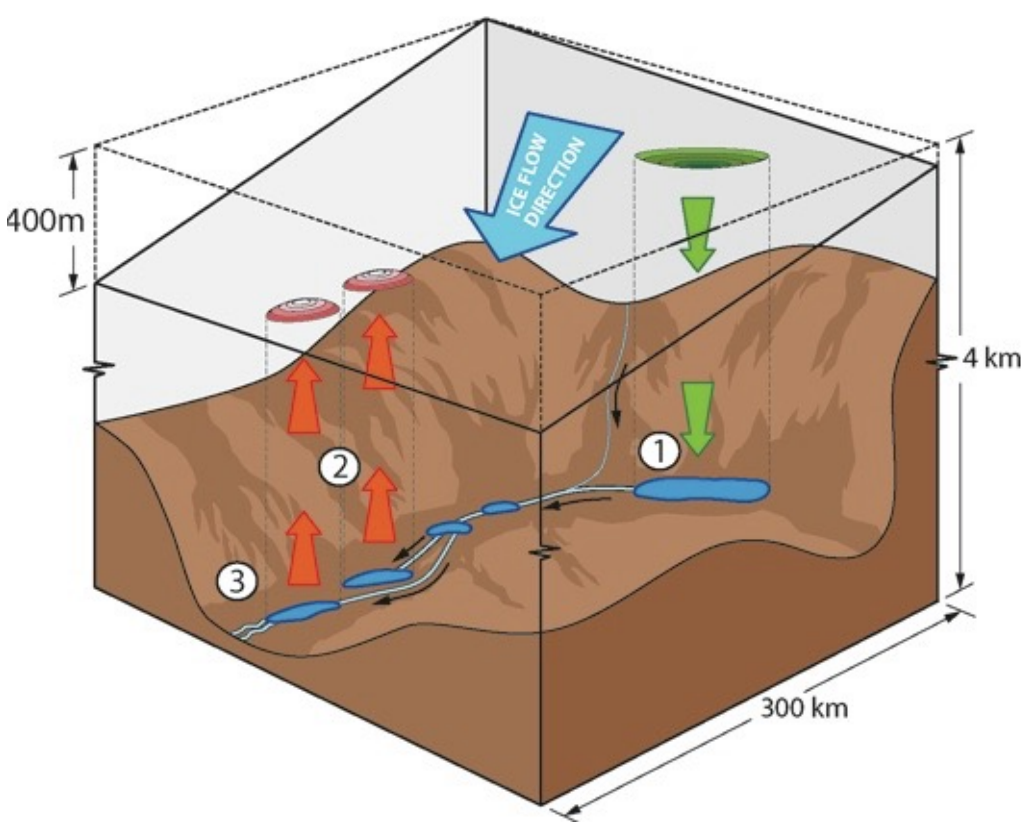
The crucial difference seems to be that erosion tends to produce a **wide cross-section**.

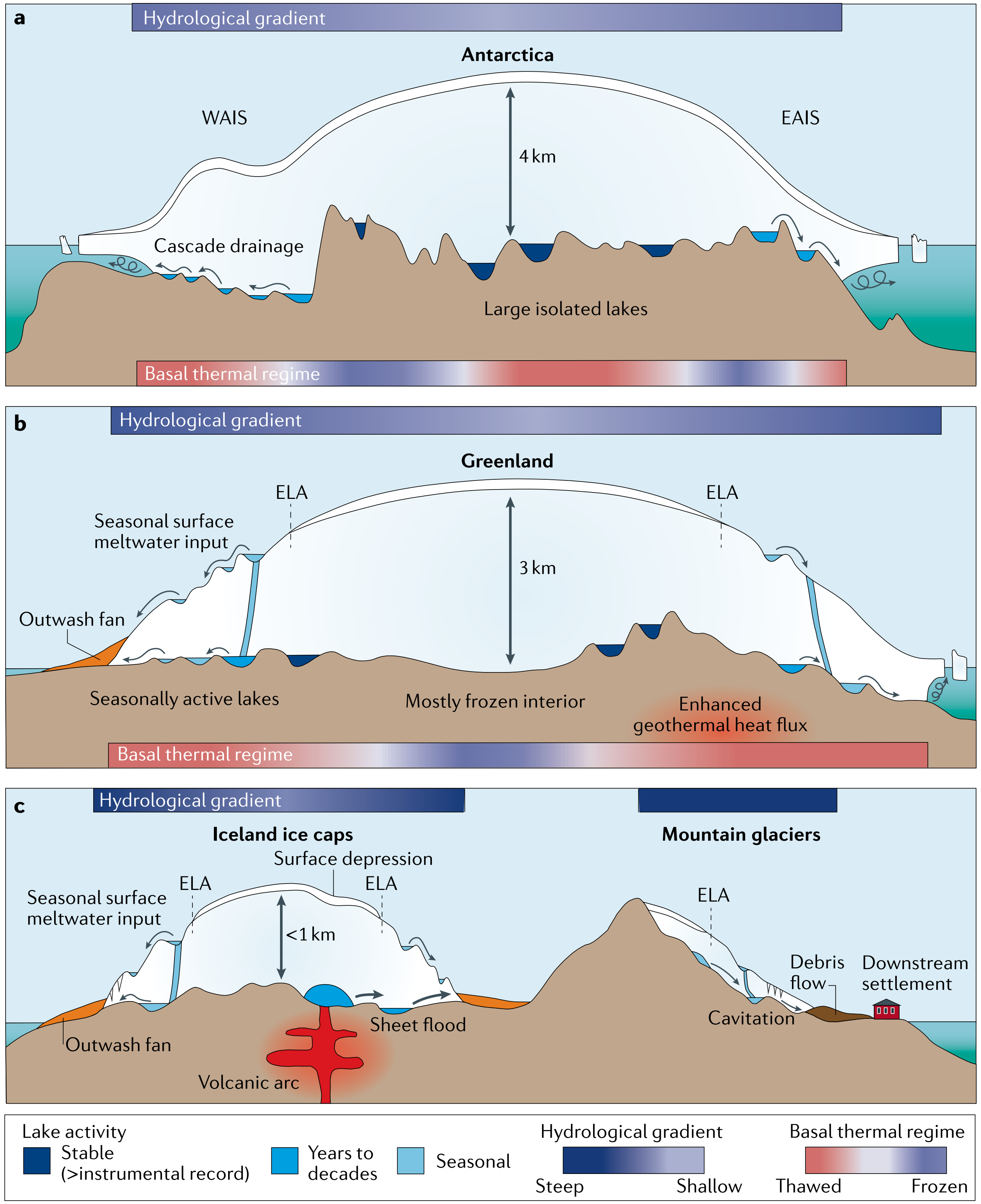
Canals are favoured when the potential gradient is small (e.g. interior of ice sheets).

# Subglacial lakes

Hundreds of lakes have been detected using radar and satellite observations.

At least some ‘active’ lakes seem to grow and drain periodically



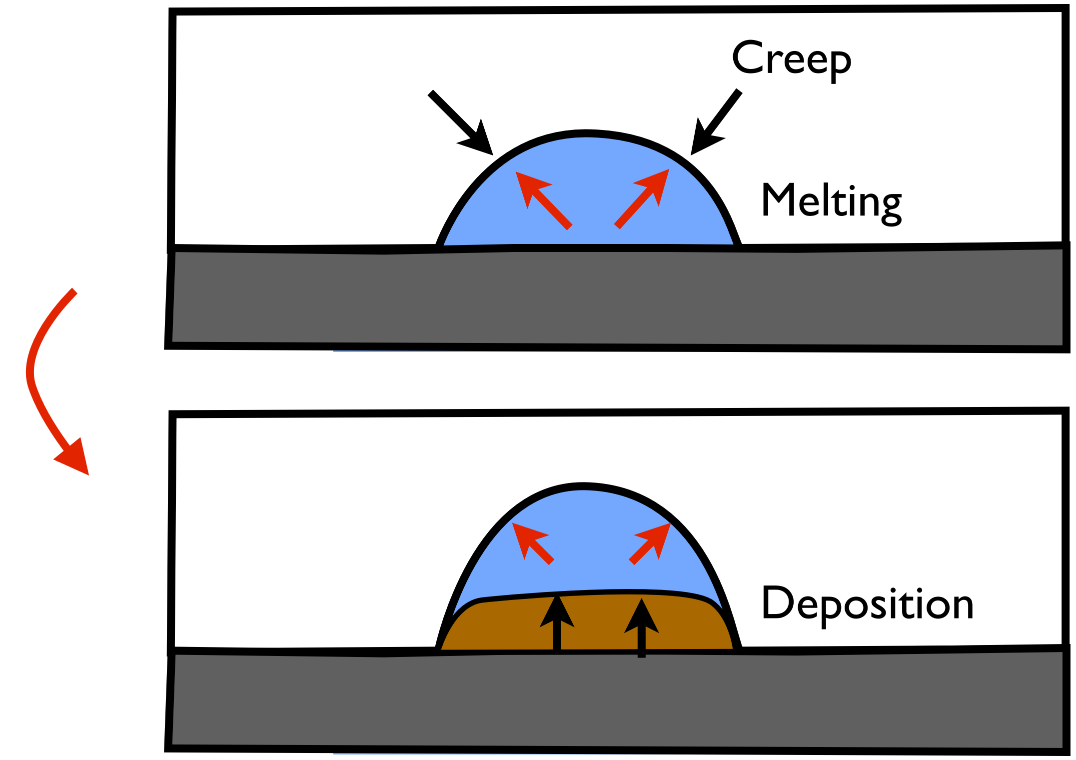

Livingstone et al 2022

# Meltwater deposits and landforms

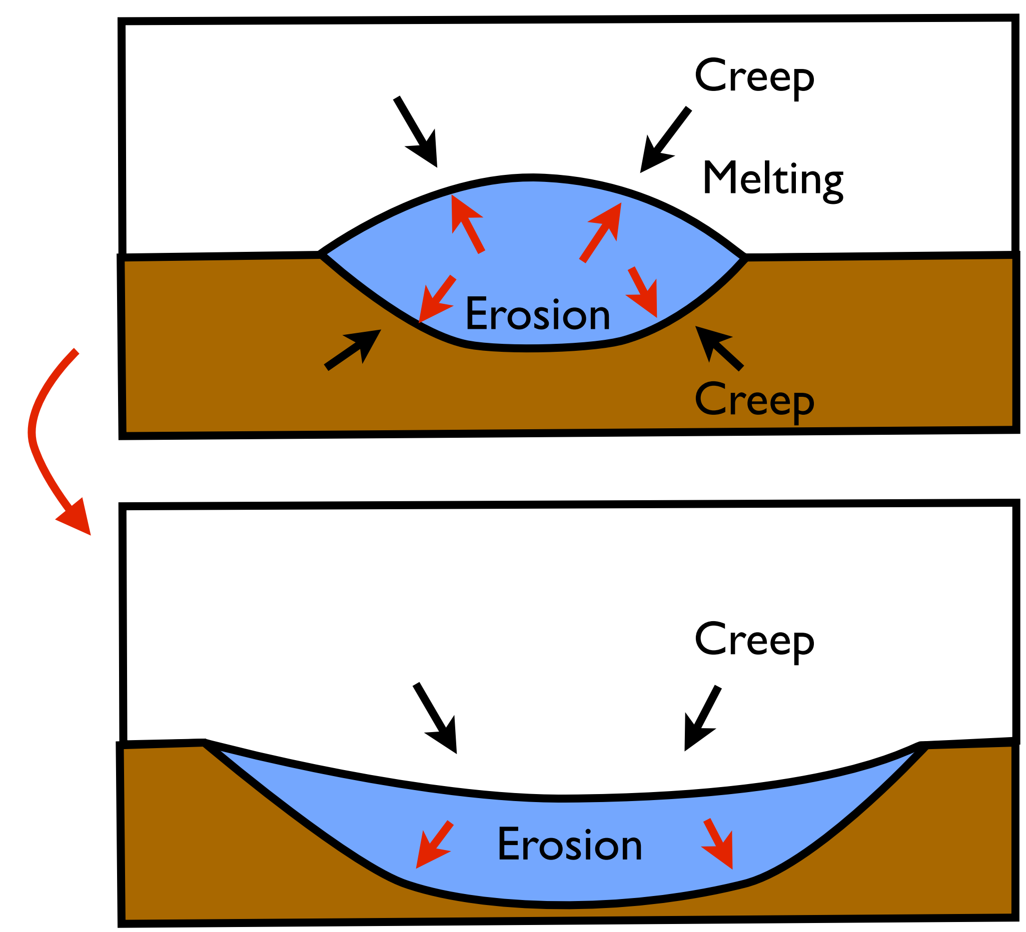
Deposition of sediments in Röthlisberger channels can build **eskers**

- Most likely under falling water speed, near margin

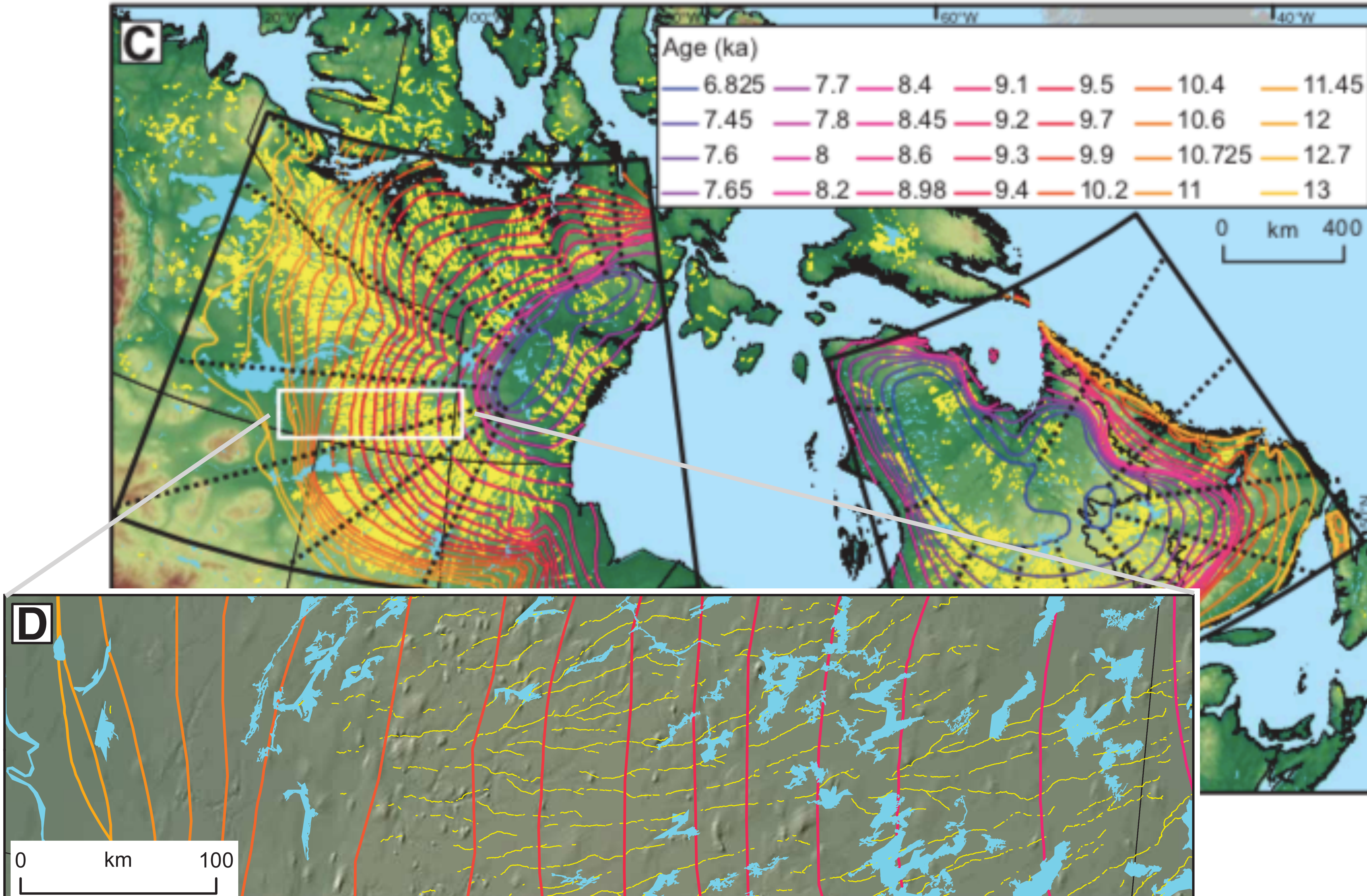
- Sediment is flushed from the surrounding bed




Erosion of sediments from canals can create **tunnel valleys**



# Eskers




Storrar et al 2014 *Geology*

# Interaction of sliding and drainage

There is the potential for a **positive feedback**

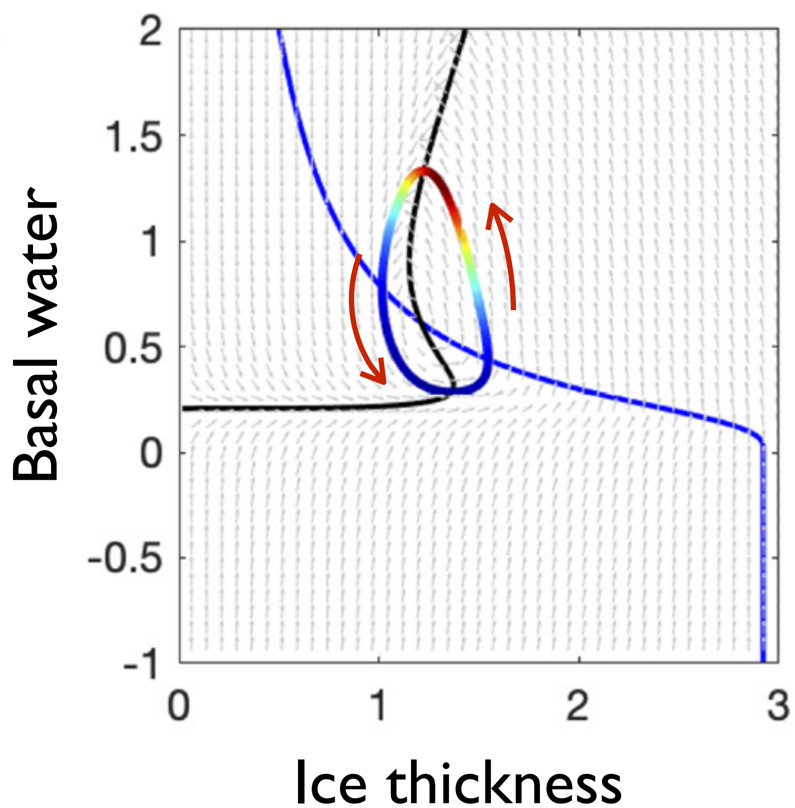
Initiation of sliding $\uparrow U_b$

→ Increased melting $\uparrow m \propto \tau_b U_b / L$

→ Increased discharge $\uparrow Q$

? → Lower effective pressure $\downarrow N$

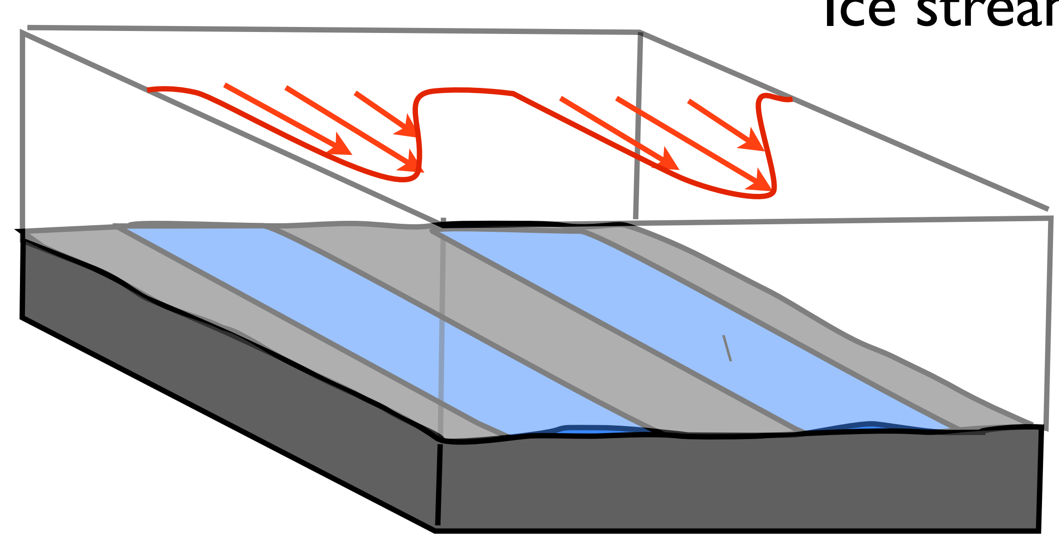
⇨ Can lead to **temporal** or **spatial** instabilities




Benn et al 2023

Glacier 'surges'

Ice streams

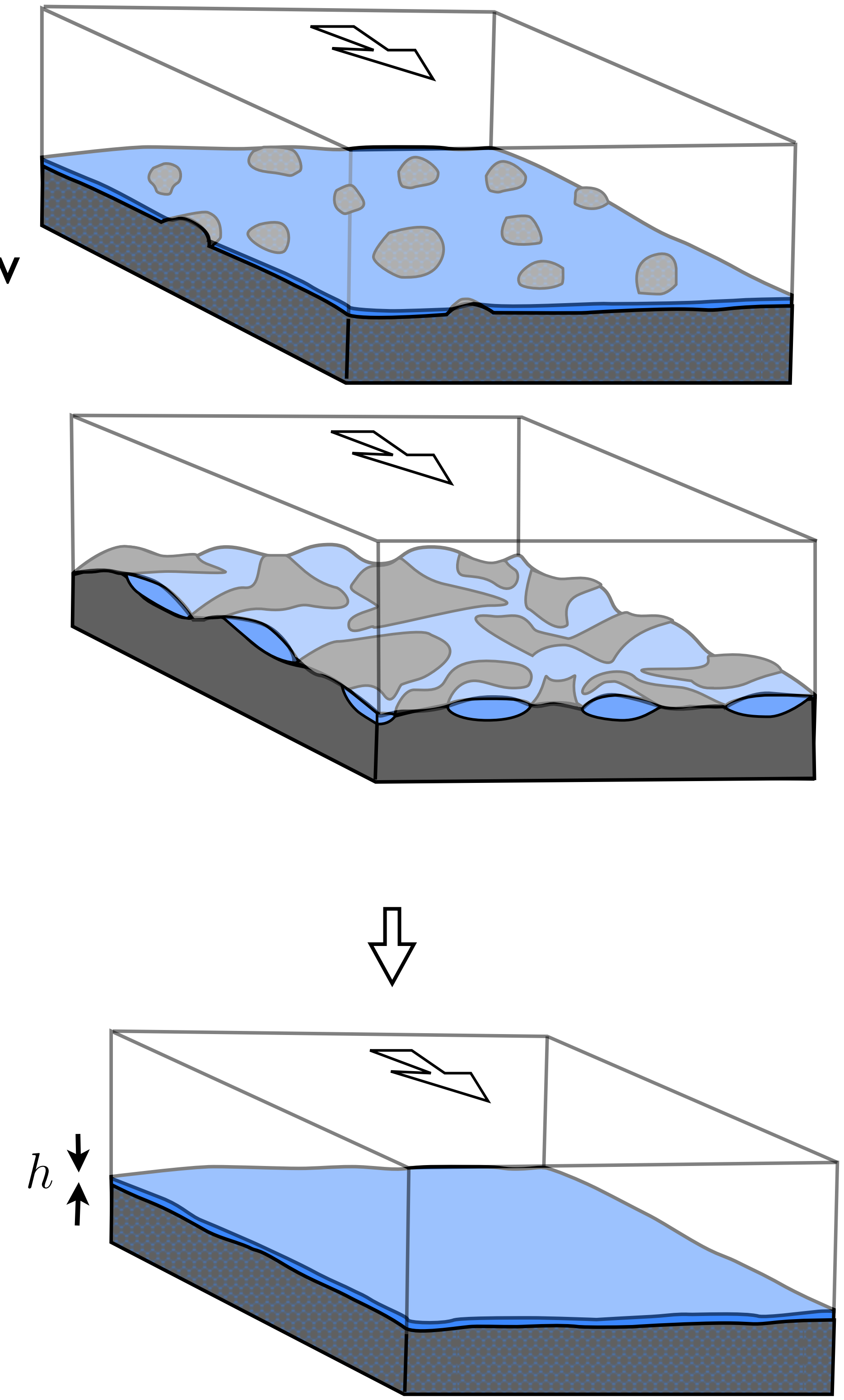


# Subglacial hydrology in ice-sheet models

see review paper by Gwenn Flowers 2015

On a large scale, distributed systems are described as a 'sheet' flow

Average water depth $h$ Average water pressure $p_w$




Average water flux $\boxed{\mathbf{q} = -Kh^{\alpha}\nabla\phi}$

Mass conservation $\boxed{\frac{\partial h}{\partial t} + \nabla \cdot \mathbf{q} = \frac{m}{\rho_w} + M}$

Basal melting

Englacial/supraglacial source

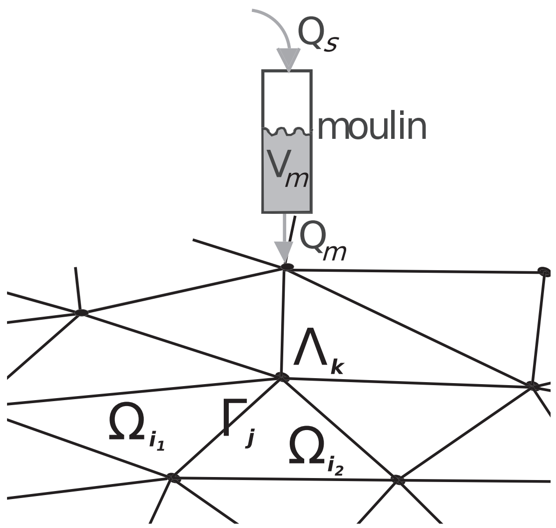
+ some additional ingredient to determine water pressure

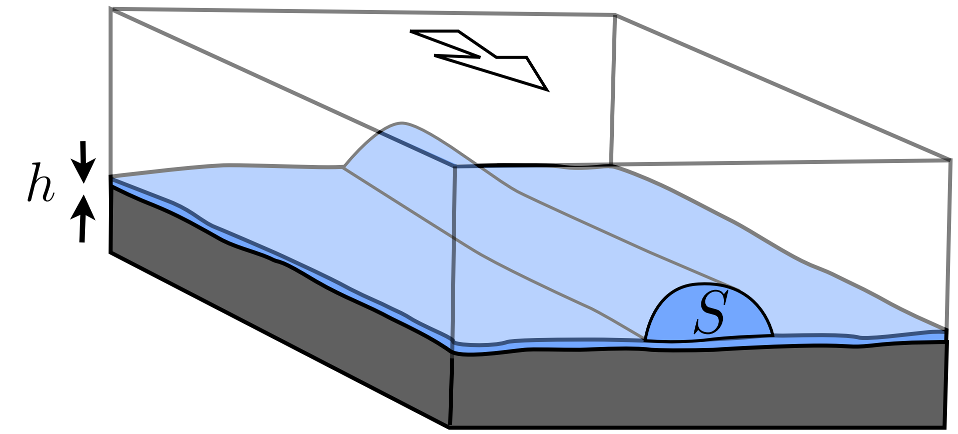
eg. water pressure = ice pressure ('routing model'),
or an equation for the evolution of the sheet permeability

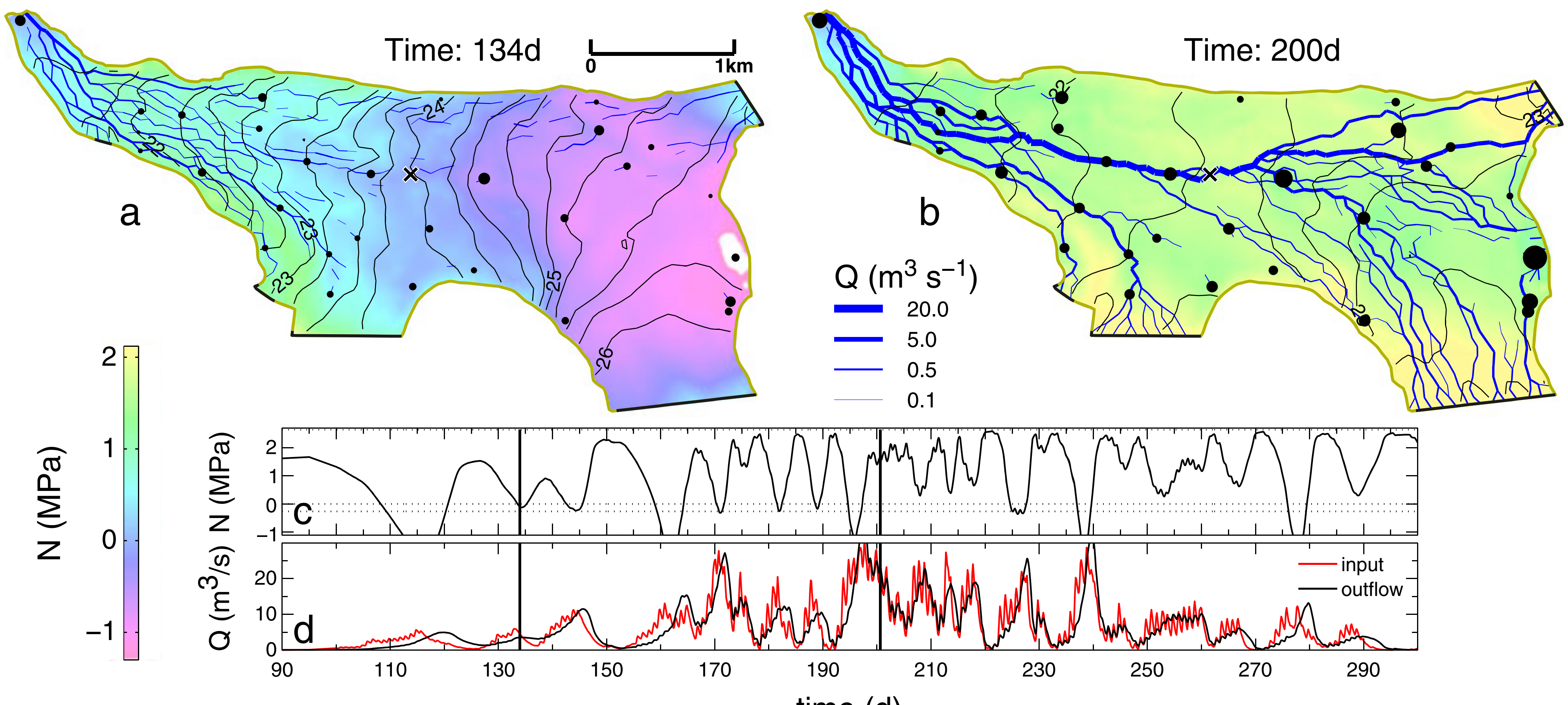
+ potential to couple to sliding law

# Subglacial hydrology in ice-sheet models

Some models couple a distributed 'sheet' with discrete 'conduits' (eg. GLaDS)



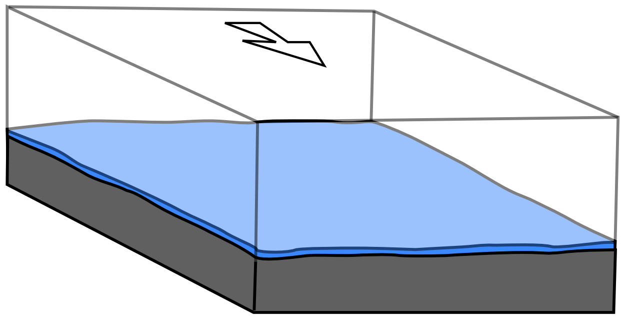



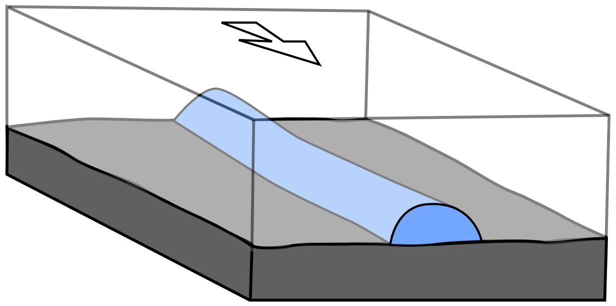

Werder et al 2013

# Summary

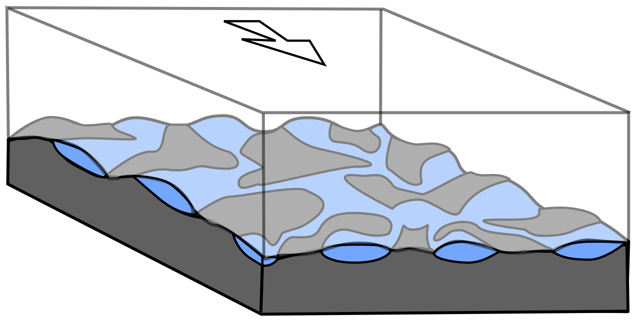
A uniform water film is **unstable**.



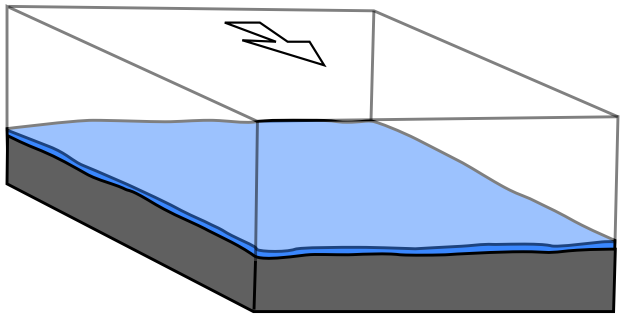
**Röthlisberger channels** form arterial networks.



Distributed flow in **linked cavities** or **sediments**.



On a large scale, the drainage system can be modelled as a **water sheet** with variable thickness and permeability.



Subglacial drainage has important **consequences for ice dynamics**, etc
(seasonal/diurnal velocity changes, surges, ice streams, grounding line dynamics, erosion,…)