

Glacier Hydrology

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Water sources

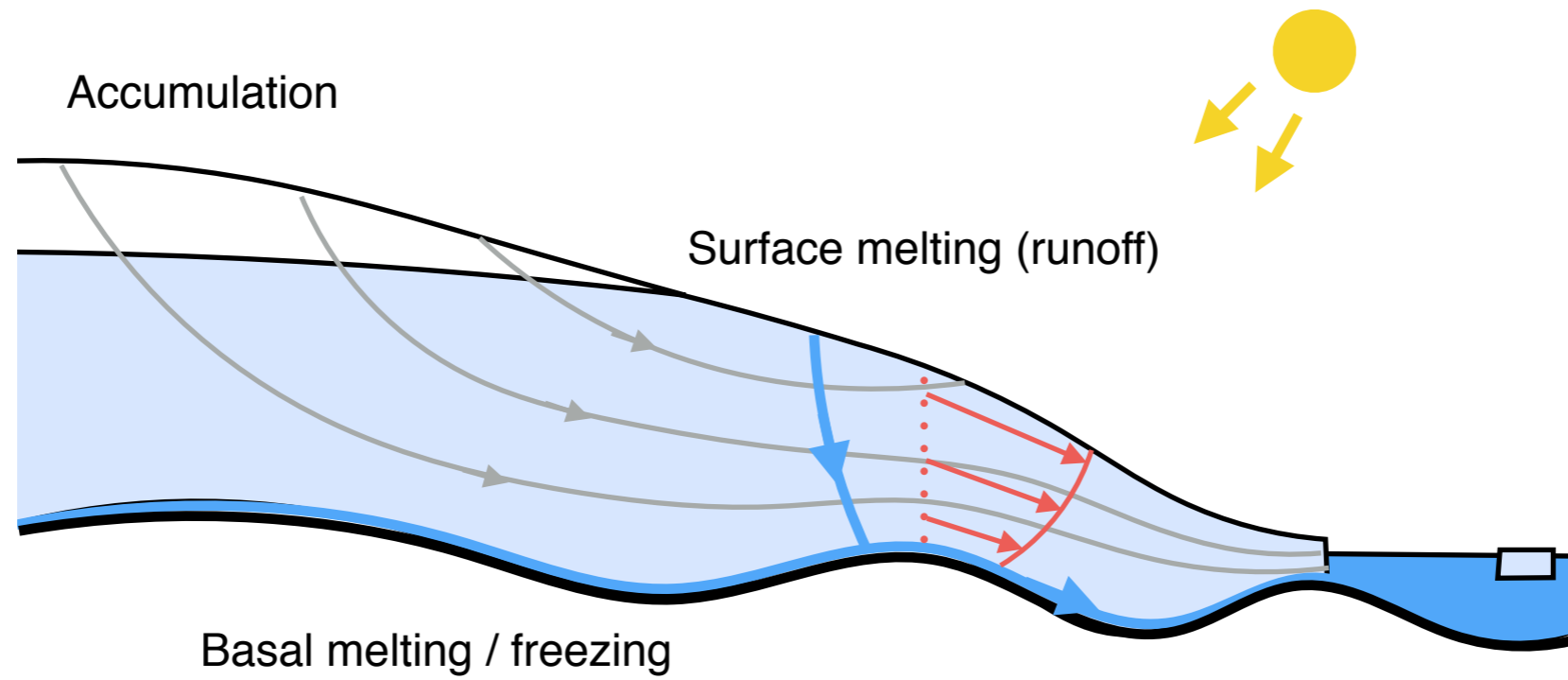
- Basal melting
- Surface melting, precipitation

Subglacial hydrology

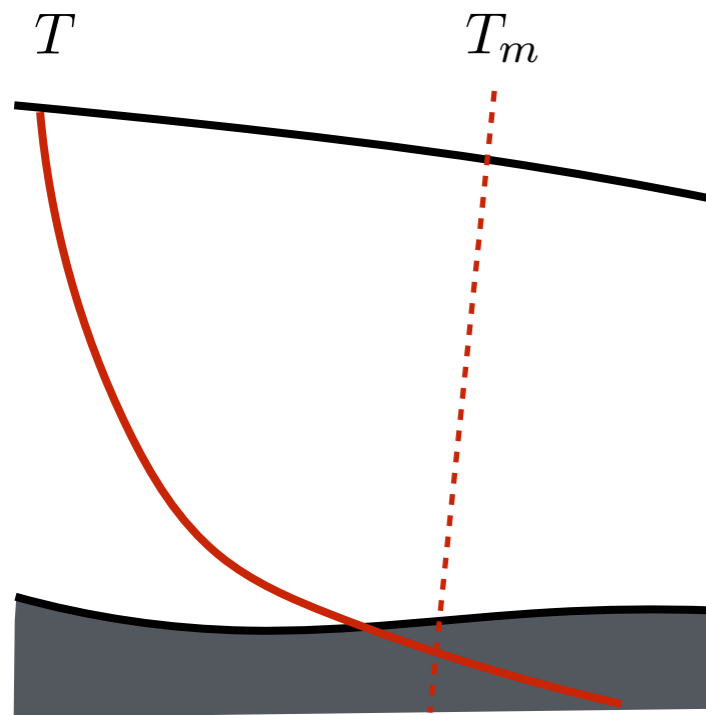
- Sheet flow
- Tunnels / channels
- Cavities
- Canals
- Lakes

Large-scale models

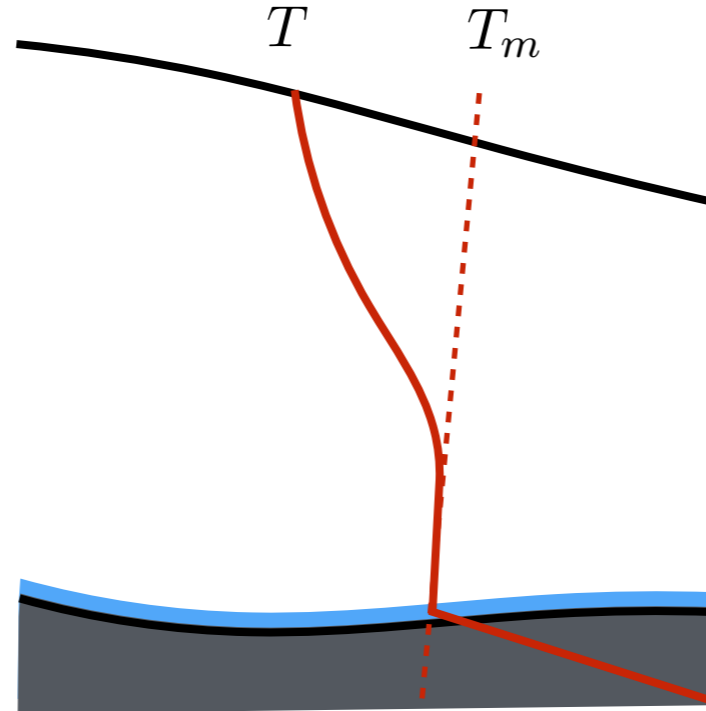
Glacier hydrology



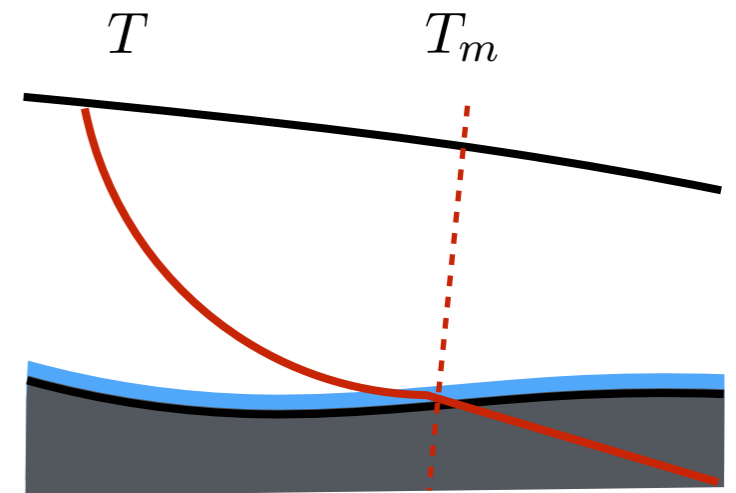
Thermal setting



Frozen bed
(no sliding?)



Basal melting



Basal freezing

Geothermal heating

$$G \sim 0.06 \text{ W m}^{-2}$$

Frictional heating

$$\tau_b \sim 100 \text{ kPa}$$

$$u_b \sim 30 \text{ m y}^{-1}$$

$$\tau_b u_b \sim 0.1 \text{ W m}^{-2}$$

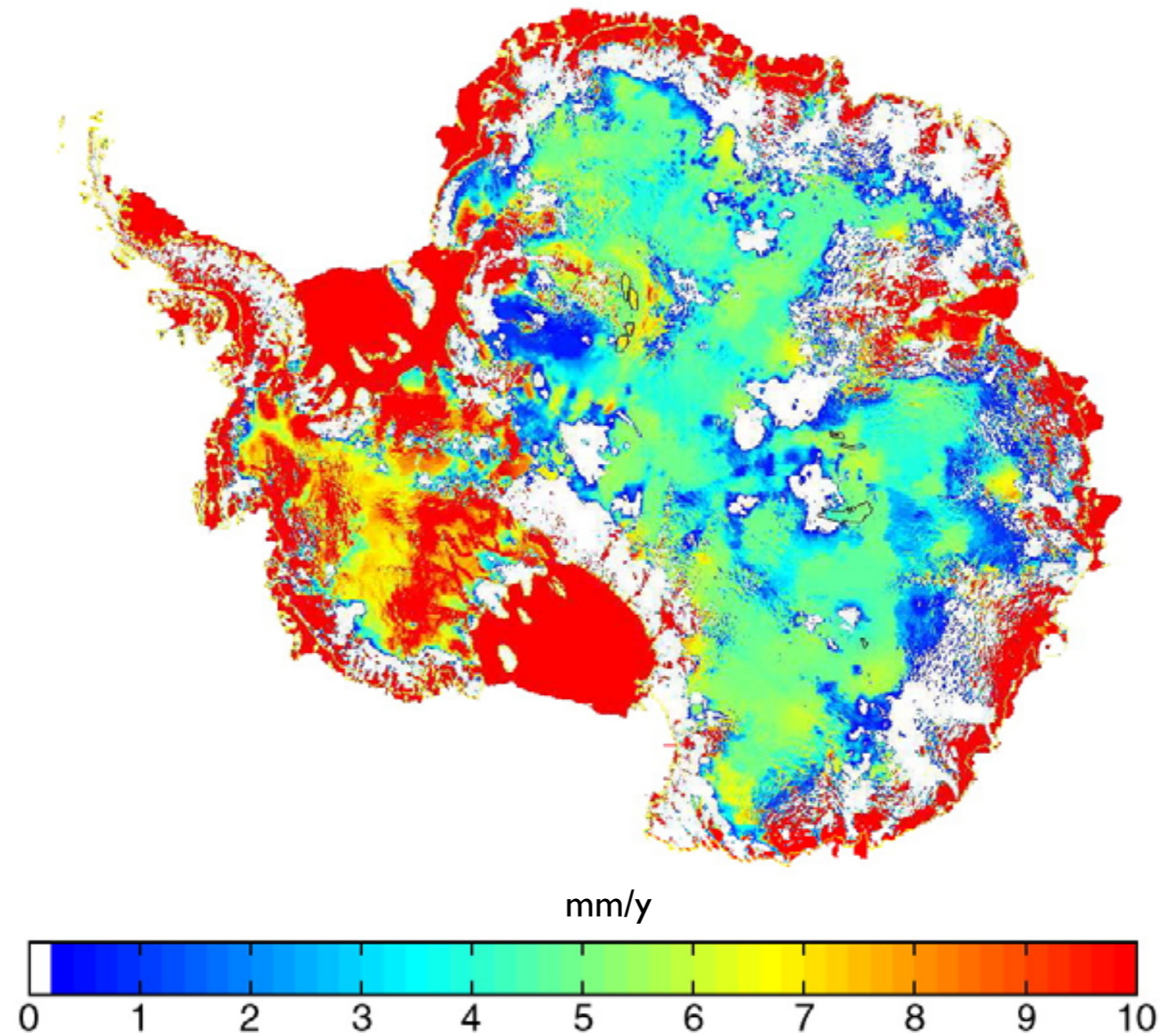


Basal melting

$$m \sim 10 \text{ mm y}^{-1}$$

Water sources in Antarctica

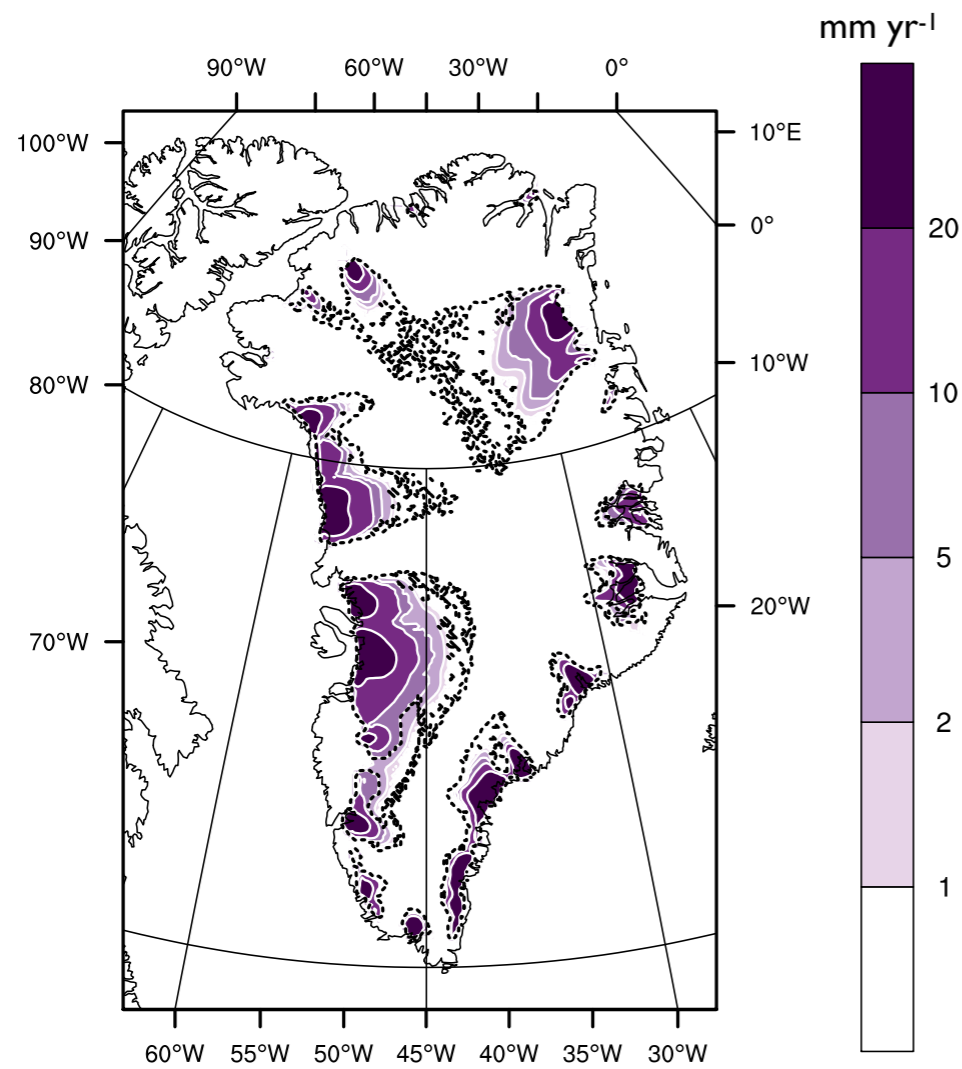
Basal melting ~ 10 mm/y



Pattyn 2010

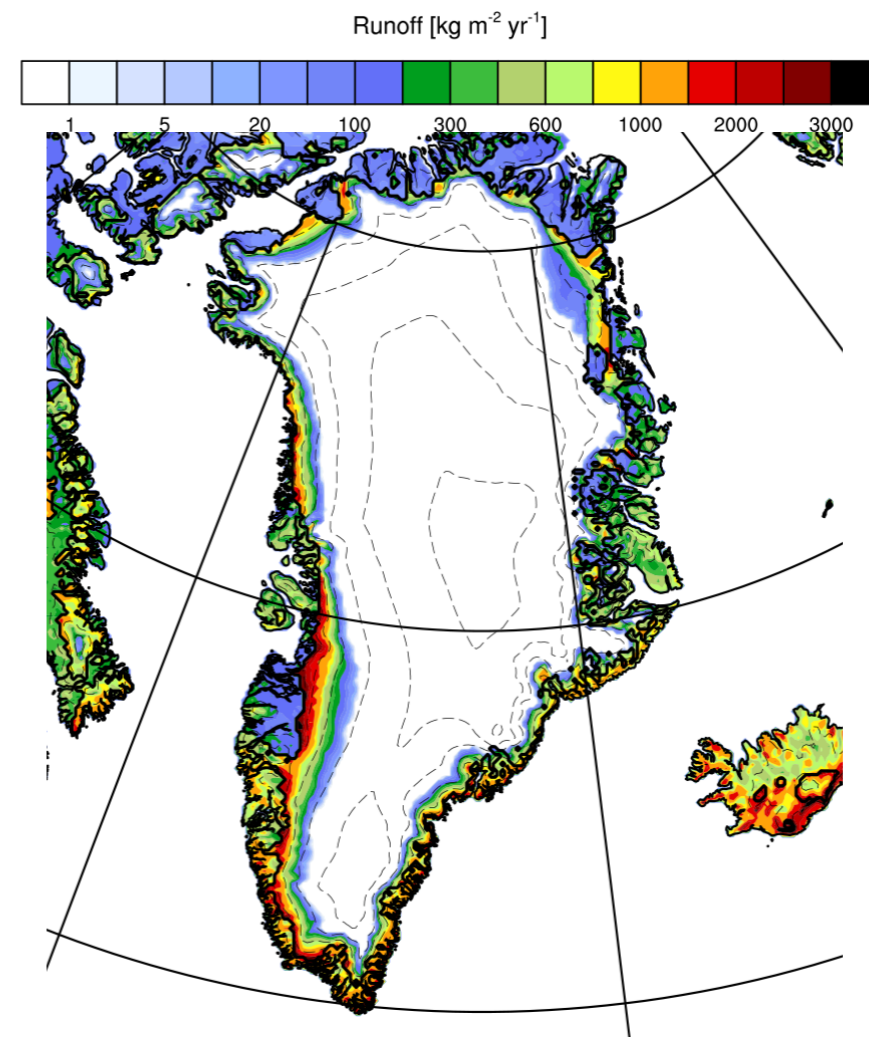
Water sources in Greenland

Basal melting ~ 10 mm/y



Aschwanden et al 2012

Surface runoff ~ 1 m/y



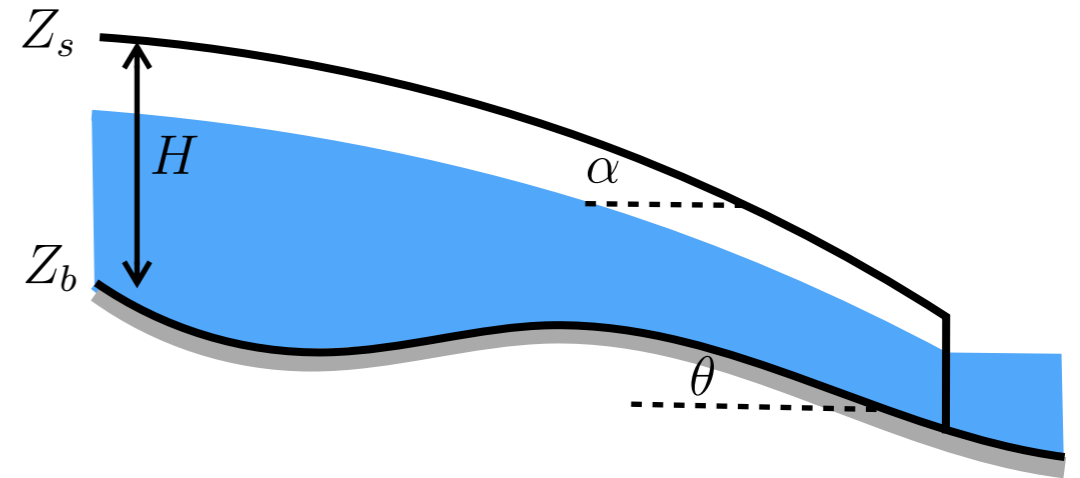
van den Broeke et al 2016

Direction of subglacial water flow

Hydraulic potential

$$\phi = \rho_w g Z_b + p_w$$

$$= \rho_w g Z_b + \rho_i g (Z_s - Z_b) - N \quad \text{in terms of **effective pressure** } N = p_i - p_w$$



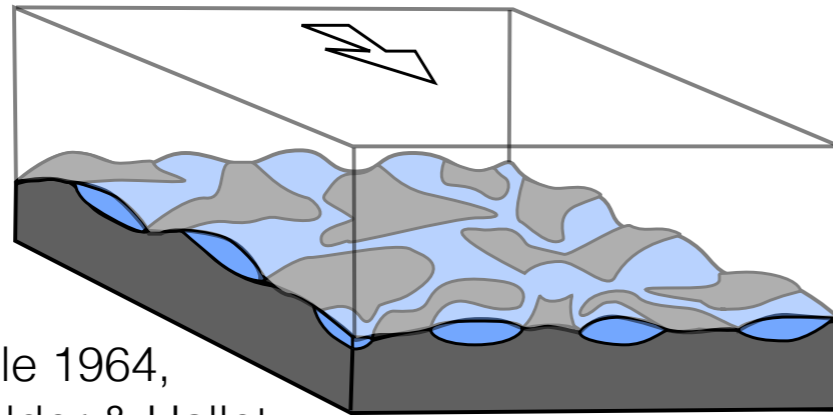
Potential gradient $-\frac{\partial \phi}{\partial x} = \Psi + \frac{\partial N}{\partial x}$

$$\Psi = \rho_i g \tan \alpha + (\rho_w - \rho_i) g \tan \theta$$

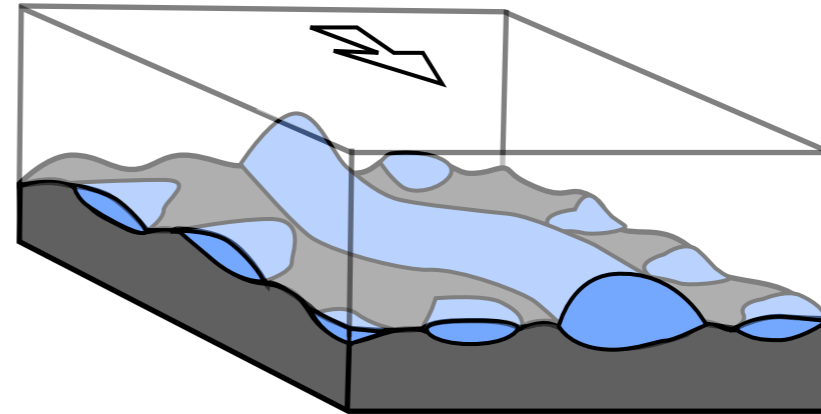
Potential gradient if basal water pressure were equal to ice pressure $N = 0$

⇒ Predominant control on water flow direction from the **ice surface slope**

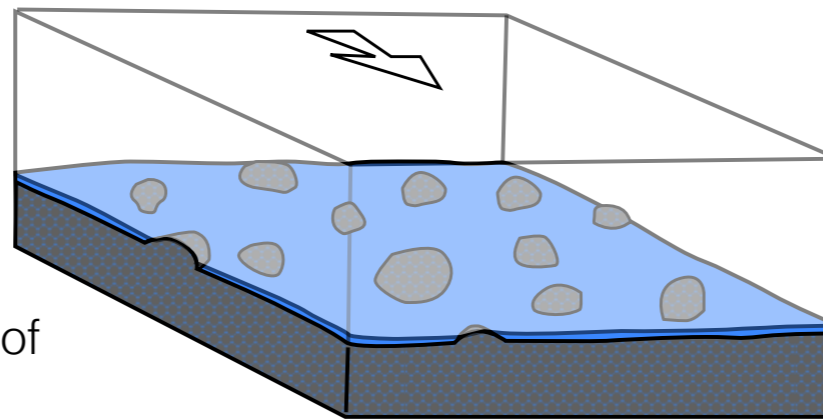
Subglacial drainage systems



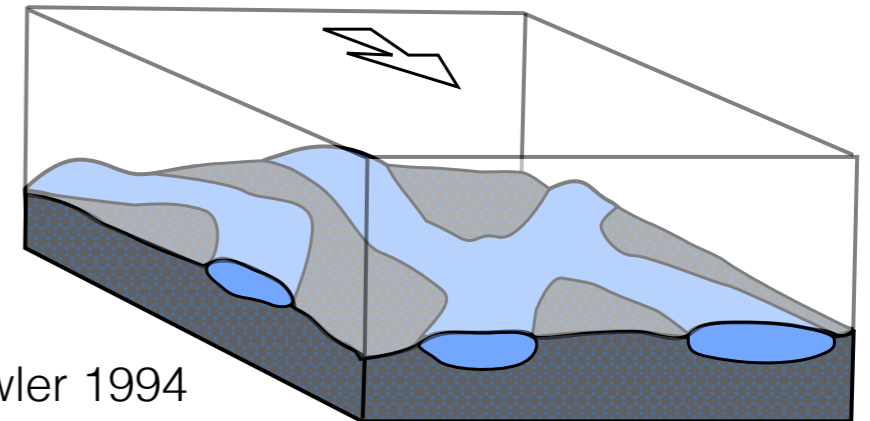
Kamb & LaChapelle 1964,
Lliboutry 1968, Walder & Hallet
1979,



Röthlisberger 1972,
Nye 1976



Alley et al 1986, Creyts & Schoof
2009



Walder & Fowler 1994

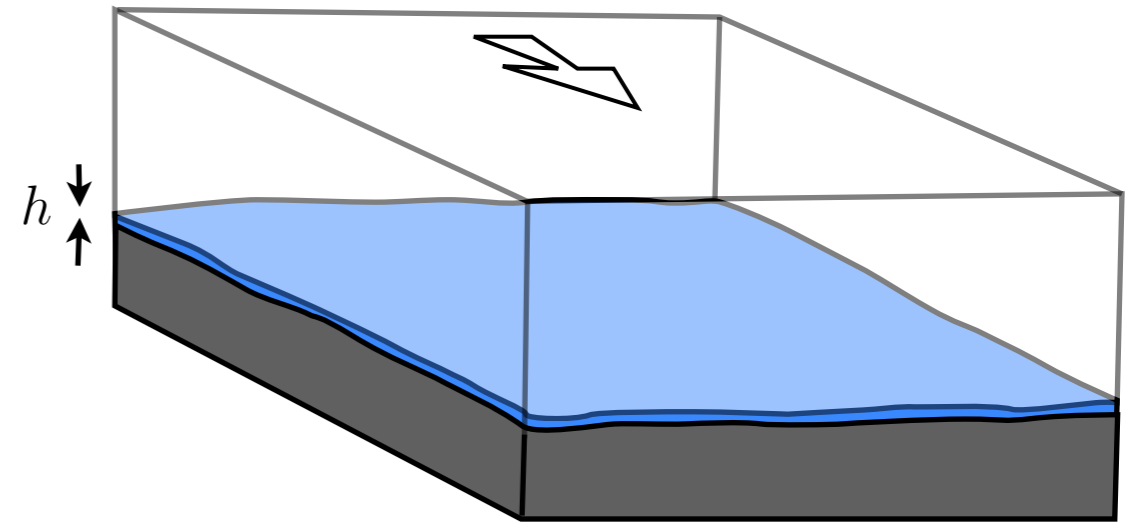


Increasing water flow

Weertman film Weertman 1972, Walder 1982

Weertman suggested water could flow as a **film**

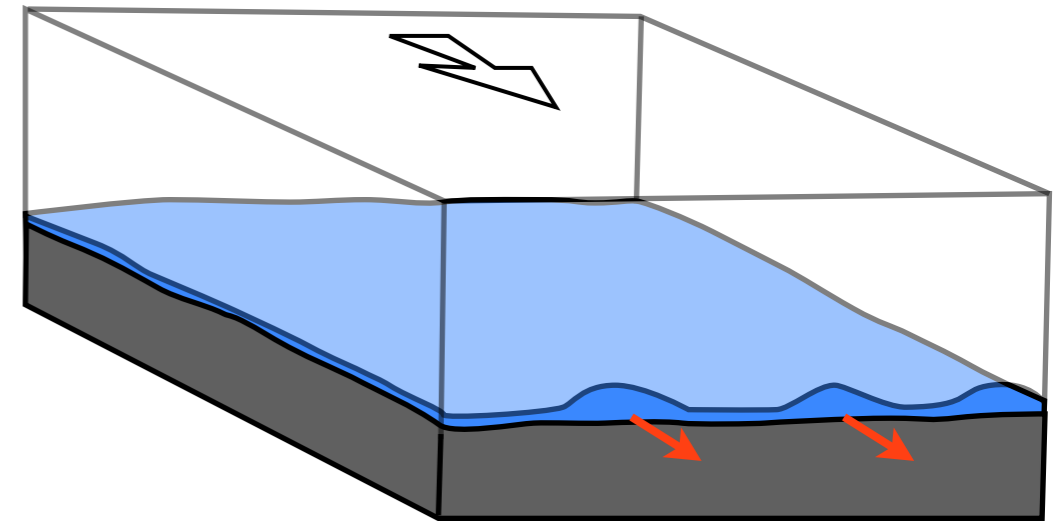
Poiseuille flux $Q = \frac{h^3}{12\mu} \left(\Psi + \frac{\partial N}{\partial x} \right)$



Water flow dissipates energy through heating

⇒ Leads to an **instability**

Larger h → Larger flux → Melting of ice roof



Flow wants to concentrate in **localized channels / tunnels**

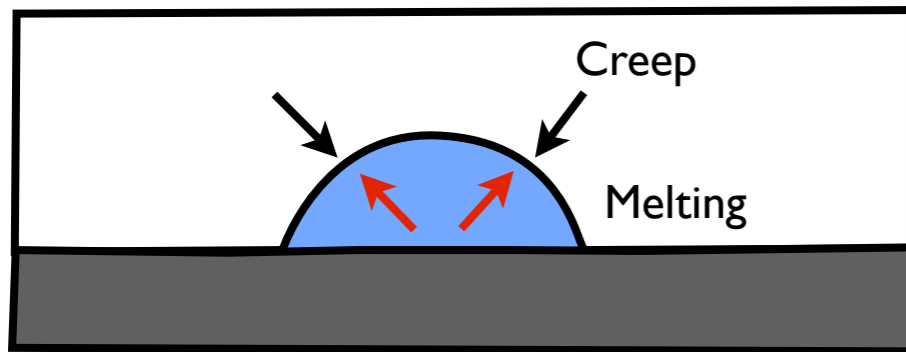
However, a **patchy film** may still exist eg. Alley 1989, Creyts & Schoof 2009



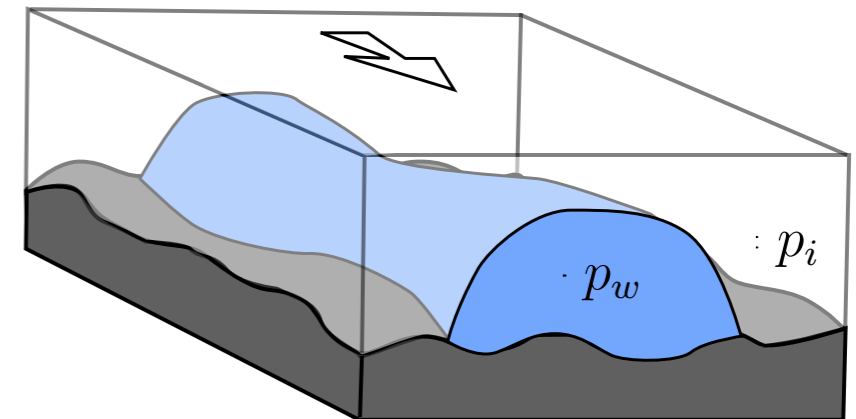
Röthlisberger channels

Röthlisberger 1972, Nye 1976

Ice wall **melting** is counteracted by **viscous creep**



$$N = p_i - p_w$$



Röthlisberger/Nye theory (ignoring pressure dependence of melting temperature)

$$\frac{\partial S}{\partial t} + \frac{\partial Q}{\partial x} = \frac{m}{\rho_w} + M$$

water mass conservation

$$\frac{\partial S}{\partial t} = \frac{m}{\rho_i} - \tilde{A} S N^n$$

wall evolution

$$mL = Q \left(\Psi + \frac{\partial N}{\partial x} \right)$$

local energy conservation

$$Q = K_c S^{4/3} \left(\Psi + \frac{\partial N}{\partial x} \right)^{1/2}$$

momentum conservation
(turbulent flow parameterization)

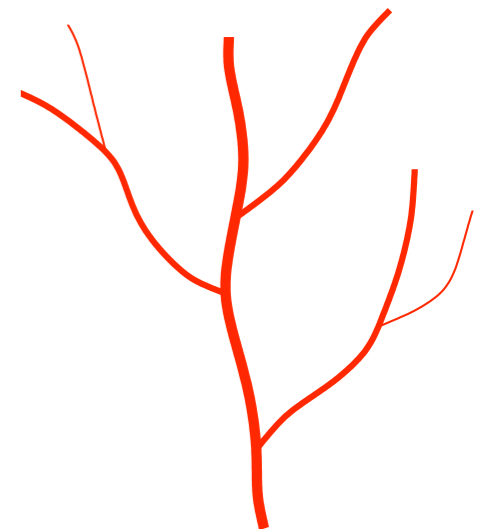
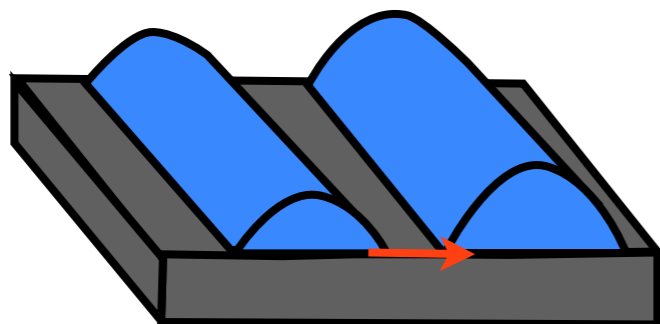
Steady state \Rightarrow

$$N \approx \left(\frac{K_c^{3/4}}{\rho_i L \tilde{A}} \right)^{1/n} \Psi^{11/8n} Q^{1/4n}$$

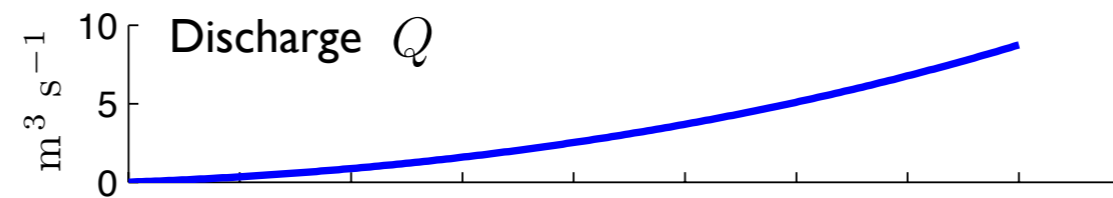
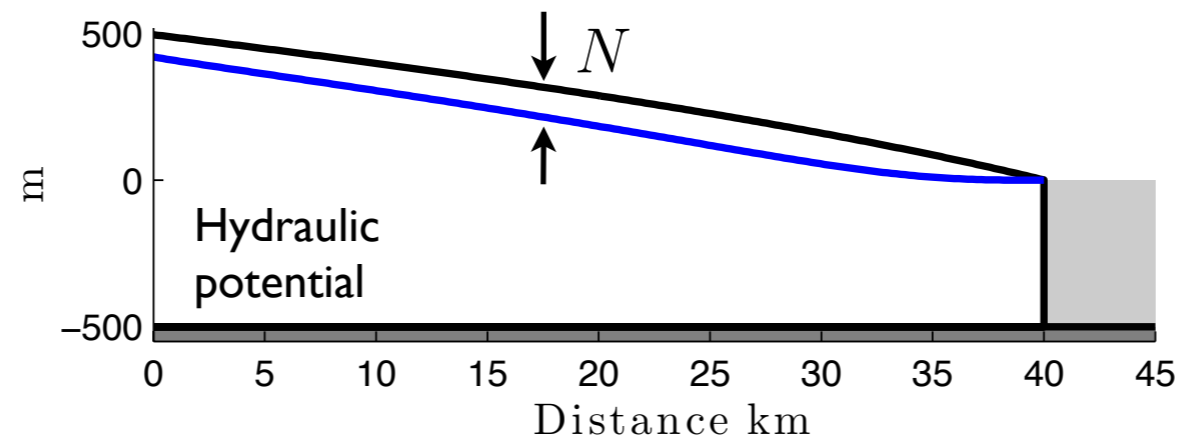
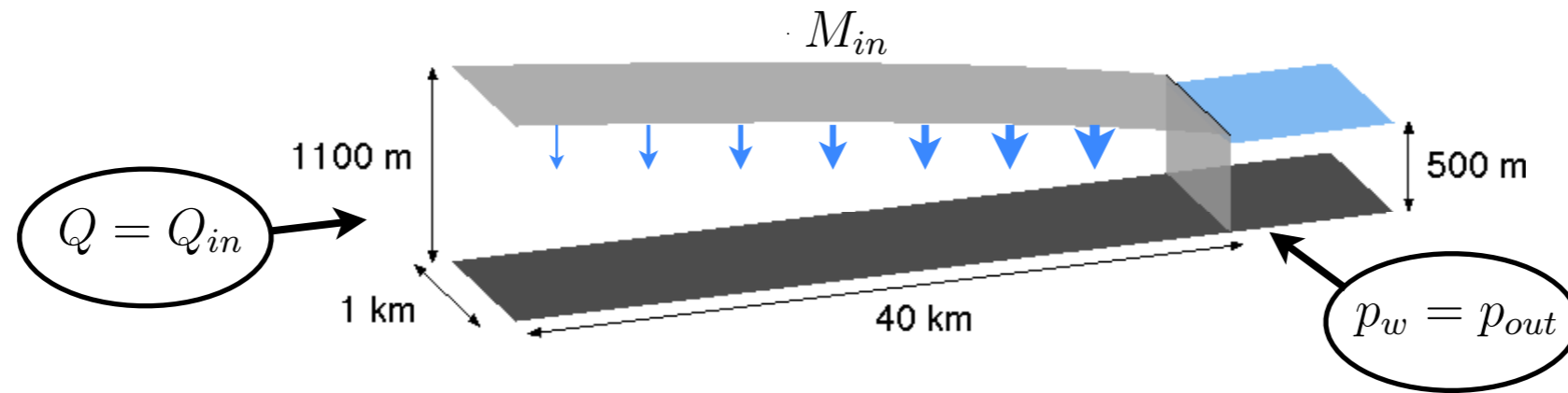
Effective pressure **INCREASES**
with discharge

Neighbouring channels compete with one another

\Rightarrow leads to an arterial network



Röthlisberger channels

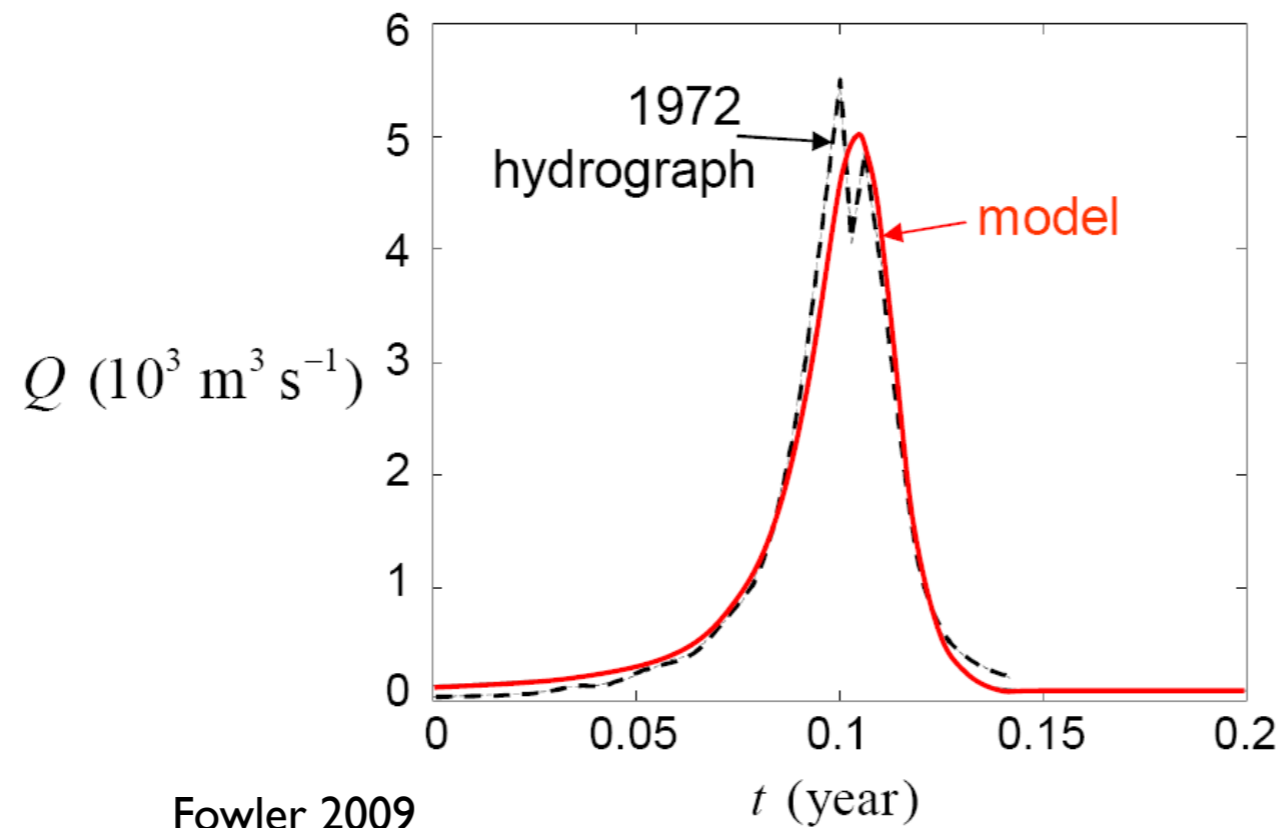


Jökulhlaups (GLOFs) Nye 1976, Spring & Hutter 1981, Clarke 2003

A success of the channel theory is the application to **floods from ice-dammed lakes**

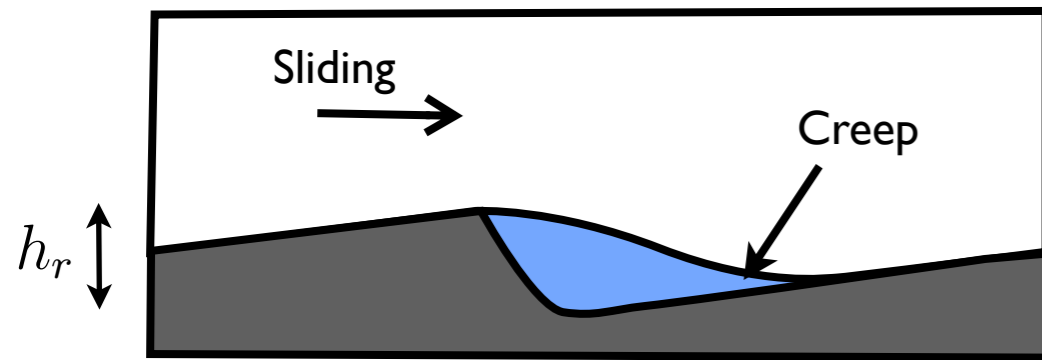
Combine **channel evolution** equation $\frac{\partial S}{\partial t} = \frac{S^{4/3} \Psi^{3/2}}{\rho_i L} - \tilde{A} S N^n$

with a **lake filling** equation $-\frac{A_L}{\rho_w g} \frac{\partial N}{\partial t} = m_L - Q$ at $x = 0$



Linked cavities Walder 1986, Kamb 1987

Cavities grow through sliding over bedrock



Model

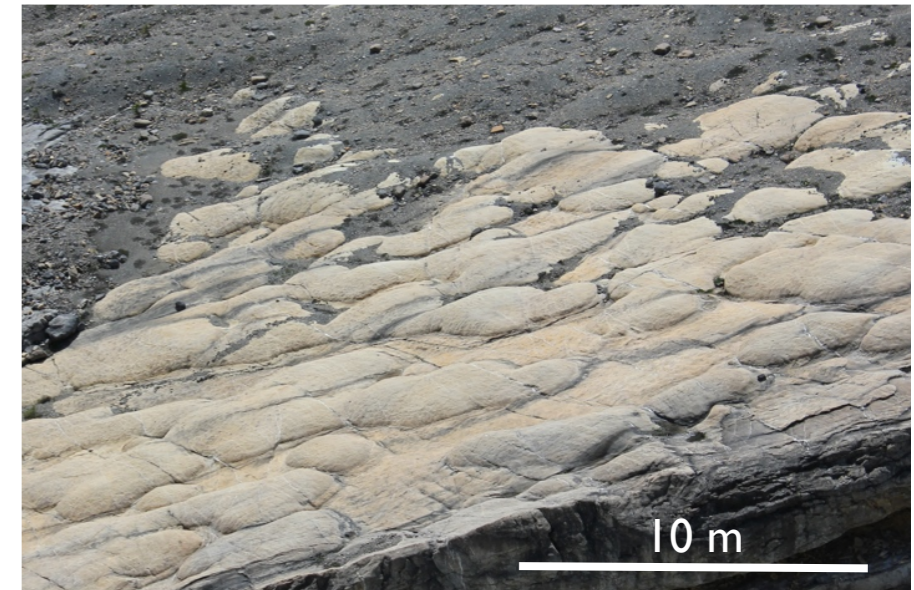
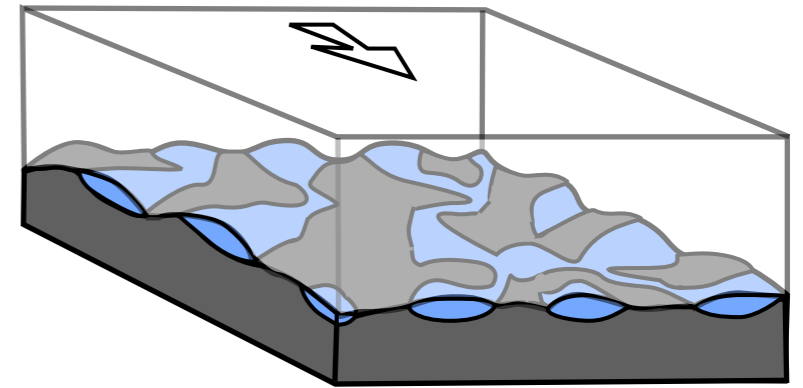
$$\frac{\partial \hat{S}}{\partial t} = U_b h_r - \tilde{A} \hat{S} N^n$$

Smaller 'orifices' control the flow

Approximate steady-state relationship

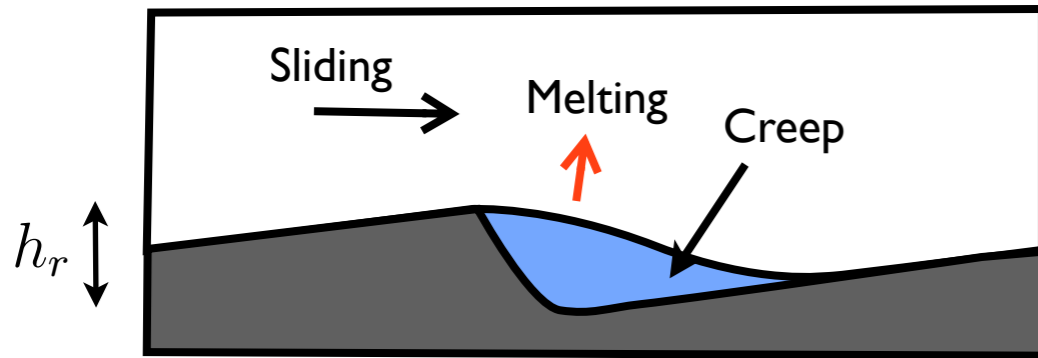
$$\Rightarrow \boxed{N(Q) \quad \frac{\partial N}{\partial Q} < 0} \Rightarrow \text{Effective pressure DECREASES with discharge} \Rightarrow \text{Flow is distributed}$$

Cavity size is controlled by parameter $\Lambda = \frac{U_b}{N^n}$ i.e. depends on **effective pressure** **and** **sliding speed**



Drainage system stability Walder 1986, Kamb 1987, Schoof 2010, Hewitt 2011

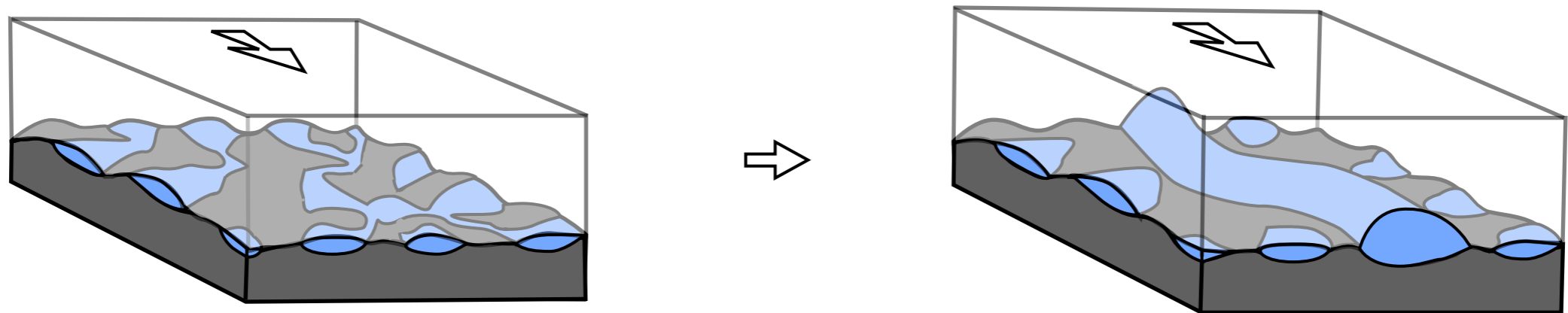
Energy is still dissipated by water flow



$$\frac{\partial S}{\partial t} = \frac{m}{\rho_i} + U_b h_r - \tilde{A} S N^n$$

A linked cavity system can become **unstable** to produce **channels**

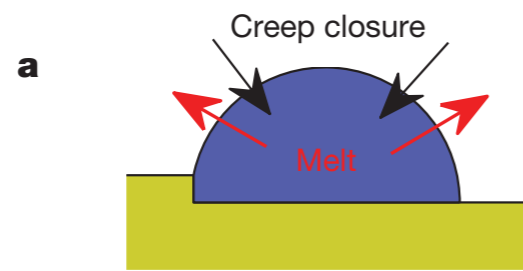
eg. if discharge becomes sufficiently large, or sliding speed sufficiently low



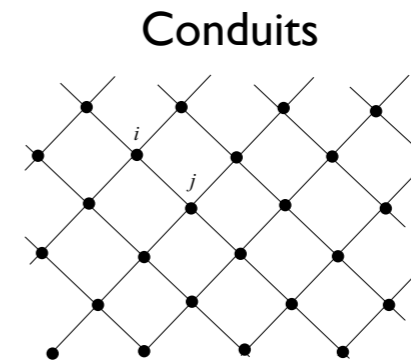
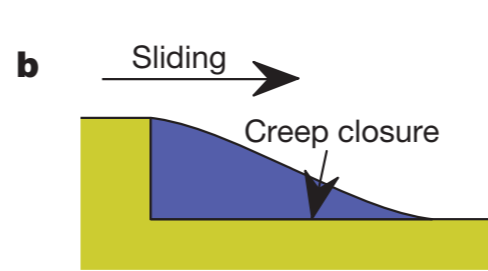
Conversely, a **channel** can become **unstable** to **cavities**

eg. if discharge low, or sliding speed sufficiently high

Seasonal evolution of drainage system



Schoof 2010

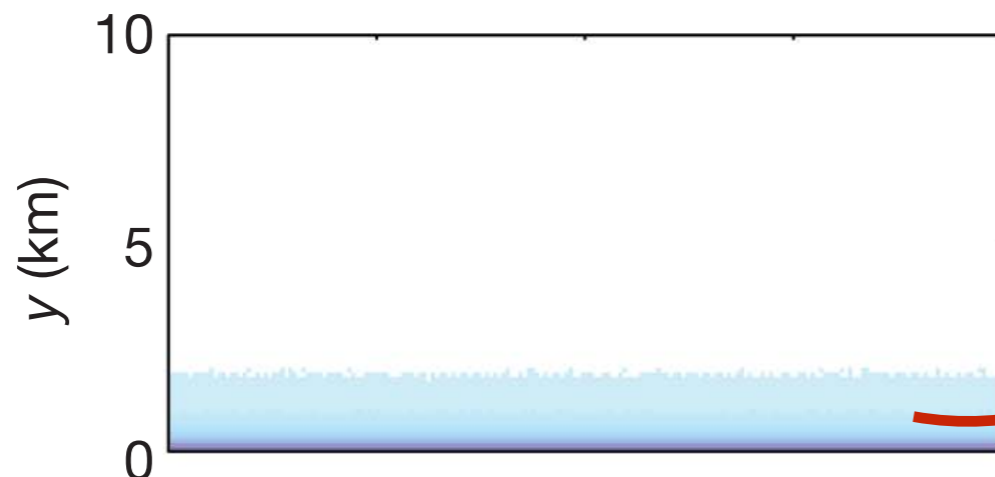


Network of 'conduits' forced by prescribed surface runoff

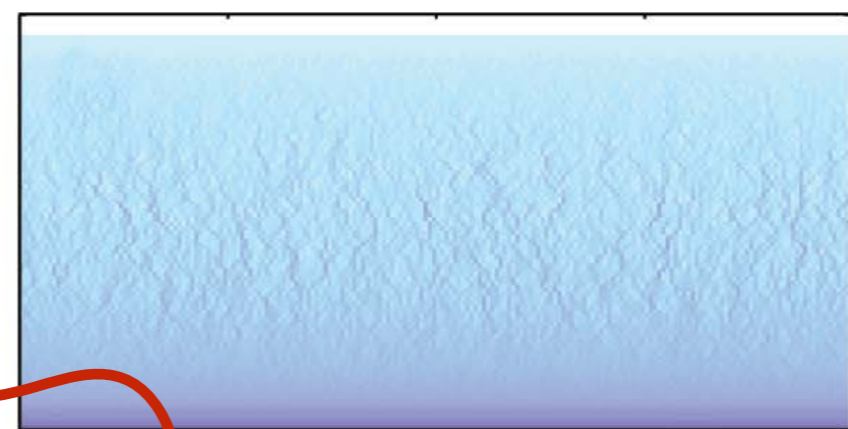
Ice flow



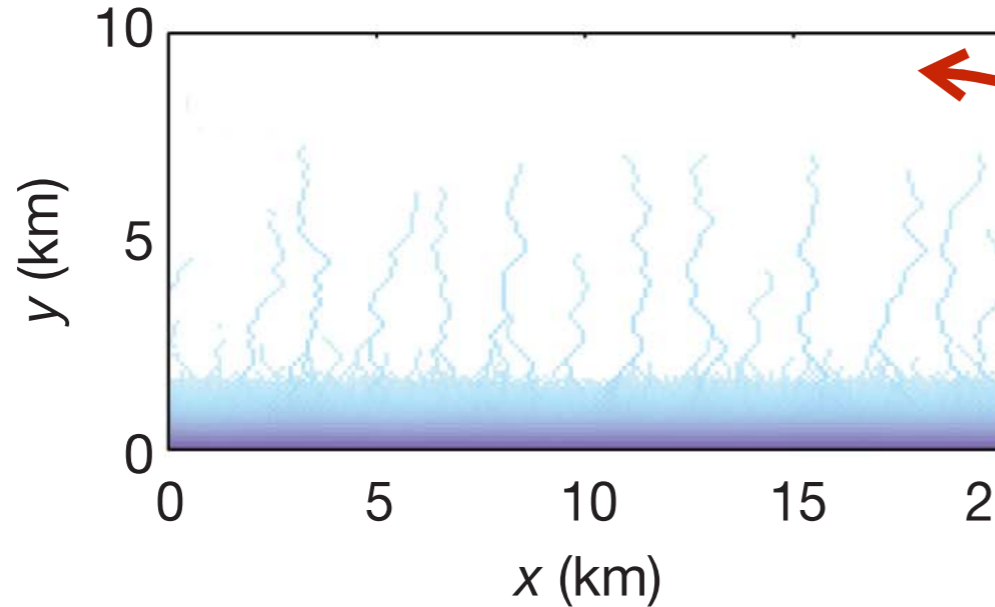
b



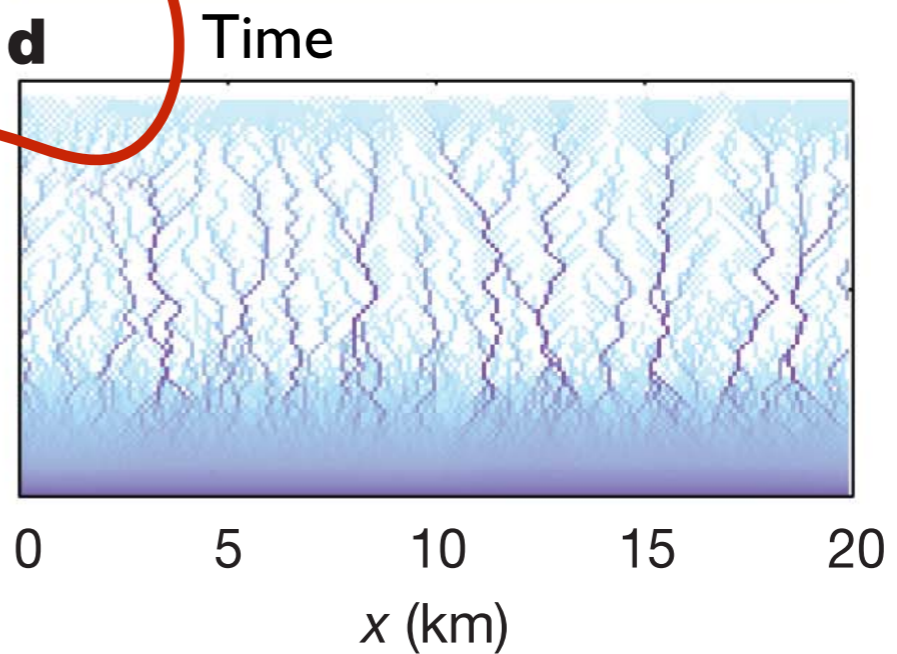
c



e



d



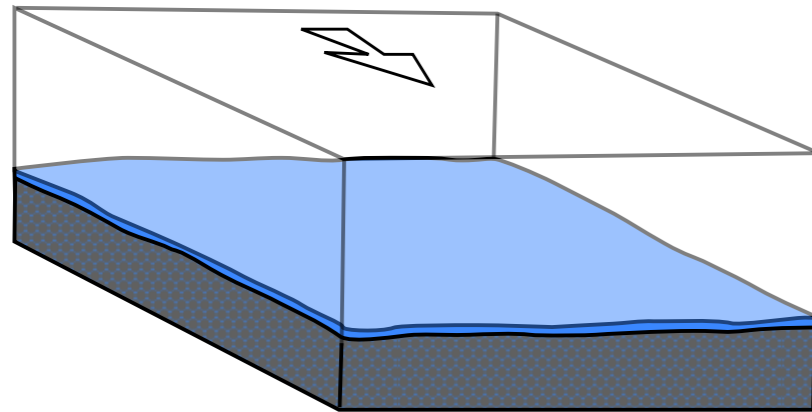
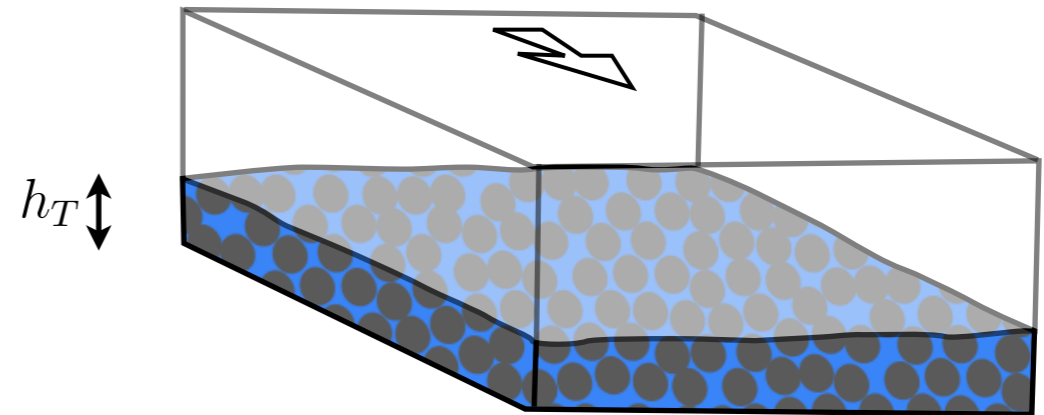
Drainage through sediments

Water can infiltrate sediment layers and moves according to a diffusion equation

$$\beta_T h_T \frac{\partial \phi}{\partial t} = \nabla \cdot (K_T h_T \nabla \phi) + m$$

↑
Compressibility

↑
Melting / freezing



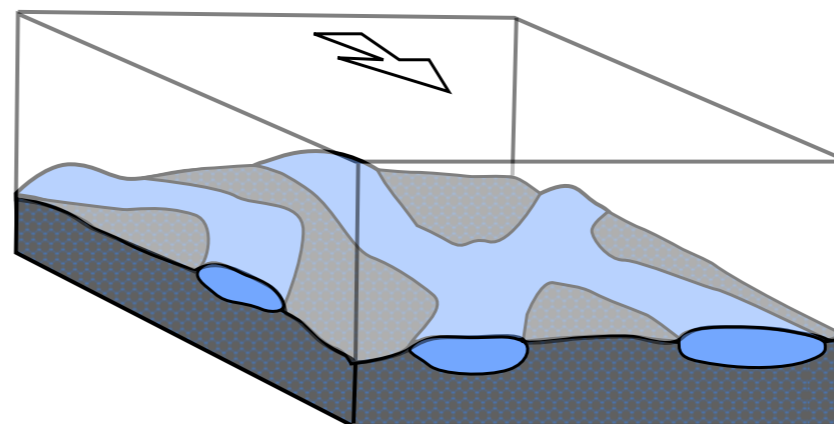
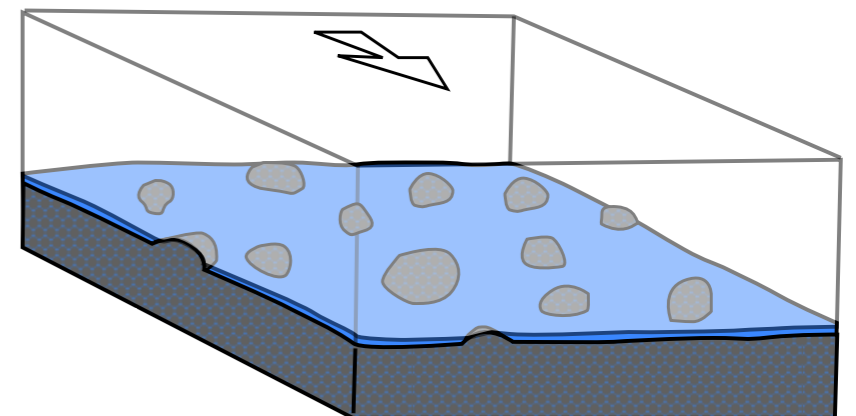
Till is relatively impermeable, so easily **saturates**

Film flow is **unstable** (Walder 1982)

Locally deep water film leads to locally more melting

Patchy sheet

Alley 1989, Creyts & Schoof 2009



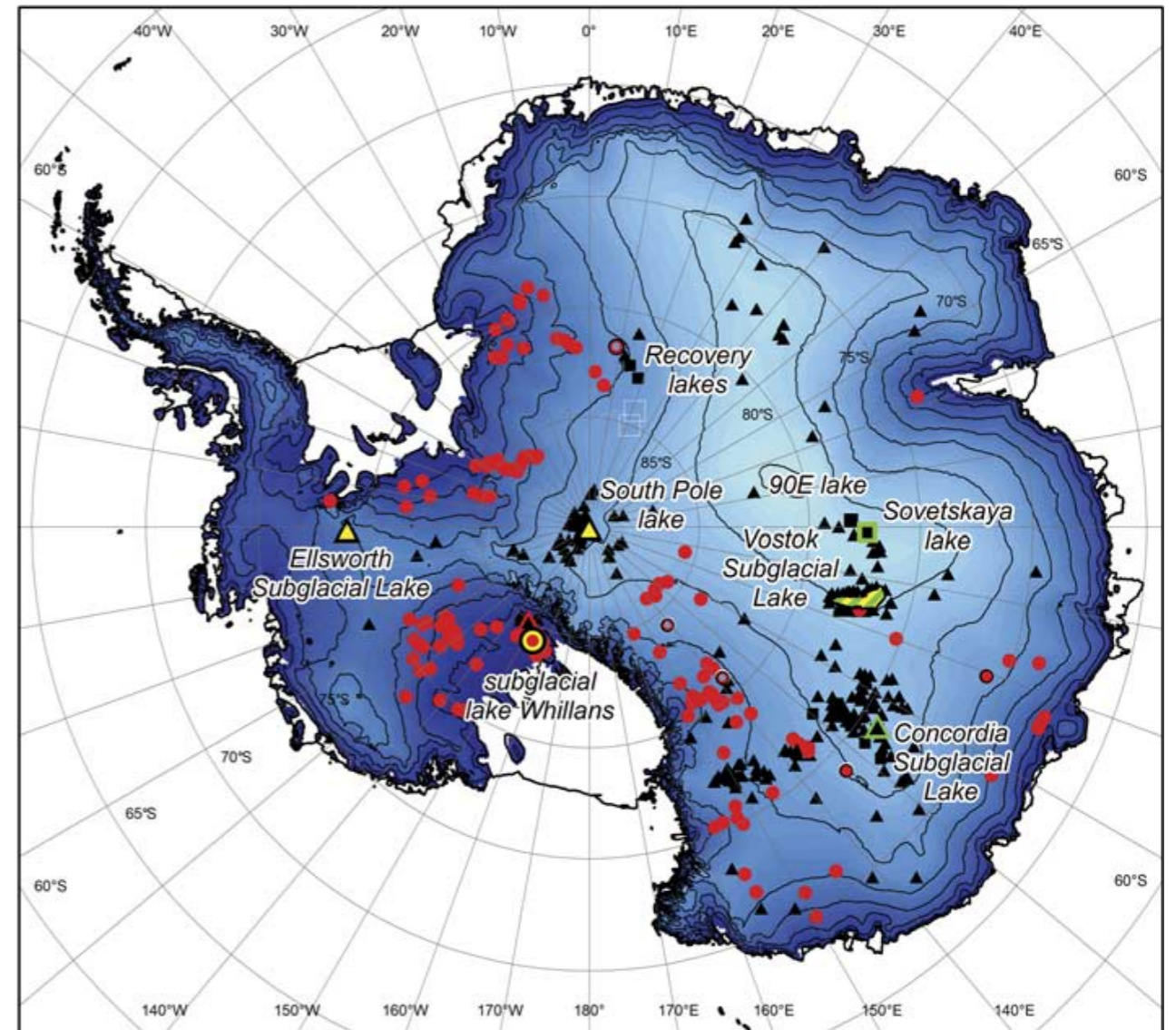
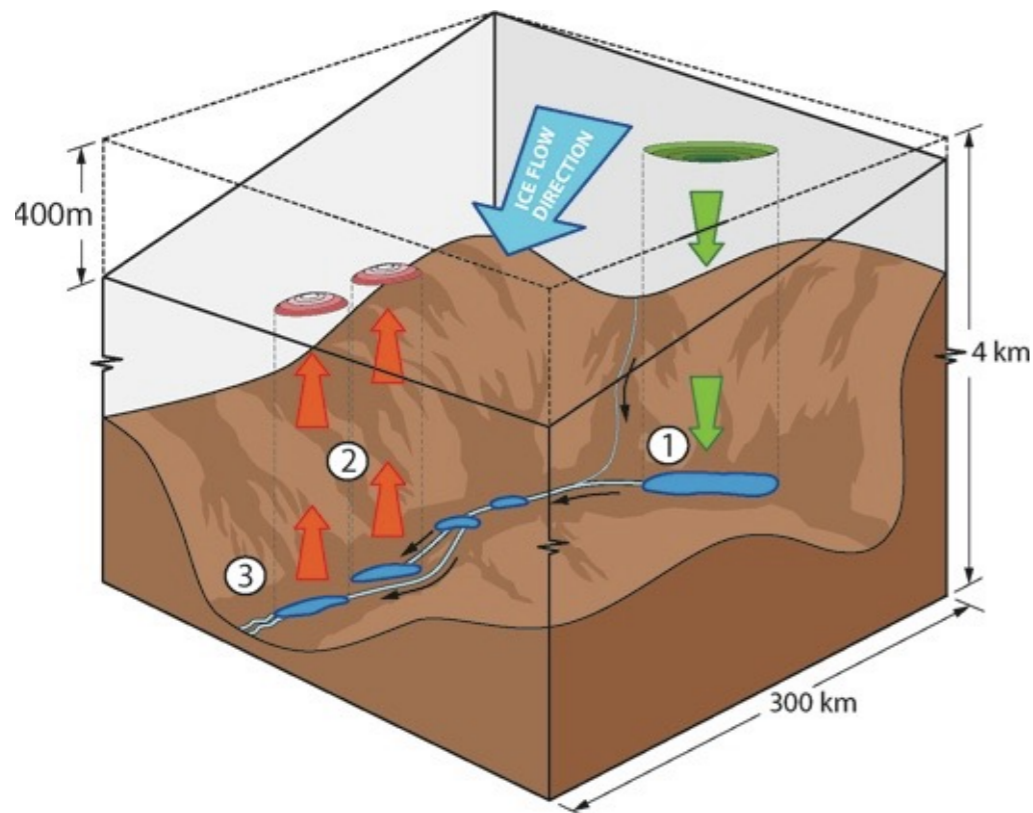
Canals

Walder & Fowler 1994

Subglacial lakes Siegert 2005, Wingham et al 2006, Fricker et al 2007, Stearns et al 2008

Hundreds of lakes have been detected using radar and satellite observations.

‘Active’ lakes grow and drain quite frequently
- through a jokulhlaup-like instability?



Wright & Siegert 2012

The formation and drainage of lakes **may** be important for ice-stream dynamics.

Ice-sheet modelling

On a large scale, distributed systems are described as a 'sheet' flow

Average water depth h Average water pressure p_w

Average water flux

$$\mathbf{q} = -Kh^\alpha \nabla \phi$$

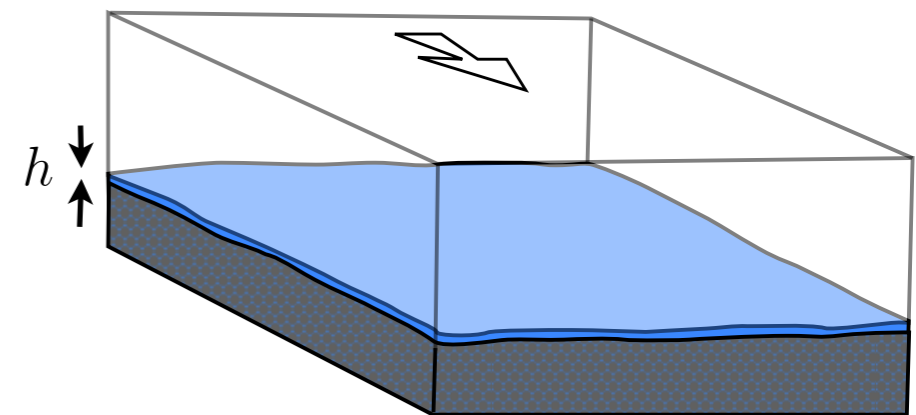
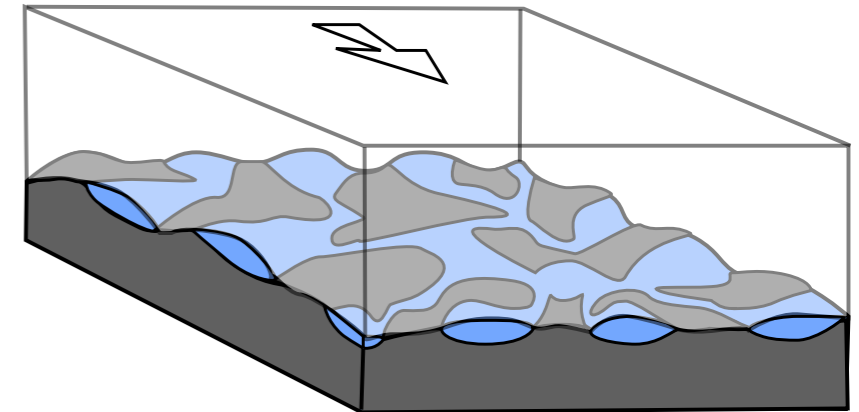
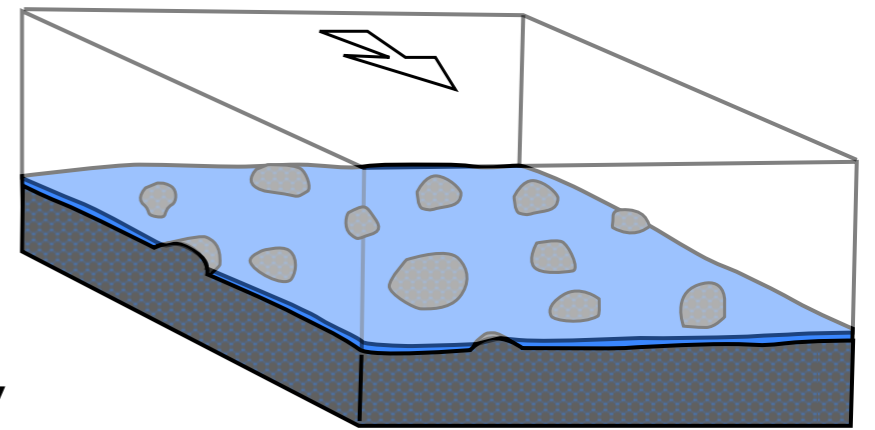
Mass conservation

$$\frac{\partial h}{\partial t} + \nabla \cdot \mathbf{q} = \frac{m}{\rho_w} + M$$

Basal melting

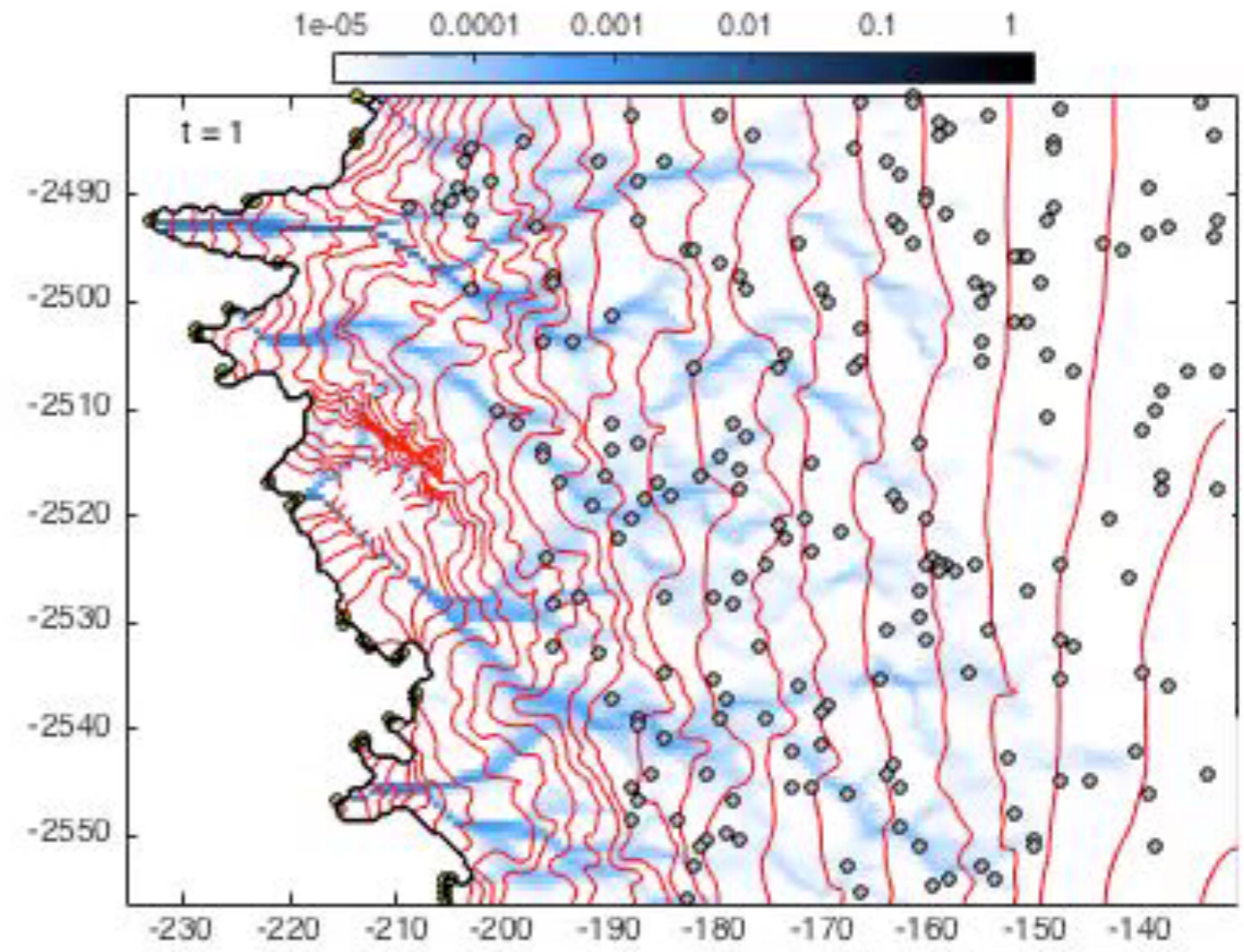
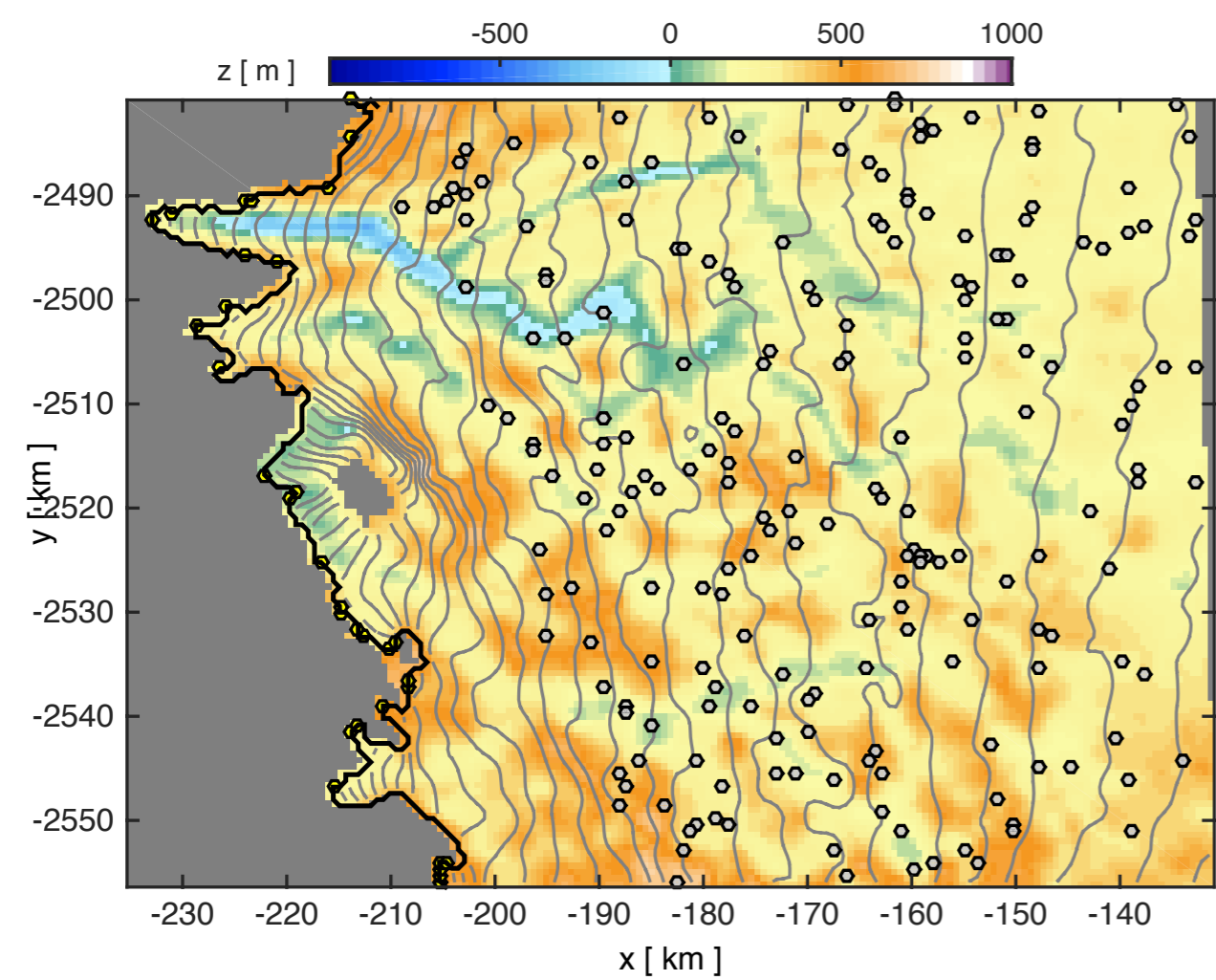
Englacial/supraglacial source

+ some additional ingredients to determine water pressure



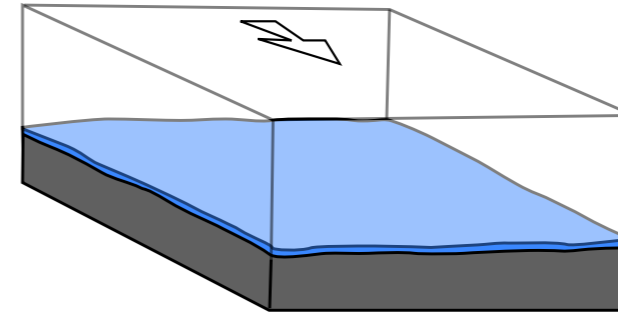
Combined sheet / channel modelling

Hewitt 2013, Werder et al 2013

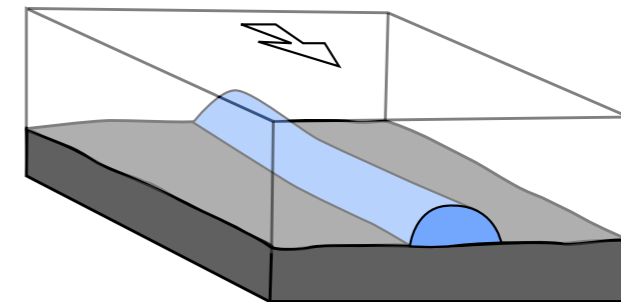


Summary

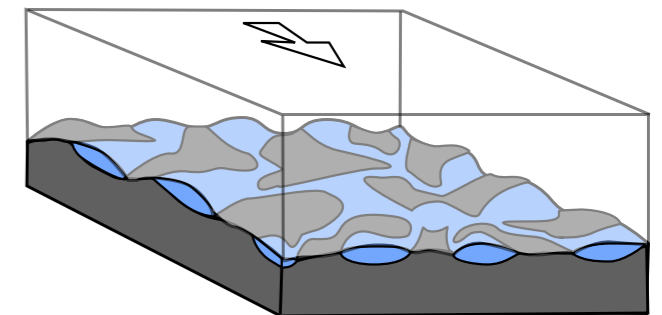
Uniform water film is **unstable**.



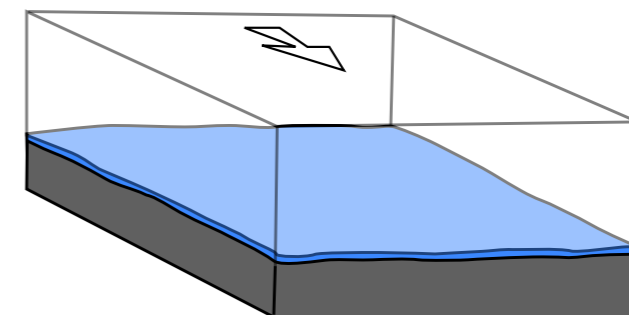
Röthlisberger channels form arterial networks.



Distributed flow in **linked cavities** or **patchy films** is possible.



On a large scale, the drainage system can be modelled as a **water layer** with variable thickness and permeability.



Evolution of the drainage system has important consequences for ice dynamics
(surges, ice streams, seasonal/diurnal velocity changes)