Rheology of ice

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Constitutive law

- Stress and strain rate
- Glen's law

Microscopic view

- Crystal structure
- Fabric
- Deformation mechanisms

Macroscopic view

- More general flow laws
- Effect of temperature and water content
- Visco-elasticity

Constitutive law

Rheology is the study of how materials flow.

We seek a constitutive law or flow law to relate stress and strain rate.

stress = force per unit area τ

strain rate = normalised stretching rate $\dot{\varepsilon}$

The general form is a tensorial relationship

deviatoric stress tensor τ_{ij}

strain rate tensor $\dot{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$

e.g. Newtonian fluid $\tau_{ij} = 2\eta\dot{\varepsilon}_{ij}$

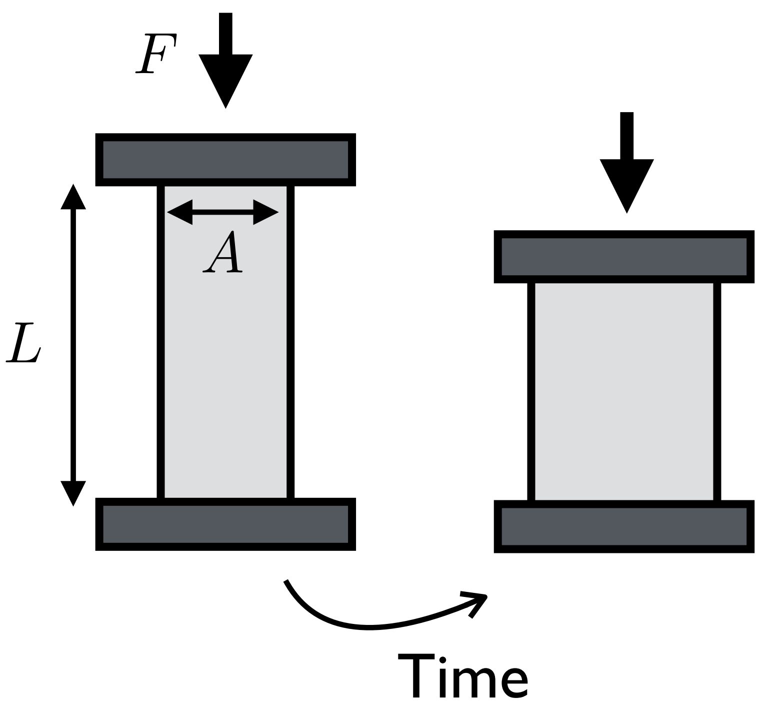
More generally $\tau_{ij} = c_{ijkl}\dot{\varepsilon}_{kl}$

c_{ijkl} is an effective viscosity tensor (4th order - 36 components) that may depend on invariants of the stress tensor, temperature, grain size, fabric, impurities, ….

If the ice is assumed to be isotropic, with stress and strain rate aligned $\dot{\varepsilon}_{ij} = \lambda\tau_{ij}$

Example modes of deformation

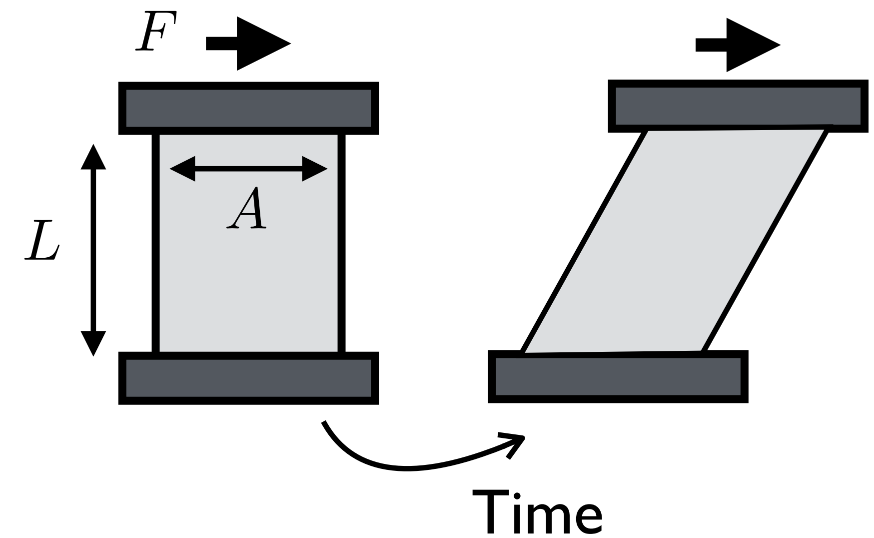
Uni-axial compression



$$\tau = \frac{F}{A}$$

$$\dot{\varepsilon} = \frac{1}{L}\frac{\mathrm{d}L}{\mathrm{d}t}$$

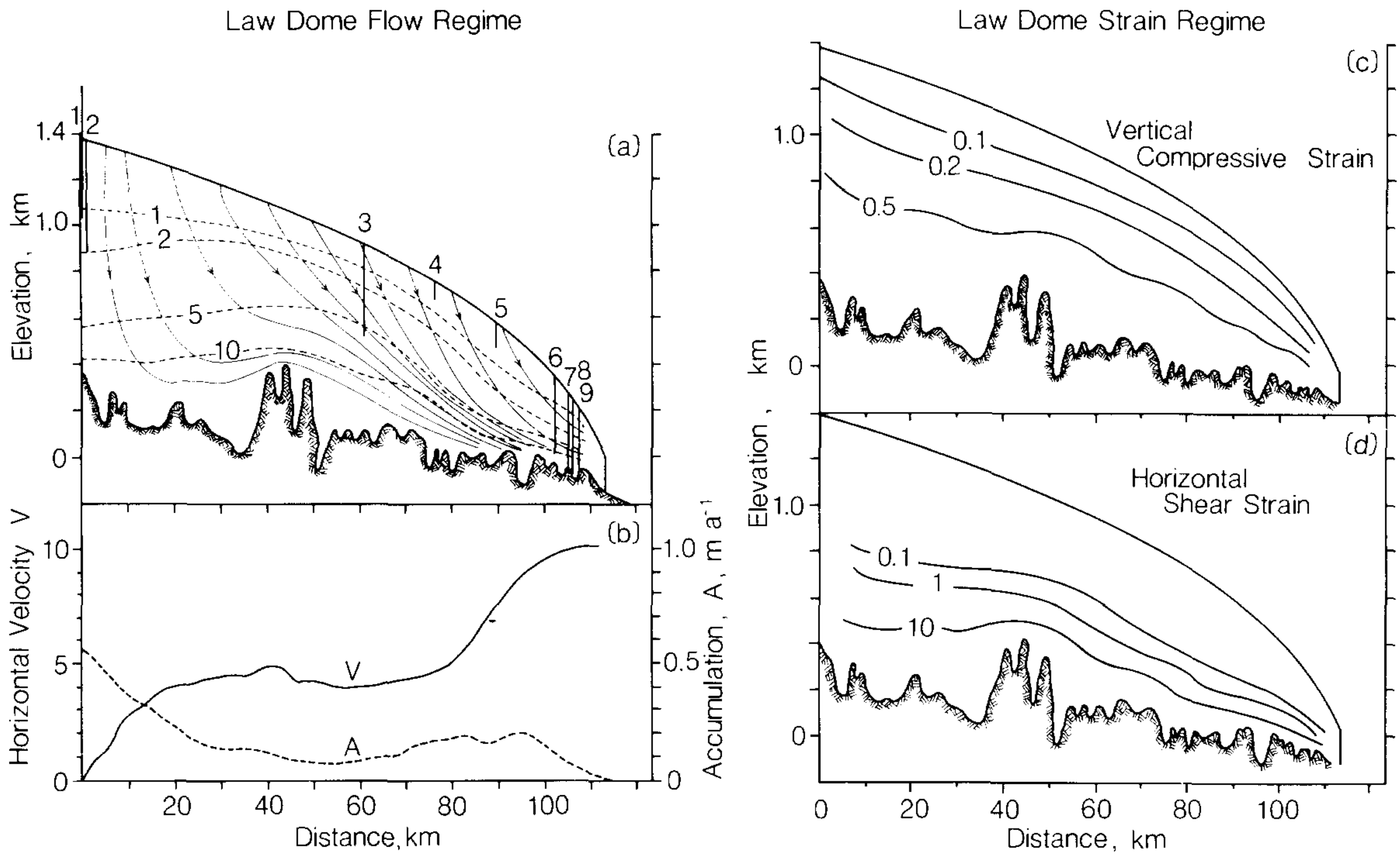
Simple shear



$$\tau = \frac{F}{A}$$

$$\dot{\varepsilon} = \frac{u}{L}$$

Example strain regimes in Antarctica



Budd & Jacka 1989

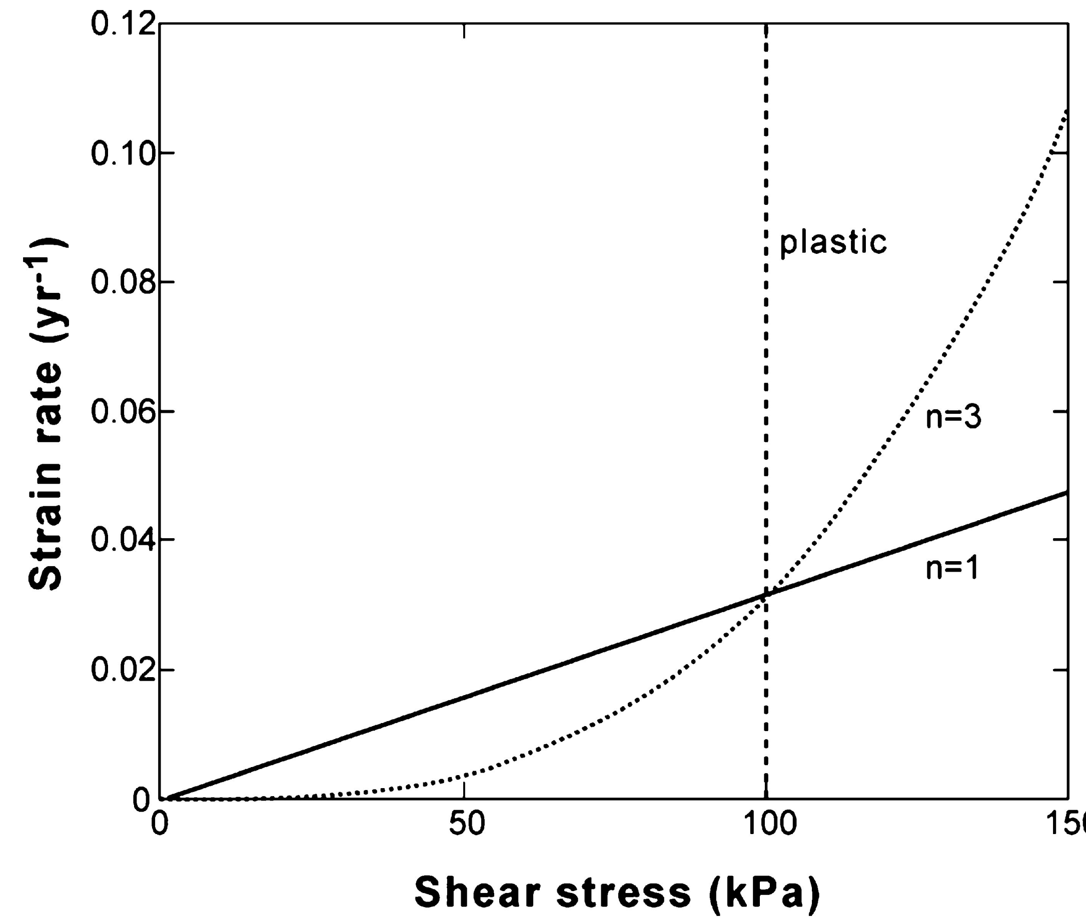
Glen's law

Glen's law is the most commonly used flow law for ice in glaciers and ice sheets.

$$\boxed{\dot{\varepsilon} = A\tau^n}$$

Usually $n \approx 3$ and $A \approx 2.4 \times 10^{-24}\ \mathrm{Pa}^{-3}\ \mathrm{s}^{-1}$ at $0^\circ\ \mathrm{C}$

But the most appropriate values in reality may depend on temperature, stress regime, grain size, etc



In tensorial form $\dot{\varepsilon}_{ij} = A\tau^{n-1}\tau_{ij}$

$$\tau^2 = \tfrac{1}{2}\tau_{ij}\tau_{ij} = \tfrac{1}{2}\left(\tau_{xx}^2 + \tau_{yy}^2 + \tau_{zz}^2\right) + \tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2$$

second invariant - 'effective stress'

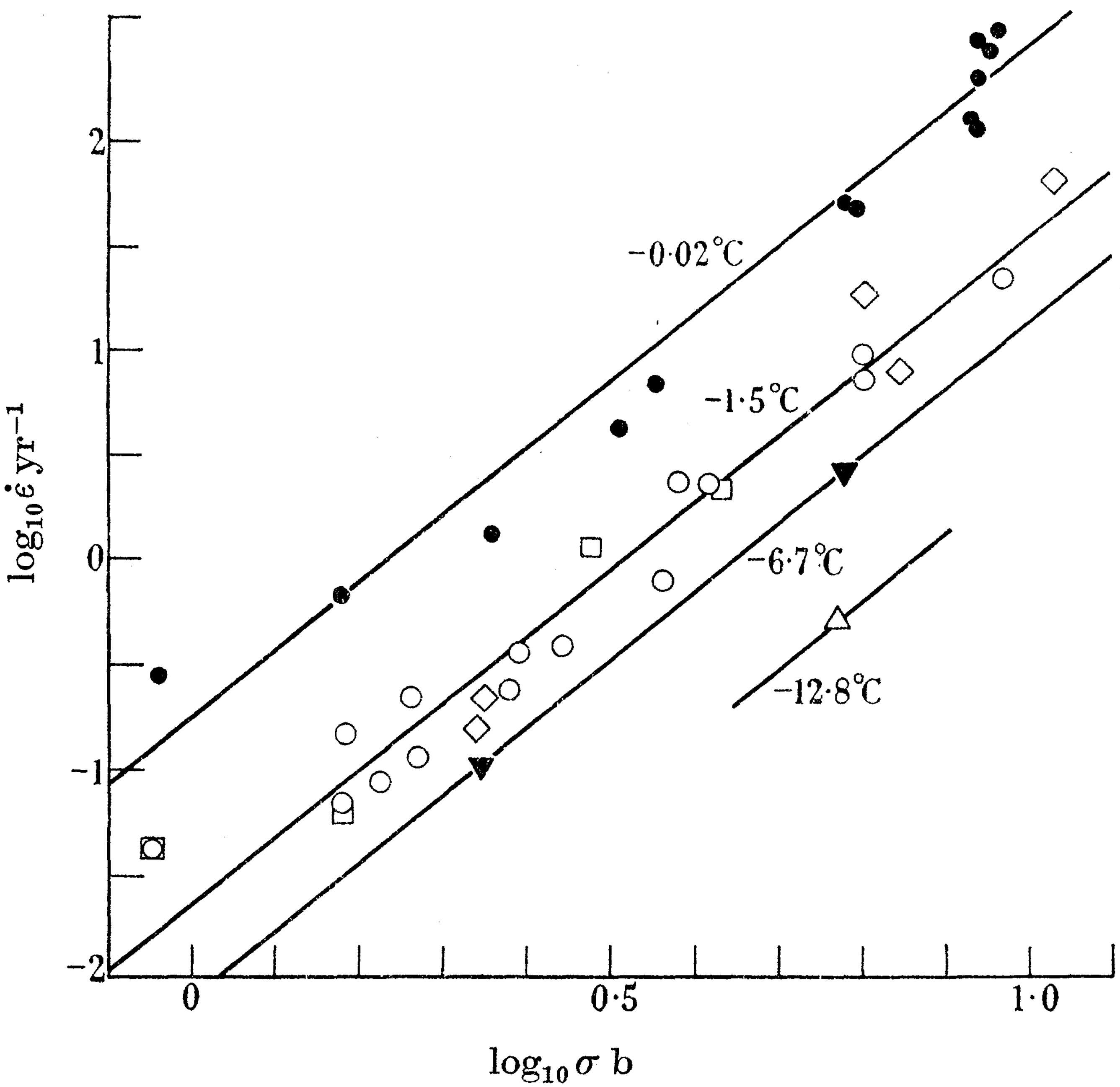
This can also be written as $\tau_{ij} = 2\eta\dot{\varepsilon}_{ij}$

$\eta = \dfrac{1}{2A\tau^{n-1}}$ is the **effective viscosity**

(In general fluid mechanics terminology Glen's law is referred to as a 'power-law').

Glen's law

Strain rate



Stress

Slope indicates $n \approx 3$

(different lines for different temperatures)

Glen 1955

Evidence for Glen's flow law

Laboratory experiments (Glen 1955, Weertman 1983, Budd & Jacka 1989)

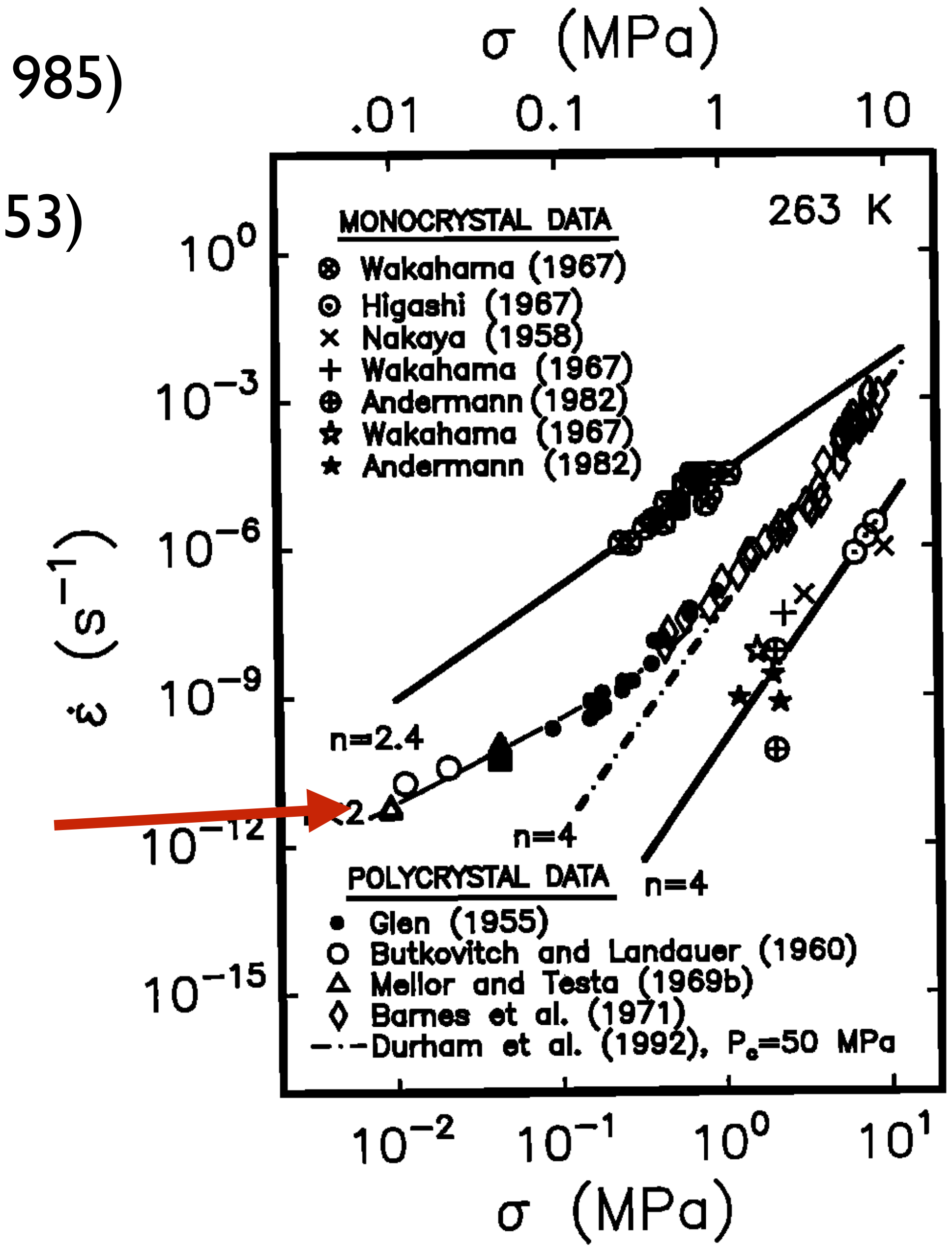
Measurements of the **stretching of ice shelves** (Jezek et al 1985)

Measurements of the **closure of subglacial tunnels** (Nye 1953)

Measurements of the **tilting of boreholes** (Paterson 1981)

Most of these studies suggest values of the power-law exponent $n \approx 2 - 4$

There is a general indication of lower exponents at lower stress (Schulson & Duval 2009).

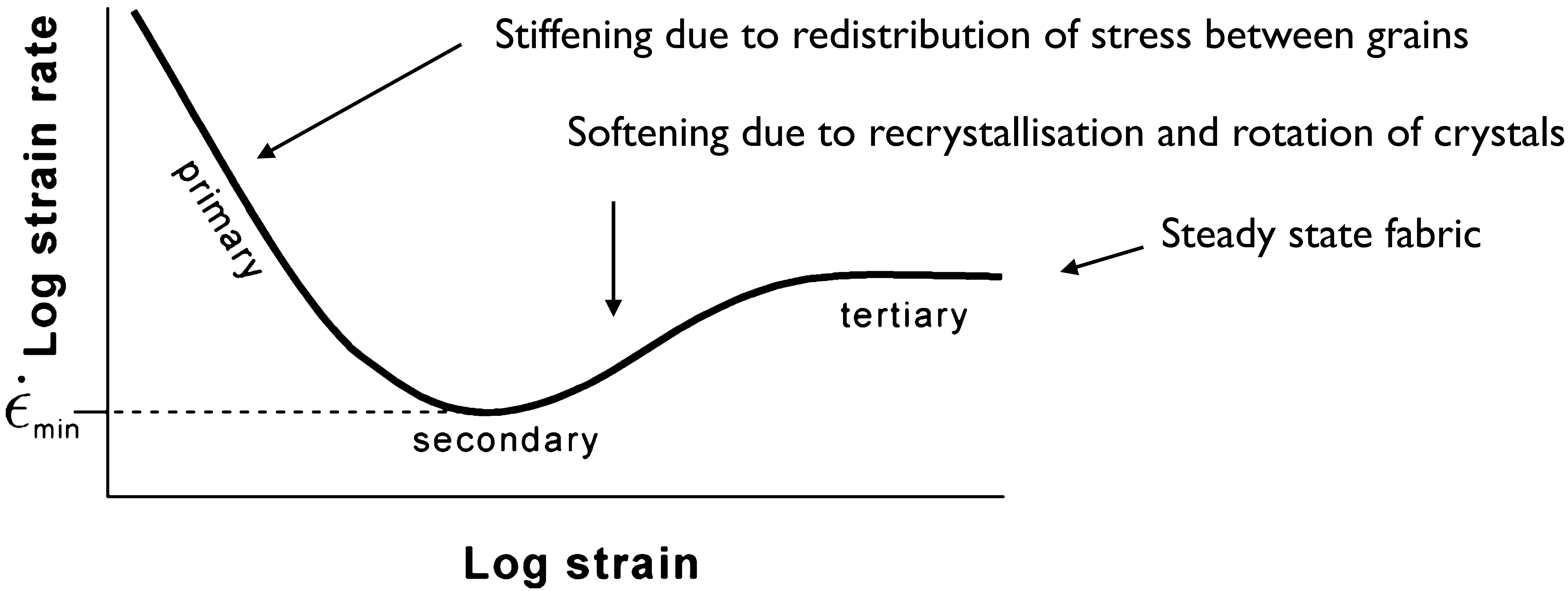


Goldsby & Kohlstedt 2001

Note: calibrating the flow law from field measurements is challenging! It is difficult to unambiguously separate out the contributions of stress, temperature and fabric.

Laboratory experiments

A typical laboratory experiment performed under constant stress conditions shows evolution of strain rate with strain (Budd & Jacka 1989).



The minimum strain rate (secondary creep) is usually used for the flow law (occurs at ~1% strain).

In contrast, most glacial ice has experienced larger strain, so is in the tertiary creep regime (?)

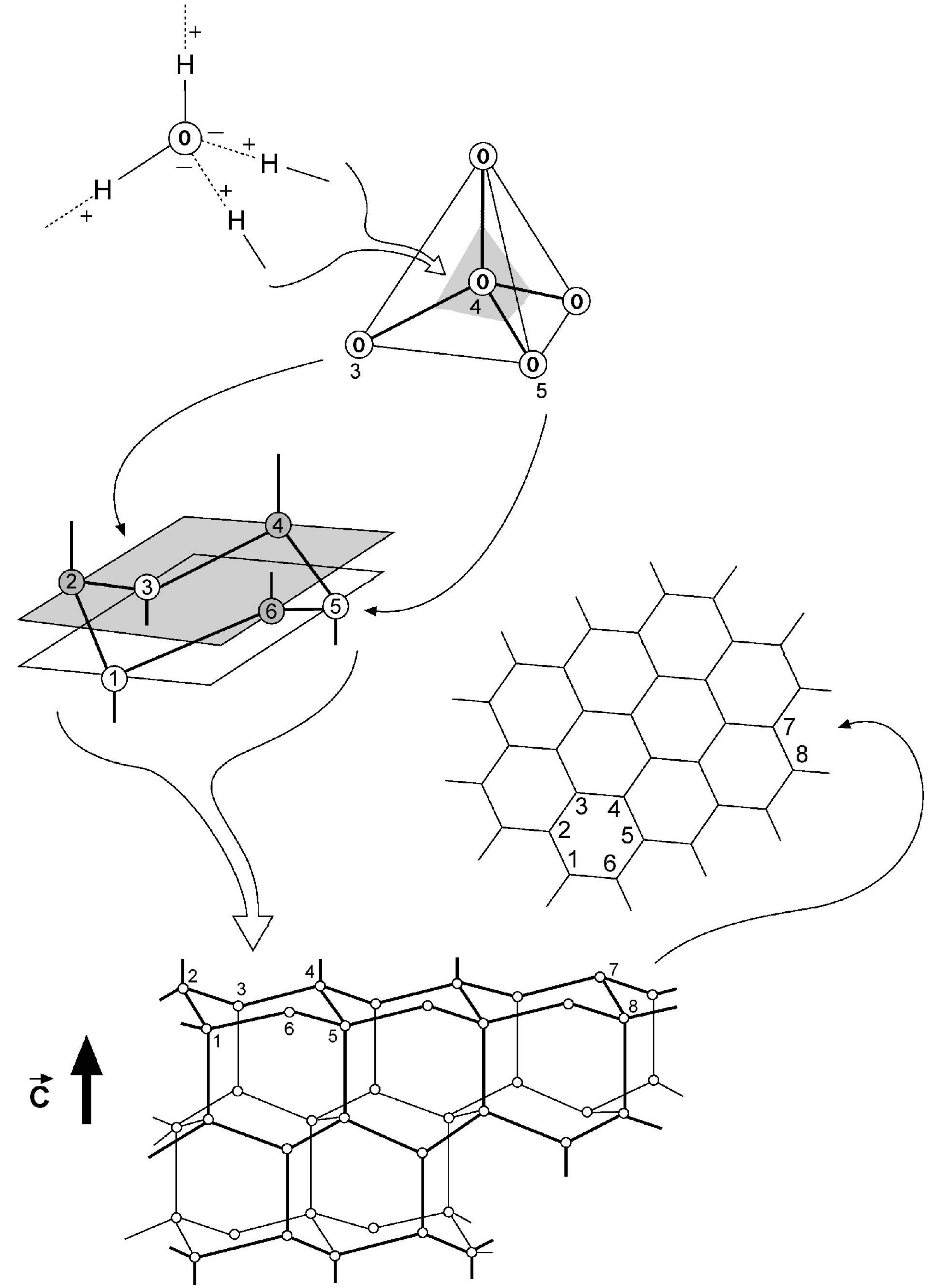
An individual crystal structure

Glacial ice is of ice type Ih (h = hexagonal)

Individual H_2O molecules are are arranged in tetrahedral patterns that tessellate to form hexagonal rings of oxygen atoms.

A single ice crystal consists of stacked layers of these rings.

The plane of the hexagons is called the basal plane, and the normal is called the c-axis.



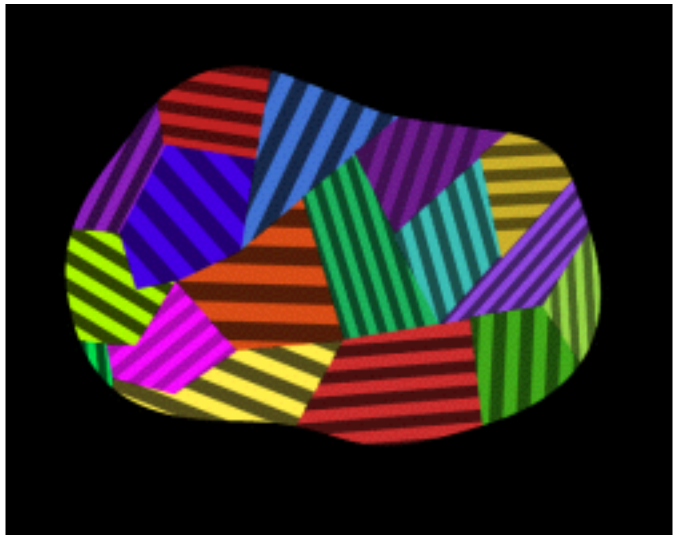
Hobbs 1974 'Ice Physics'

Cuffey & Paterson 2010

Polycrystalline ice

Polycrystalline ice contains many grains (crystals), with different orientations of their c-axes.

Individual grains in glacial ice are typically 1–10 mm in size.



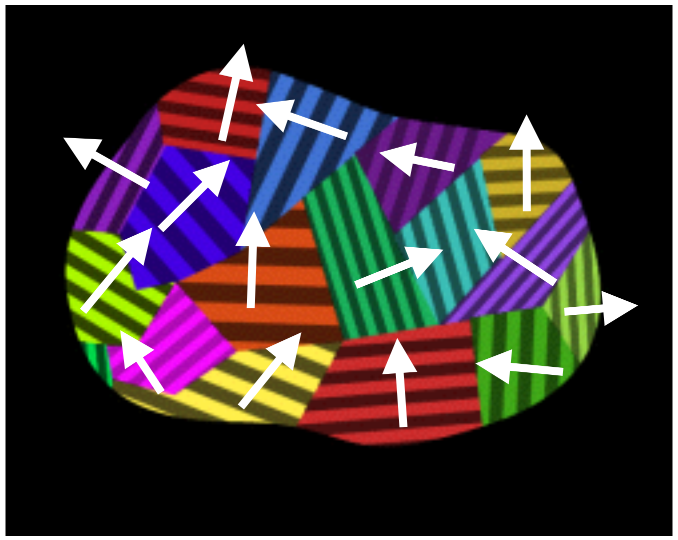
http://www.iceandclimate.nbi.ku.dk/

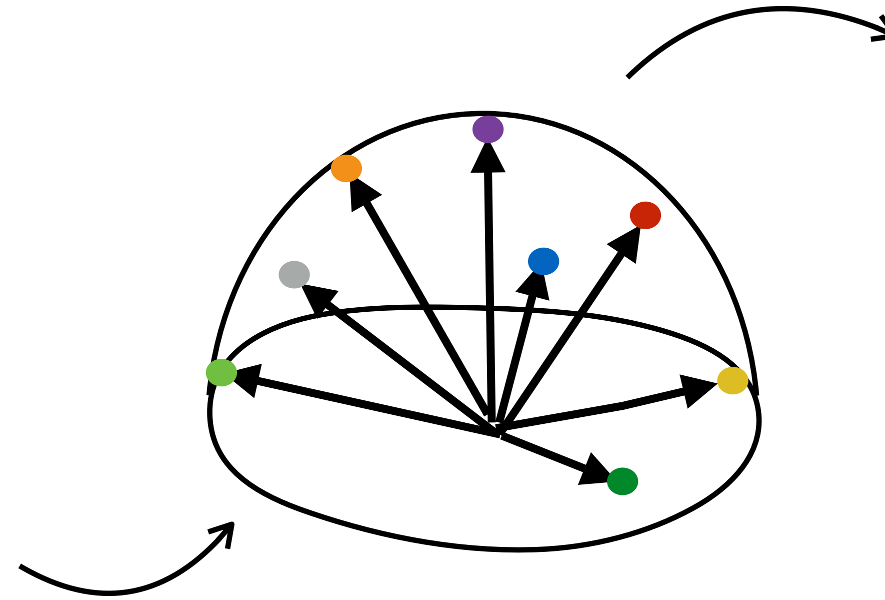
In cross-polarised light, thin-sections of ice cores show different orientations of the c-axis as different colours.

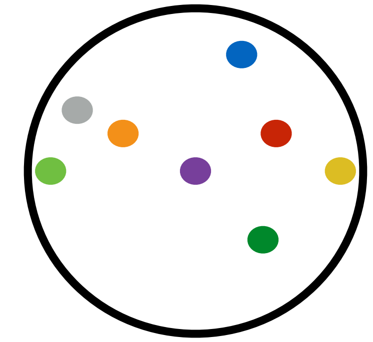
The ensemble of c-axis orientations is referred to as the fabric of the ice - it can evolve, as grains grow and deform, and as new crystals form.

Schmidt diagrams

The fabric is visualised with a **Schmidt diagram**:



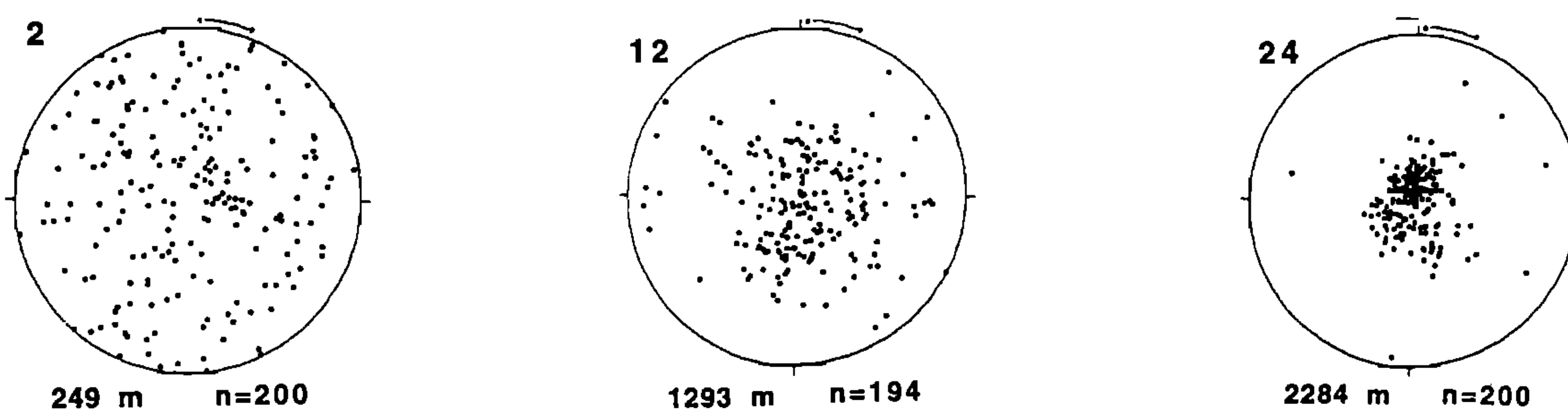




Viewed from above, each c-axis is a dot

Plot projection of each c-axis vector onto hemisphere

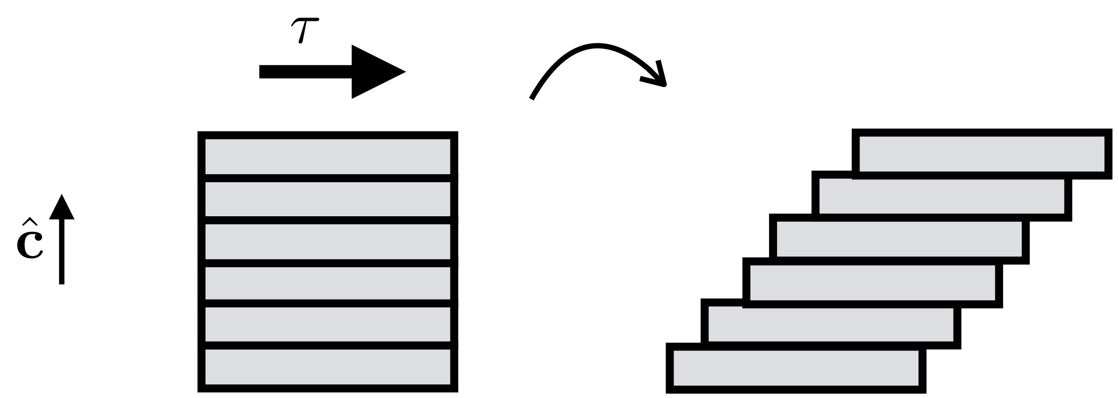
With a larger samples of crystals (from thin-sections of NGRIP ice core):



Thorsteinsson et al 1997

Deformation of a single crystal

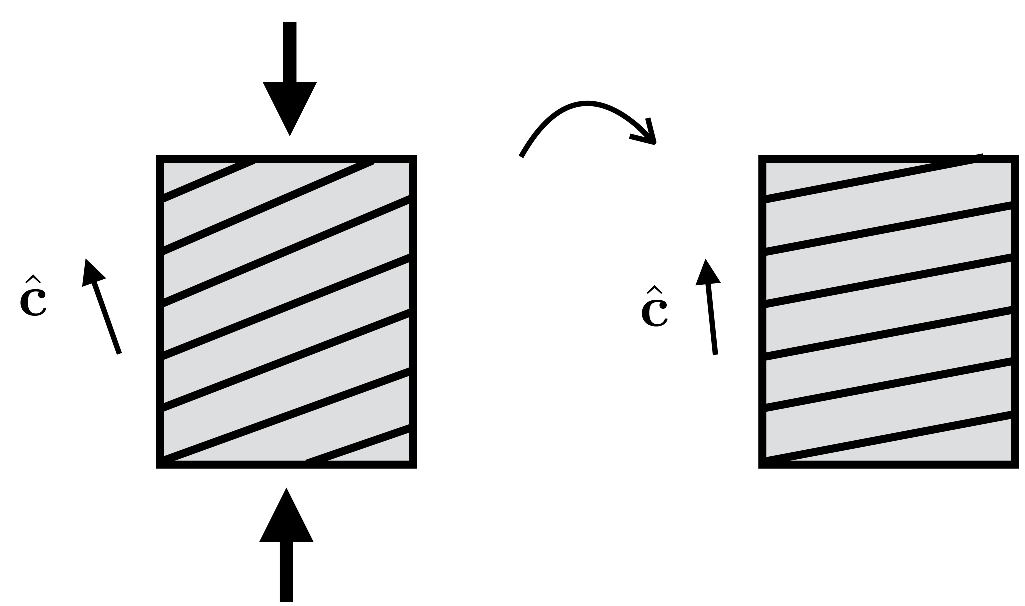
A single crystal deforms easily if shear stress is applied along its basal plane - such deformation is termed basal glide.



Deformation is much harder if shear stress is applied along a different plane (Duval et al 1983).

Deformation is achieved through the motion of dislocations in the crystal lattice, along basal planes (dislocation glide), and across basal planes (dislocation climb).

Compressive stress applied to individual crystals causes their c-axes to rotate towards the compressive axis.



Deformation of polycrystalline ice

Most of the deformation in polycrystalline ice occurs by **basal glide**. But the different orientations of crystals mean that this is not usually the rate-limiting process.

The rate limiting process, responsible for controlling the macroscopic strain rate (described by the flow law) depends on magnitude of stress, temperature, and grain size.

Dislocation creep - dislocation climb enables non-basal-plane motion.

$n \approx 3 - 4$

- favoured at high stress.
- independent of grain size.

Grain boundary sliding - favoured at low stress

$n \approx 1.8 - 2.4$

- sensitive to grain size.

Diffusion creep - favoured at very low stress.

$n \approx 1$

- molecules diffuse through crystals or along grain boundaries
- sensitive to grain size.

Grain size and fabric evolution

Normal grain growth occurs in the absence of deformation - grain boundaries are energetically unfavourable.

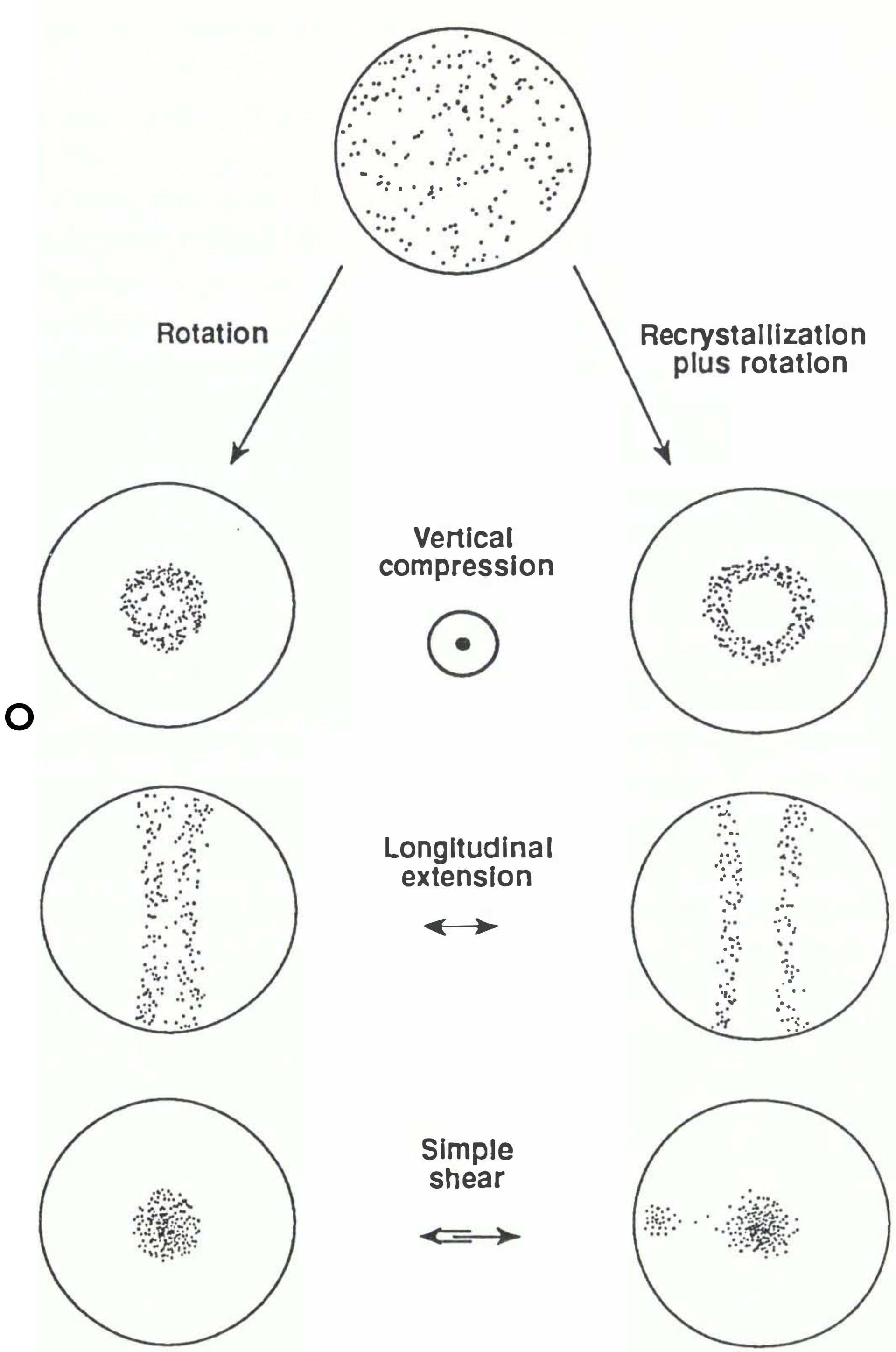
Deviatoric stress causes individual c-axes to rotate towards the compression axis.

Dynamic recrystallisation occurs during deformation - this includes polygonisation (subdivision of grains resulting from alignment of dislocations) and nucleation of new grains (with no initial strain energy and c-axes at ~45° to compression axis).

Under constant stress, a steady-state balance between grain growth, rotation, and recrystallisation may be possible.

In general, grain size, fabric, and strain rate, all co-evolve.

A favoured orientation of c-axes yields an anisotropic response of strain rate to stress.



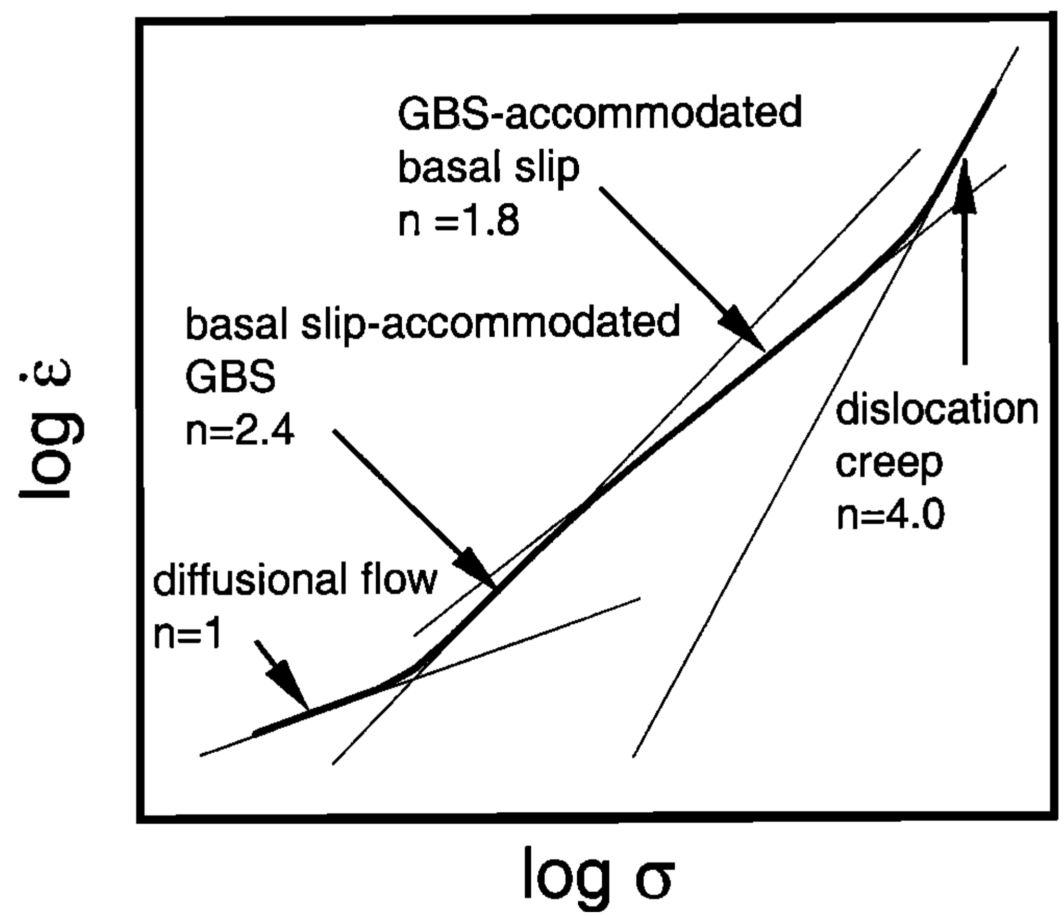
Alley 1992

Alternative flow laws

Combining deformation mechanisms suggests a flow law like

$$\dot{\varepsilon} = \dot{\varepsilon}_{diff} + \left(\dot{\varepsilon}_{basal}^{-1} + \dot{\varepsilon}_{gbs}^{-1}\right)^{-1} + \dot{\varepsilon}_{disl} \qquad\qquad \dot{\varepsilon}_{(\cdot)} = A_{(\cdot)}\tau^{n_{(\cdot)}}$$

This allows different mechanisms to dominate at different stresses, temperatures, and grain sizes.



Goldsby & Kohlstedt 2001

Glen’s law - parameter dependence

Return to Glen’s law

$$\boxed{\dot{\varepsilon}_{ij} = A\tau^{n-1}\tau_{ij}}$$

Effect of temperature

Appears to be reasonably described with an Arrhenius law $A = A_0 \exp\left(-\frac{Q}{RT}\right)$

(varies by a factor of ~1000 over range of glacial temperatures -55C—0C)

Apparent activation energy increases above ~-10C - perhaps due to pre-melted films on grain boundaries, facilitating grain boundary sliding (Barnes et al 1971).

Effect of water content

For temperate ice (at the melting point), inter-granular water content W softens the ice (Duval 1977)

$$A = (3.2 + 5.8W) \times 10^{-24}\ \mathrm{Pa}^{-3}\ \mathrm{s}^{-1}$$

(varies by a factor of ~3 for W in range 0–1%)

Effect of impurities

Impurities likely soften ice by facilitating the motion of dislocations and enhancing pre-melting on grain boundaries. Their effect is not usually included explicitly in flow laws.

Enhancement factors

An **enhancement factor** E is sometimes introduced into the flow law to account for un-resolved effects of grain size, fabric and impurities.

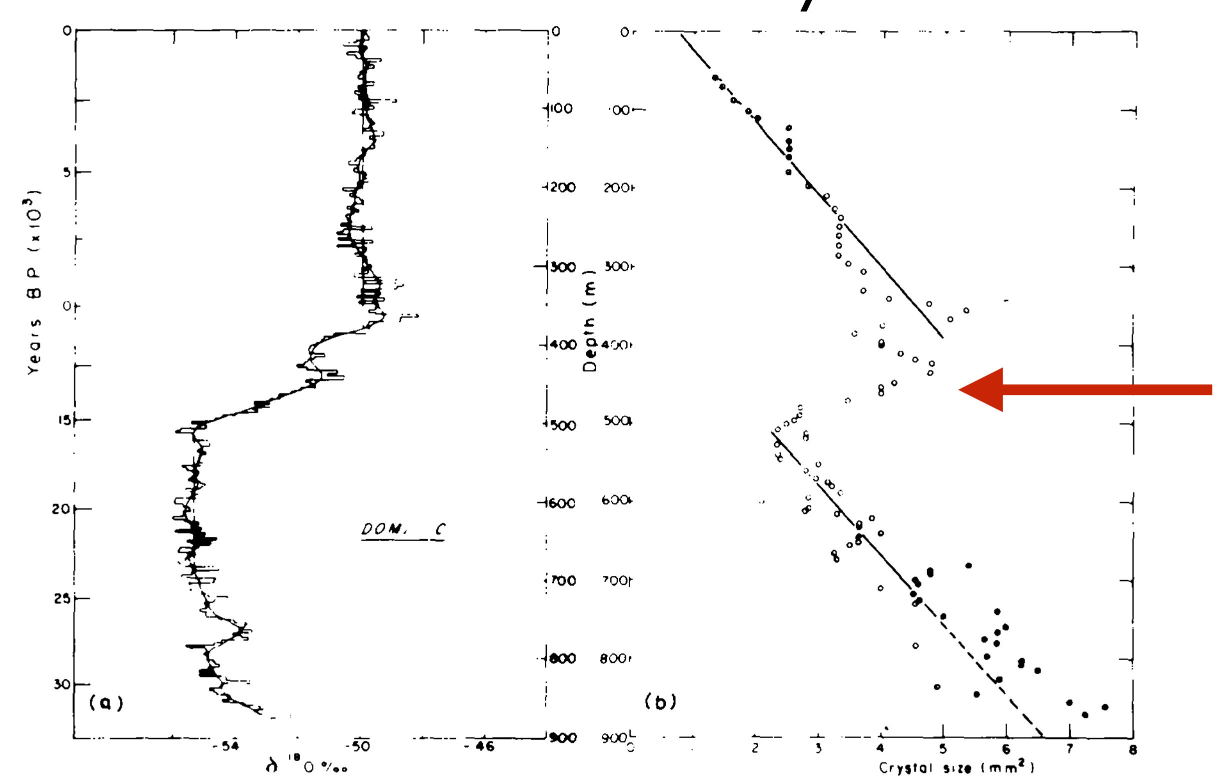
$$\dot{\varepsilon}_{ij} = EA(T)\tau^{n-1}\tau_{ij}$$

The enhancement factor should not be treated as a known parameter; ideally is should be fitted to observations at each point in the ice (e.g. using inverse methods).

Example: an enhancement factor is often applied to ice-age ice, which is observed to be softer than neighbouring Holocene ice (due to smaller grain size).

δ ^{18}O

Crystal size



Duval Lorius 1980

Elastic deformation

Creep deformation occurs when stress is applied for a sufficiently long time (longer than the Maxwell time, around a day).

The response to short time-scale forcing is **elastic** - this is particularly important for the tidal flexure of ice shelves.

Elastic deformations are described by a constitutive law relating stress and strain

$$\sigma_{ij} = \frac{E}{1+\nu}\left(\frac{\nu}{1-2\nu}\varepsilon_{kk}\delta_{ij} + \varepsilon_{ij}\right)$$

Young's modulus $E \approx 10 \text{ GPa}$

Poisson's ratio $\nu \approx 0.3$

To describe both elastic and creep deformations, a viscoelastic constitutive law can be used, such as a Maxwell model

$$\sigma_{ij} = -p\delta_{ij} + \tau_{ij} \qquad \frac{1-2\nu}{E}\dot{p} = -\frac{1}{3}\dot{\varepsilon}_{kk} \qquad \frac{1+\nu}{E}\dot{\tau}_{ij} + \frac{1}{2\eta}\tau_{ij} = \left(\dot{\varepsilon}_{ij} - \tfrac{1}{3}\dot{\varepsilon}_{kk}\delta_{ij}\right) \qquad \eta = \frac{1}{2A\tau^{n-1}}$$

This encompasses linear elasticity on short timescales, and Glen's law on long timescales $t \gtrsim \eta/E$

Summary

Glacial ice has a polycrystalline structure that evolves in response to flow.

Macroscopic deformation occurs predominantly by basal glide, accommodated and rate-limited by a combination of dislocation creep and grain boundary sliding.

Strain rates are particularly sensitive to temperature. They also depend on grain size, impurities, and water content.

Glen's law is the standard rheology used for ice-sheet modelling - but it does not account for the complex evolution of fabric and resulting anisotropy.

The most appropriate parameters depend on the ice under consideration and its deformation history.