Glacier and Ice-Sheet Sliding

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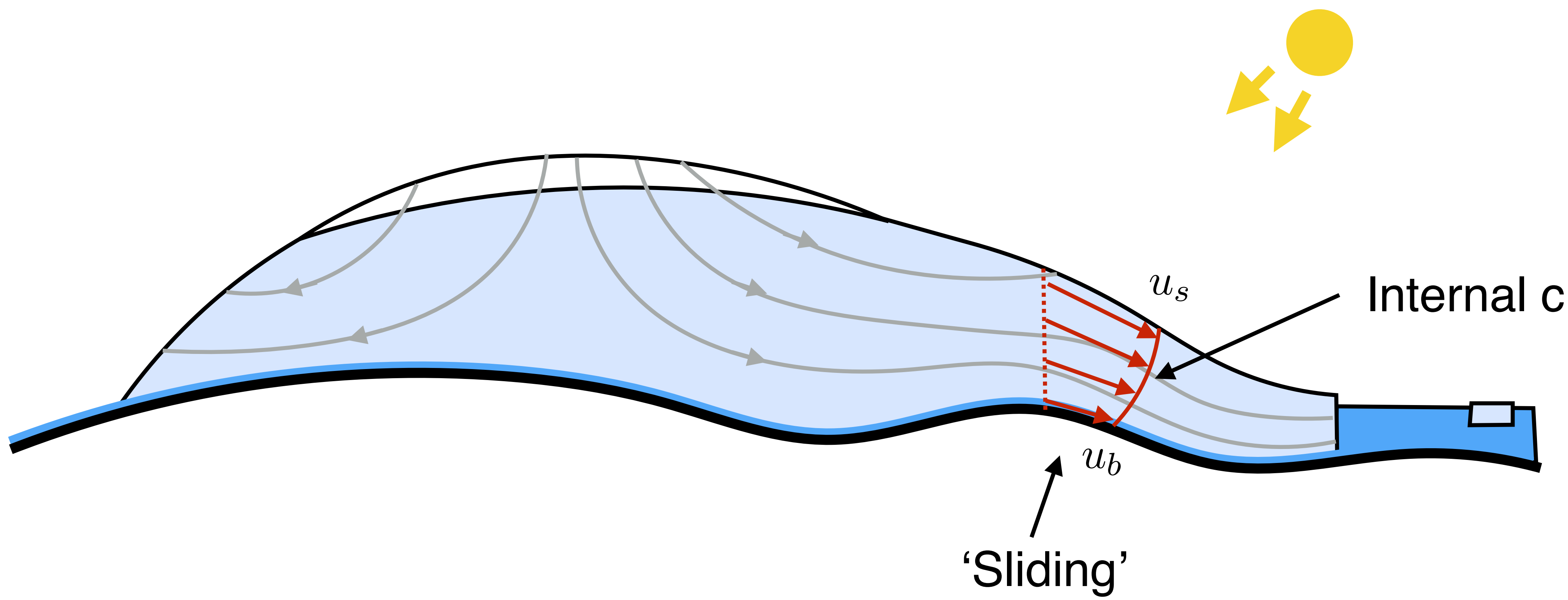
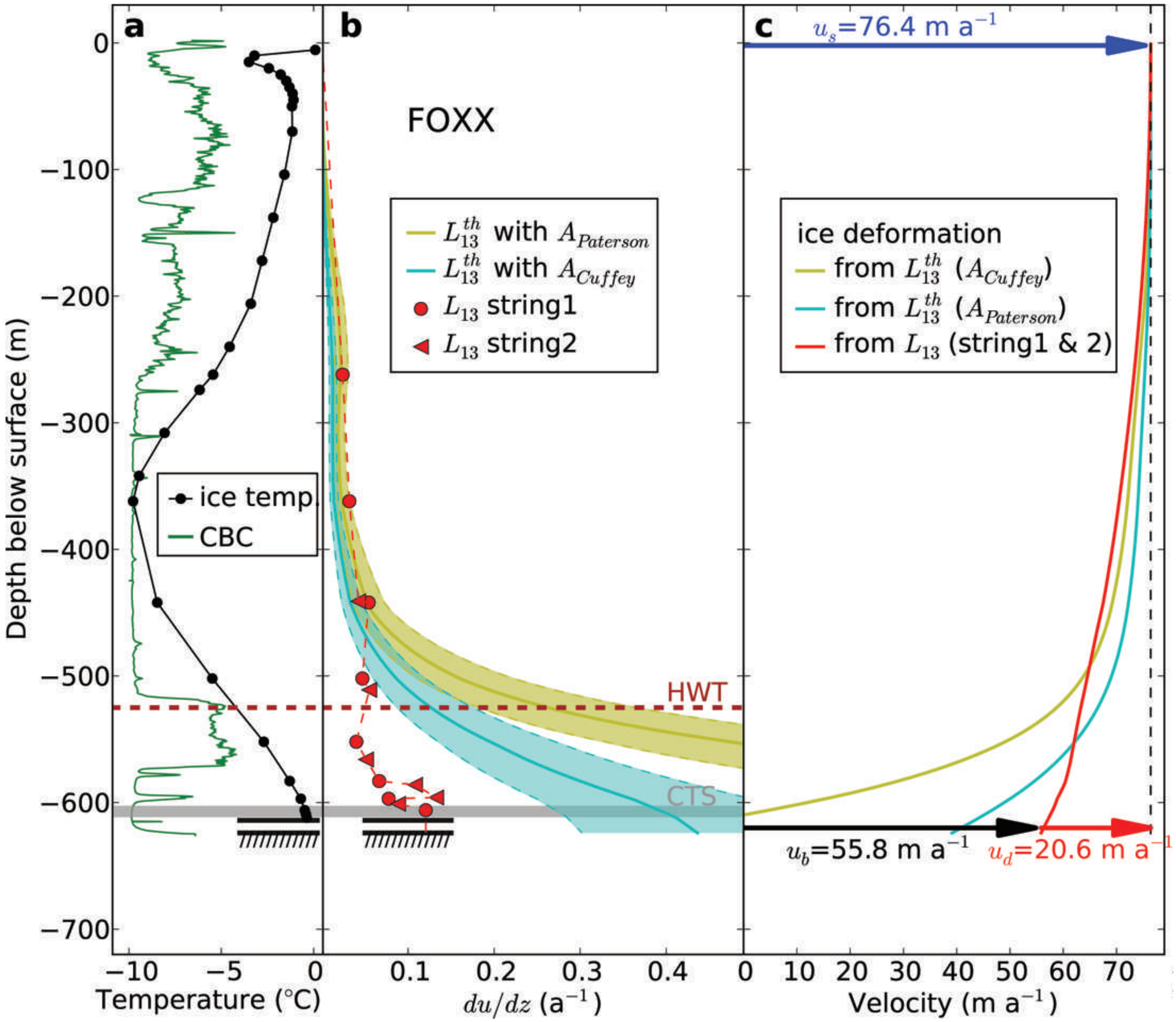
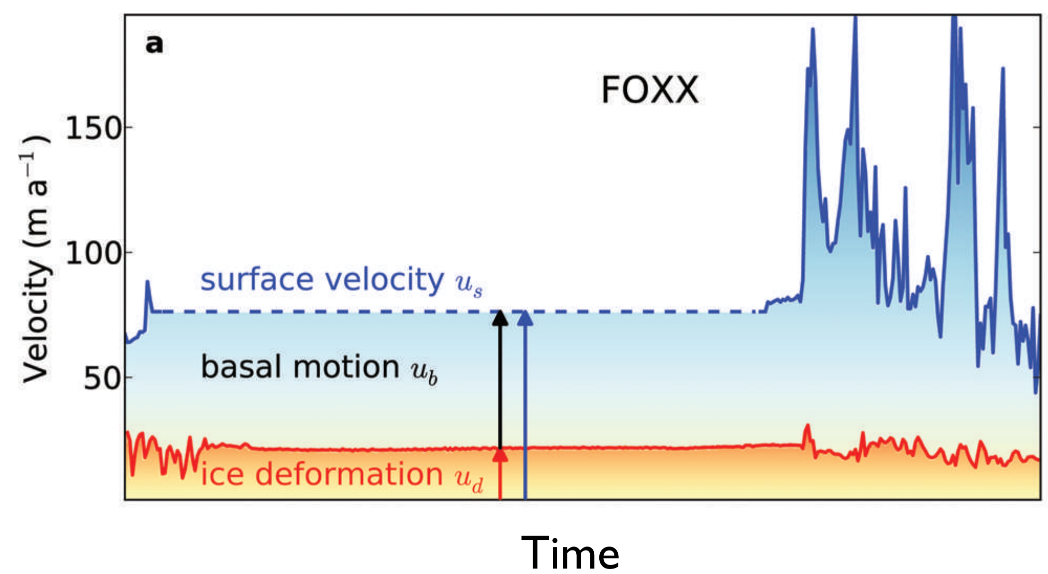


u_s
Internal creep
u_b
'Sliding'

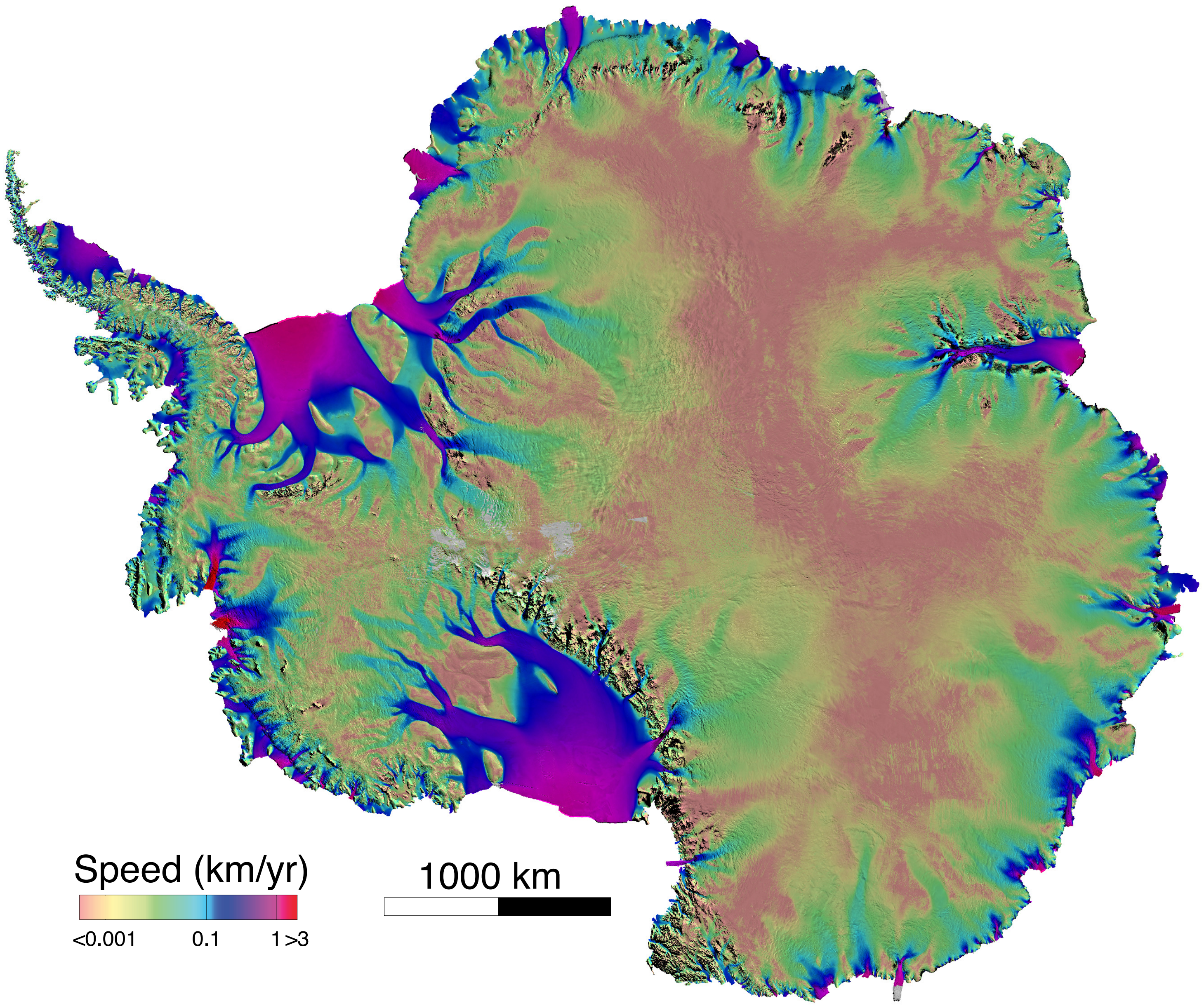
GPS and borehole-derived ice speeds



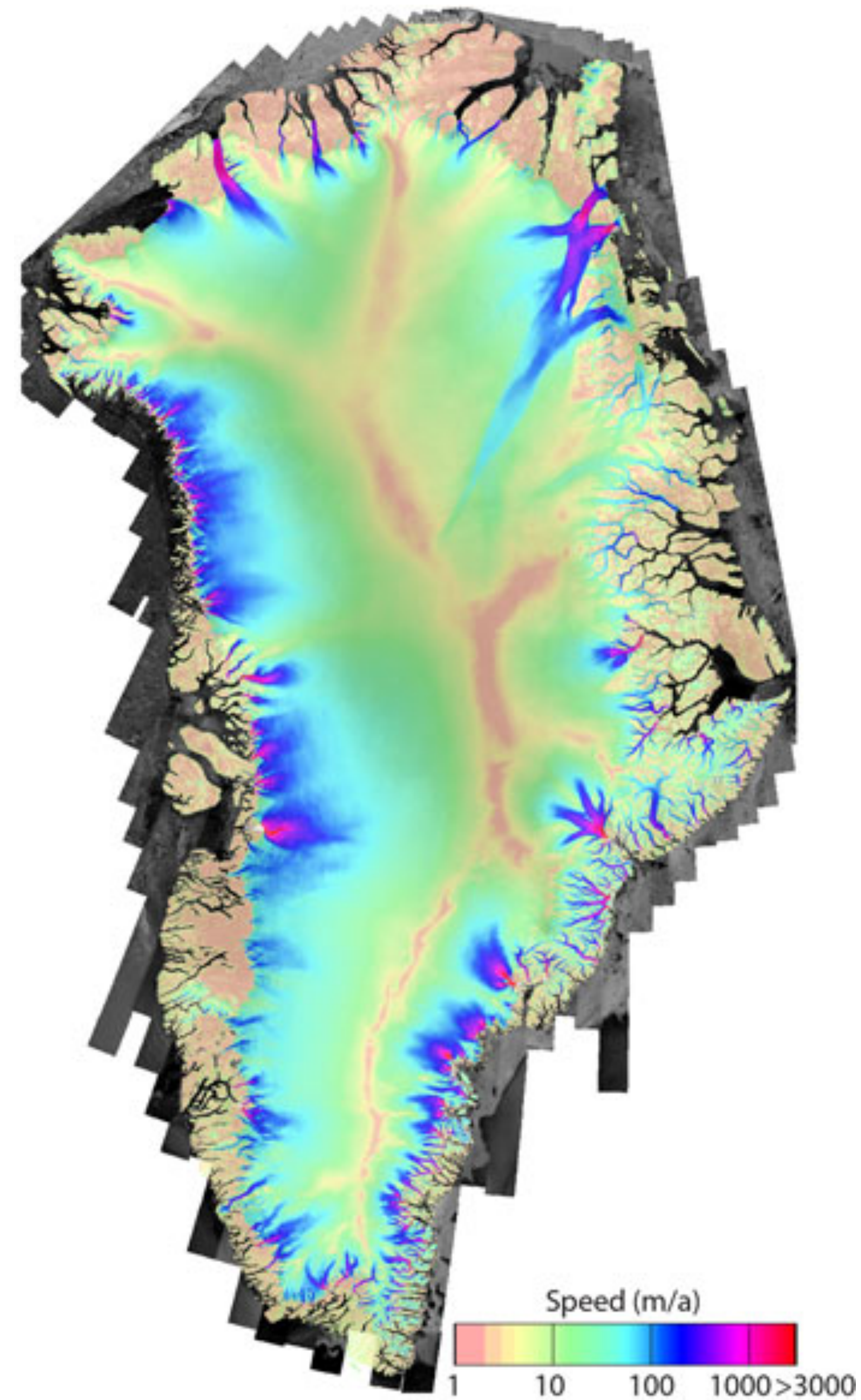


Ryser et al 2014

Satellite-derived ice surface speeds



Mouginot et al 2019



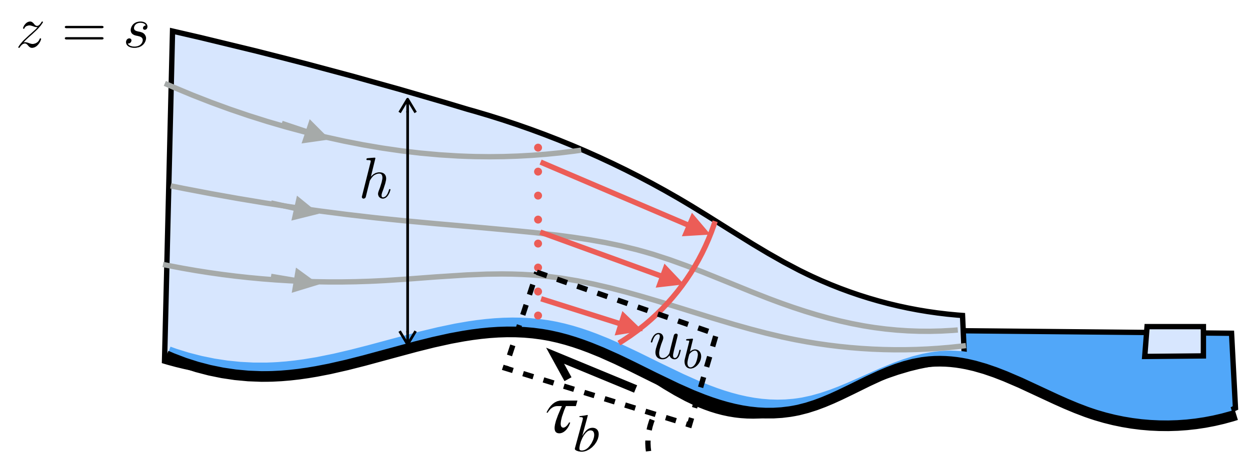
Joughin et al 2018

What controls how fast a glacier or ice sheet slides?

What physical processes enable it to slide?

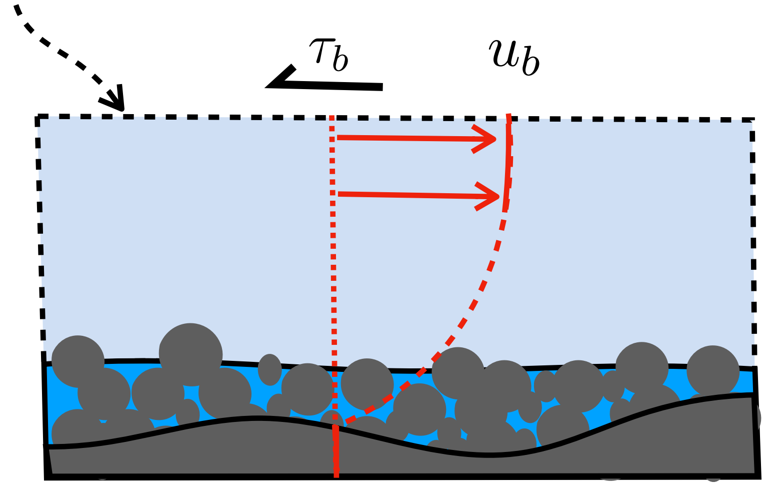
How do we describe sliding in an ice-sheet model?

Sliding law / Friction law



Stokes flow $$0 = -\nabla p + \nabla \cdot \boldsymbol{\tau} + \rho_i \mathbf{g}$$
$$\nabla \cdot \boldsymbol{u} = 0$$

To calculate ice flow we need a **basal boundary condition**
which relates **basal shear stress** $\tau_b = |\boldsymbol{\tau}_b|$ and **basal speed** $u_b = |\mathbf{u}_b|$



$$\tau_b = f(u_b, \cdots)$$

This is a parameterization of **unresolved** processes close to the bed.

Historically thought of as 'sliding' law $u_b = F(\tau_b, \cdots)$

→ May be multi-valued

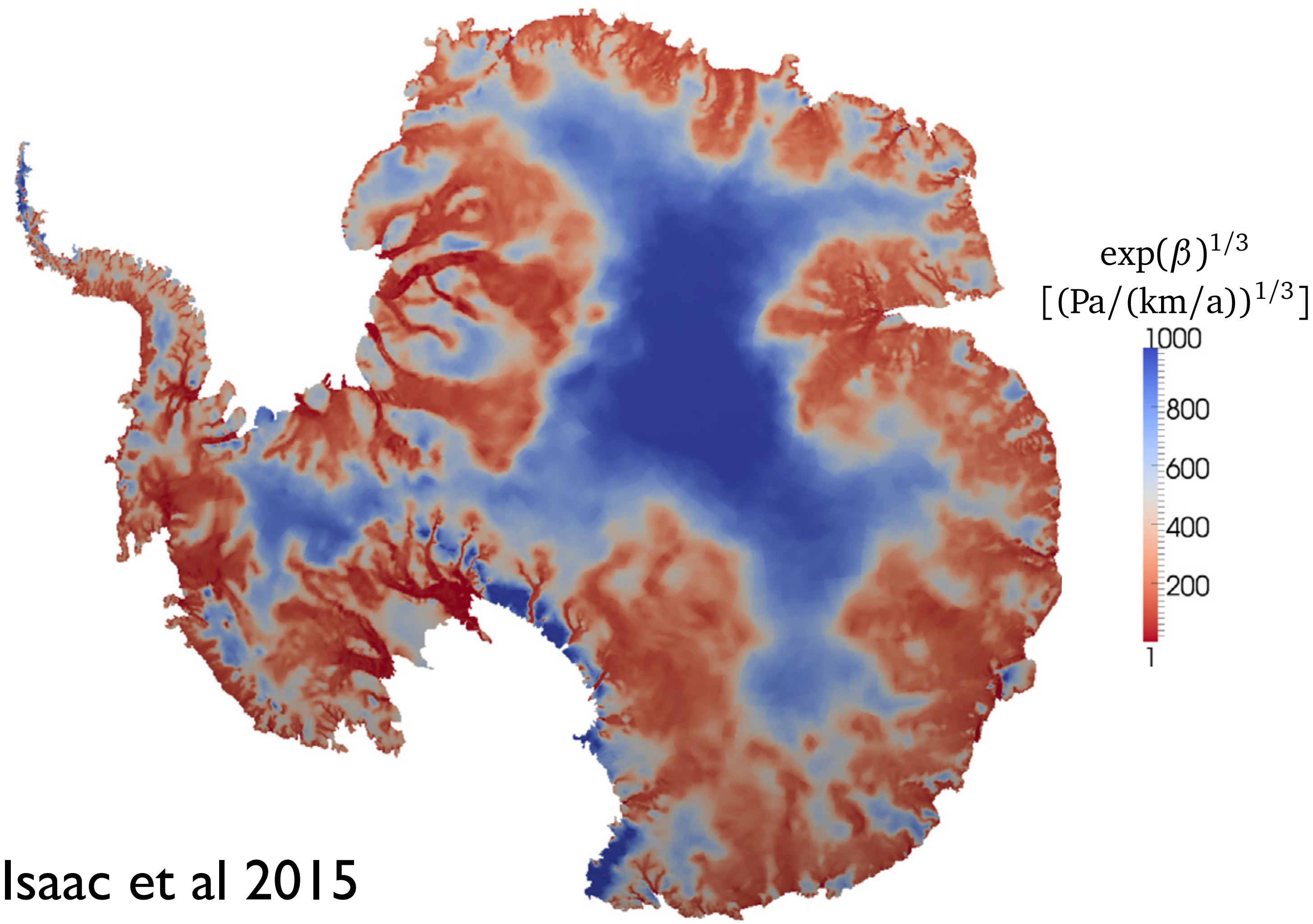
Shallow ice approximation $\boldsymbol{\tau}_b \approx -\rho g h \nabla s$

Modern view point is as a 'friction' law $\tau_b = f(u_b, \cdots)$

Numerical ice-sheet models

Many numerical models use a **friction law** of the form $\boldsymbol{\tau}_b = C|\mathbf{u}_b|^{m-1}\mathbf{u}_b$ ('Weertman law')

The coefficient $C = C(x, y)$ is usually treated as a fitting parameter(s), chosen to achieve a good fit with observations of surface velocities.

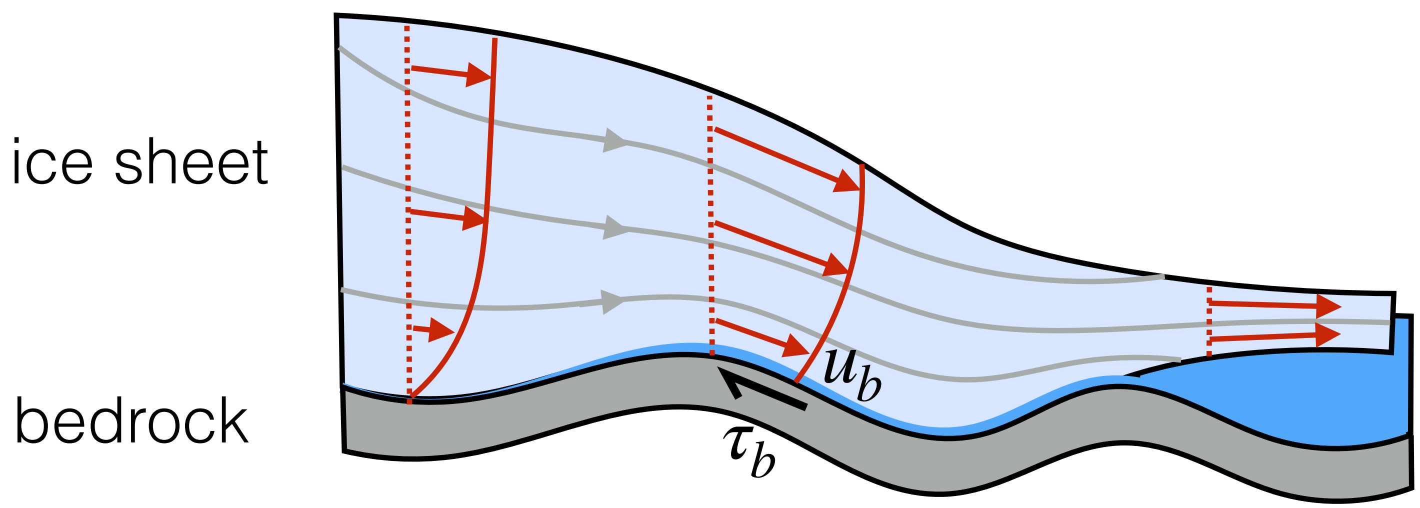


Isaac et al 2015

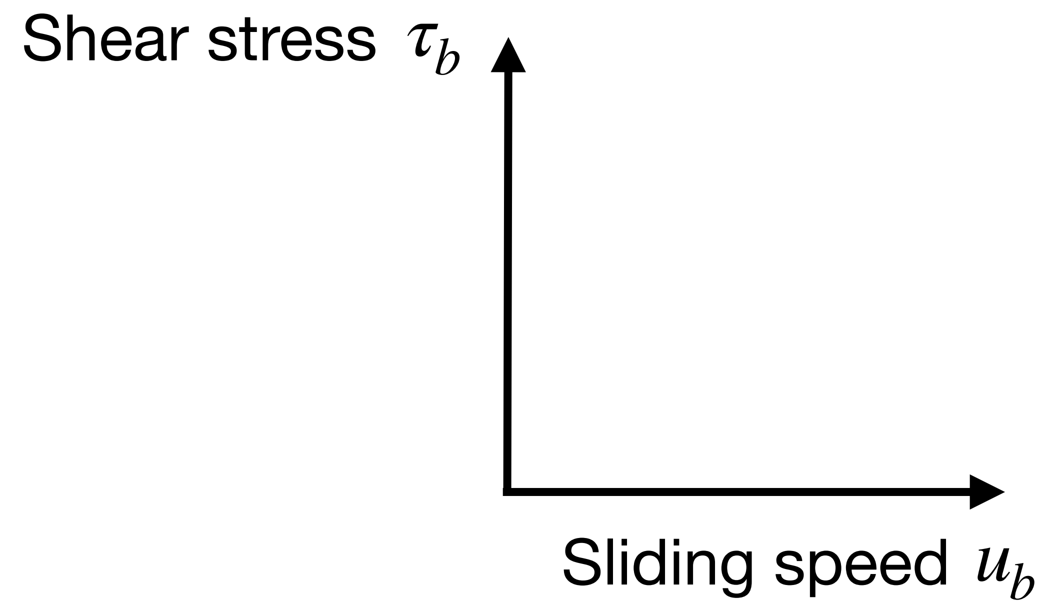
But the coefficient reflects properties of the bed that may vary with time.

⇨ We want to understand what **physical processes** govern the friction law.

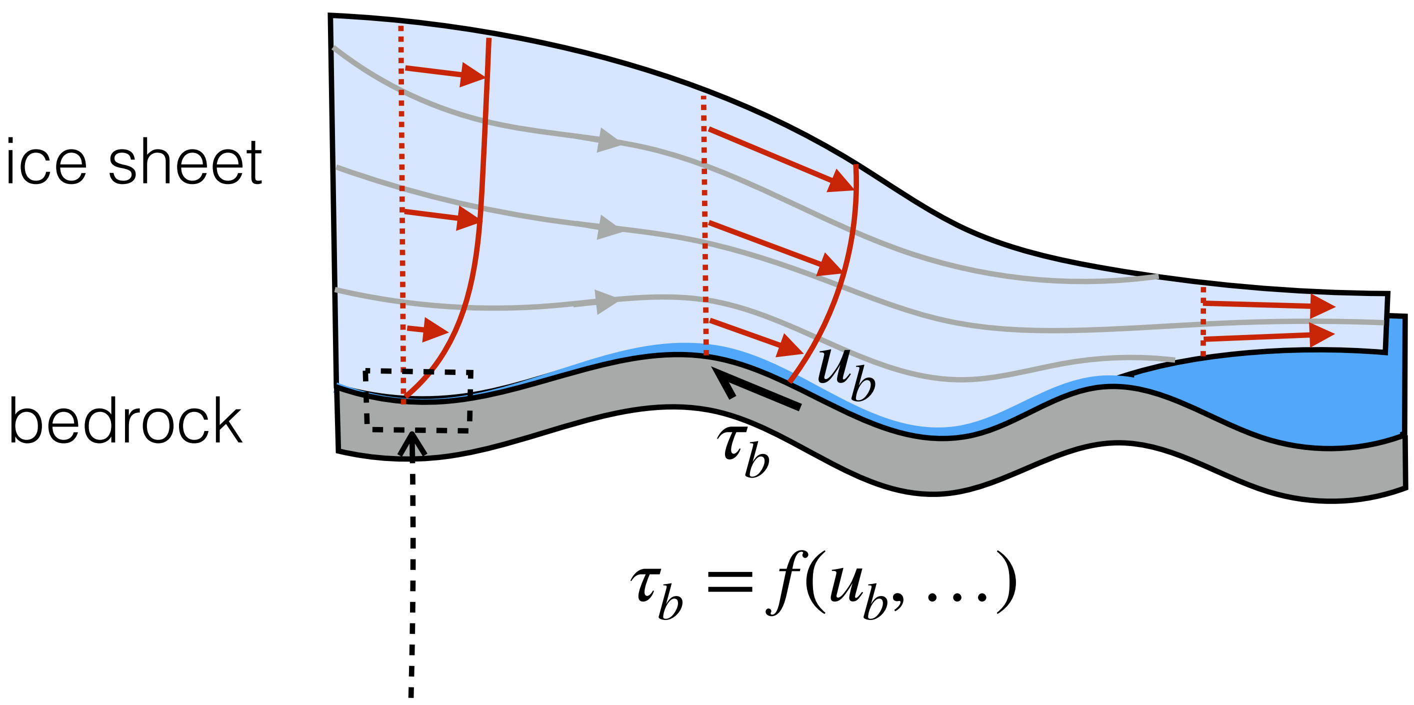
Friction law

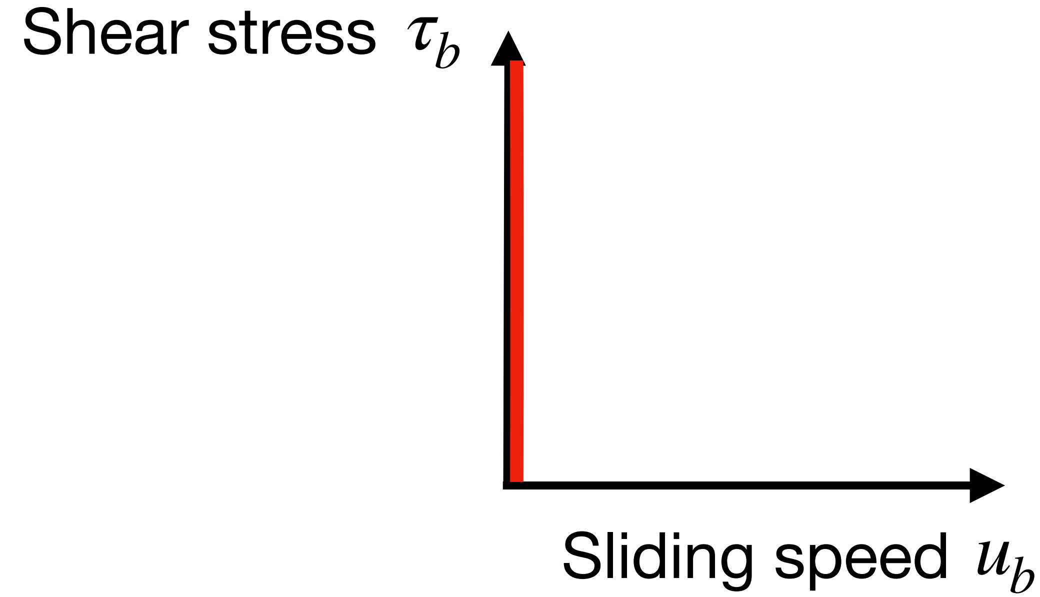


$$\tau_b = f(u_b, \ldots)$$

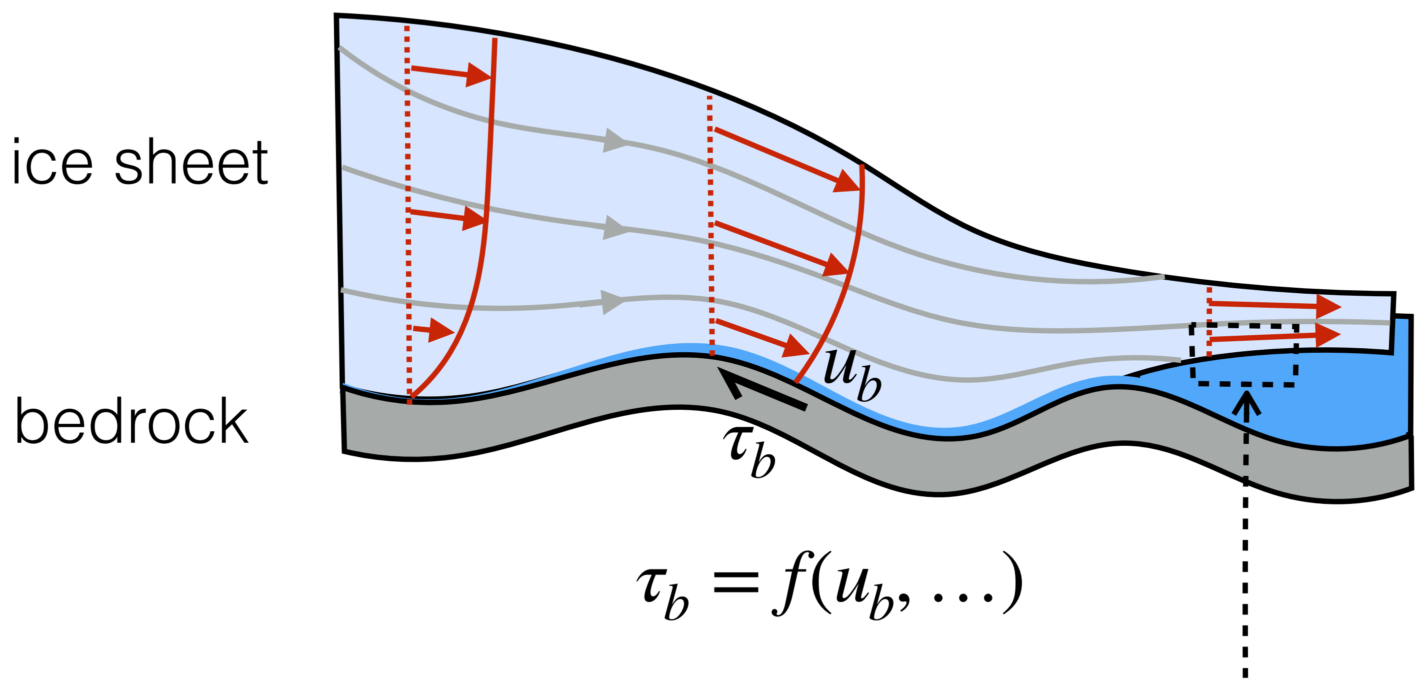


Friction law





Friction law



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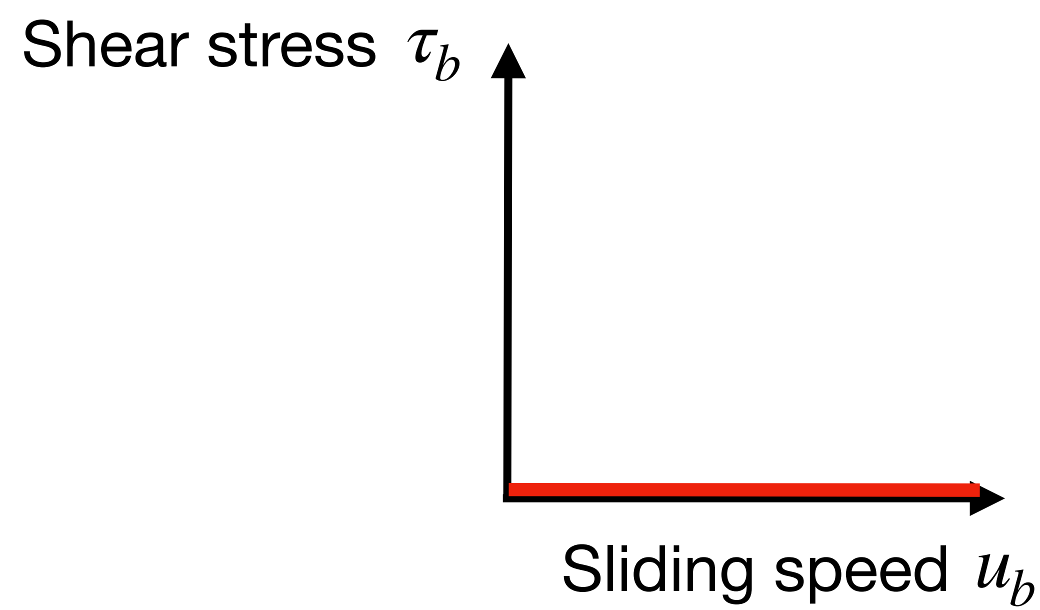
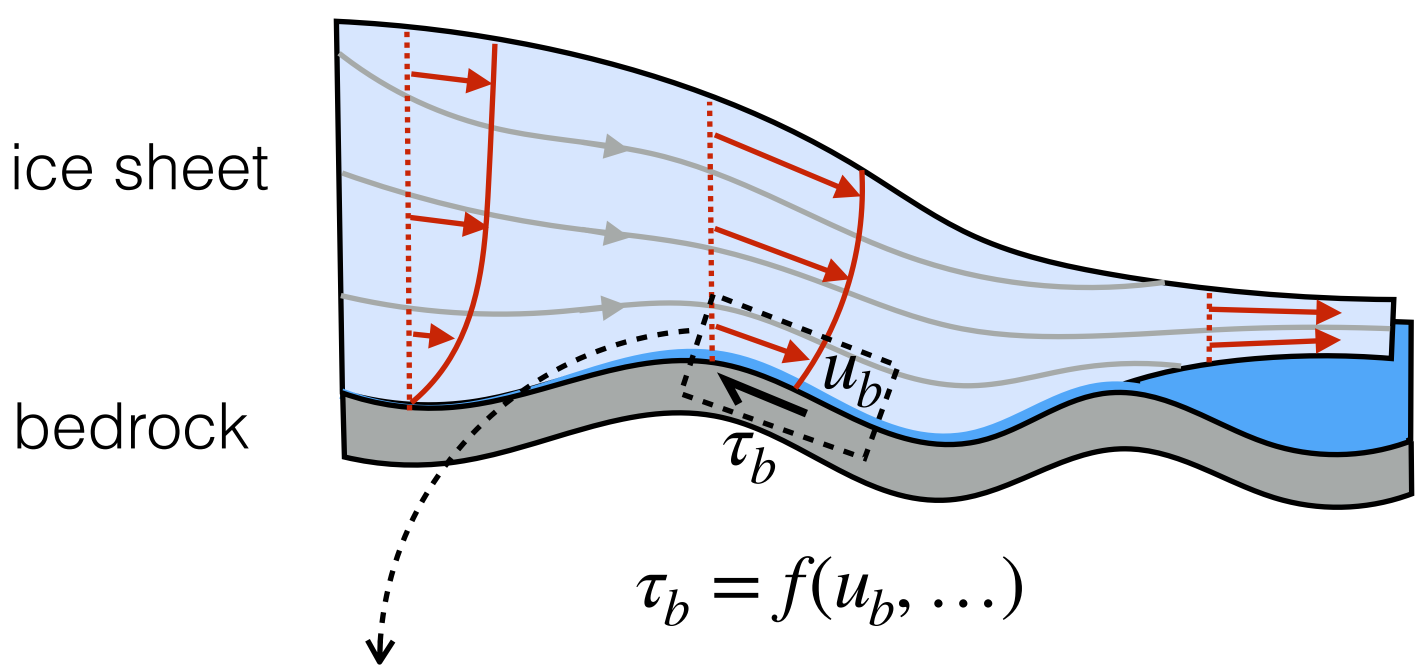
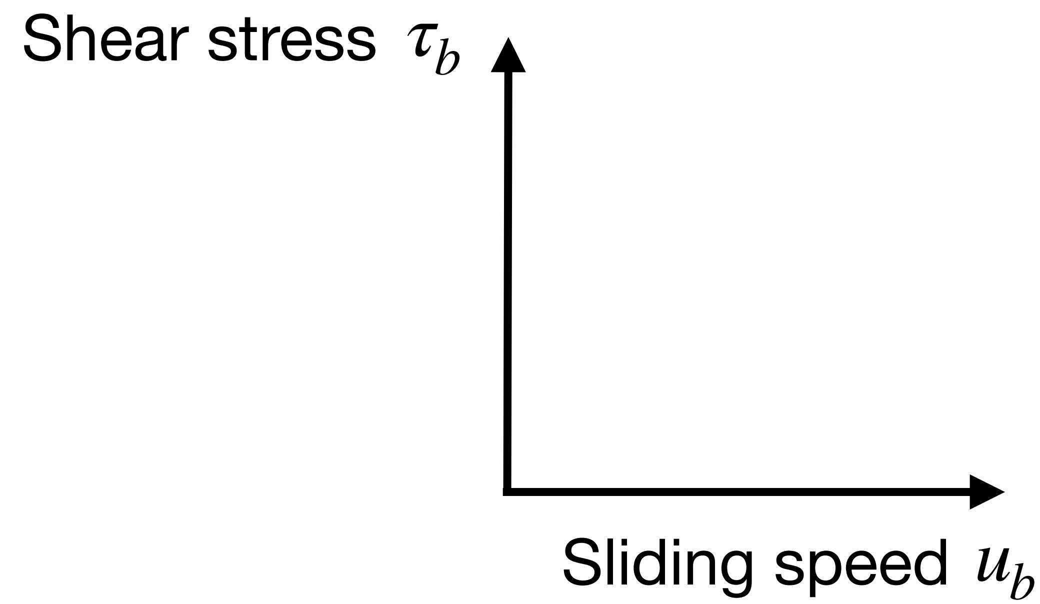
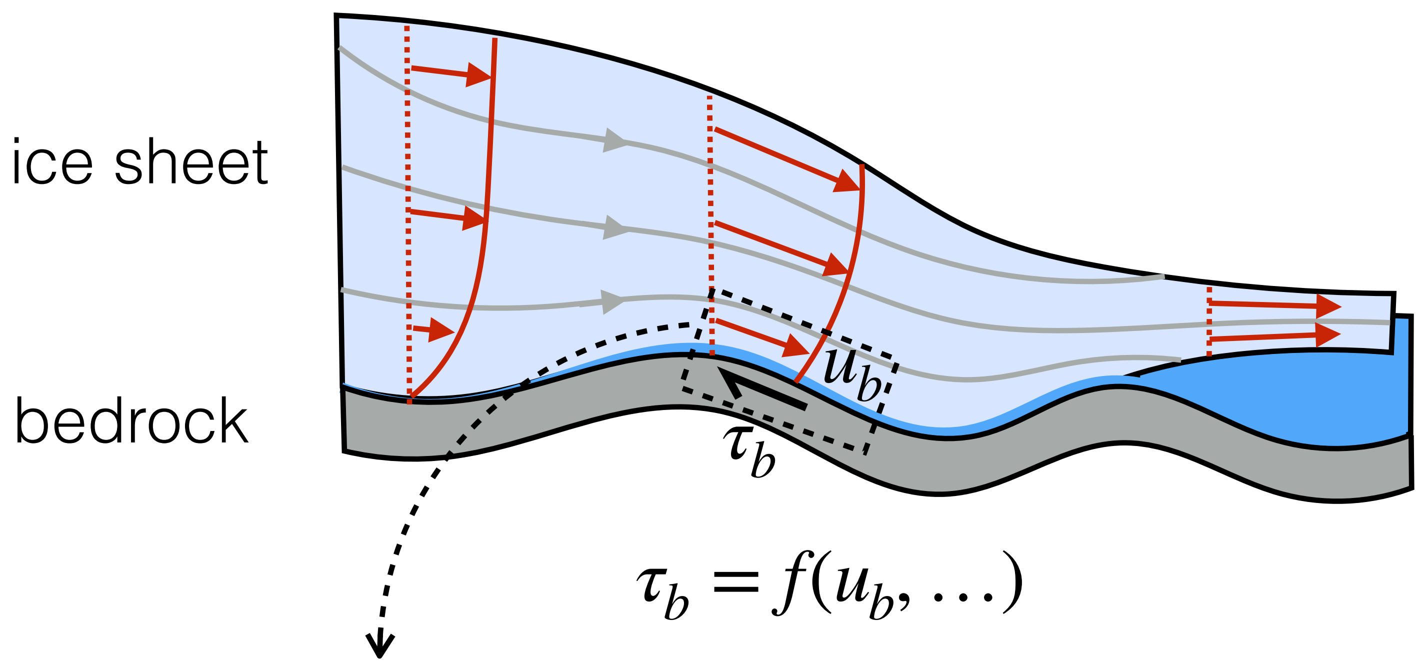


Friction law

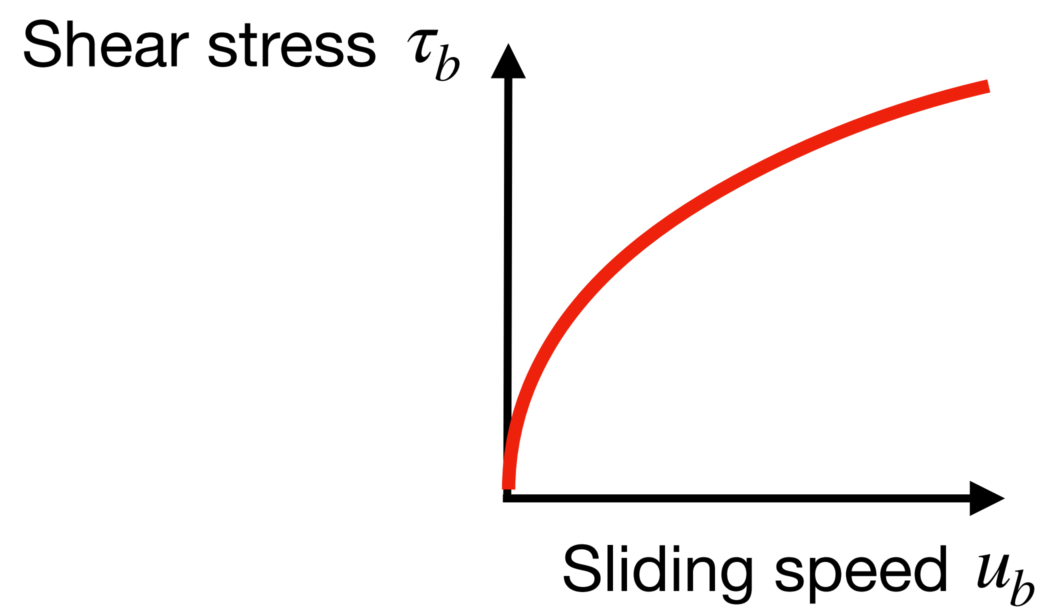


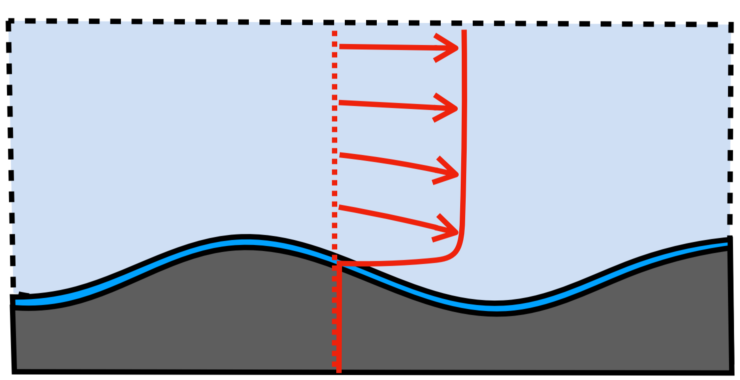


Friction law



$$\tau_b = f(u_b, \ldots)$$

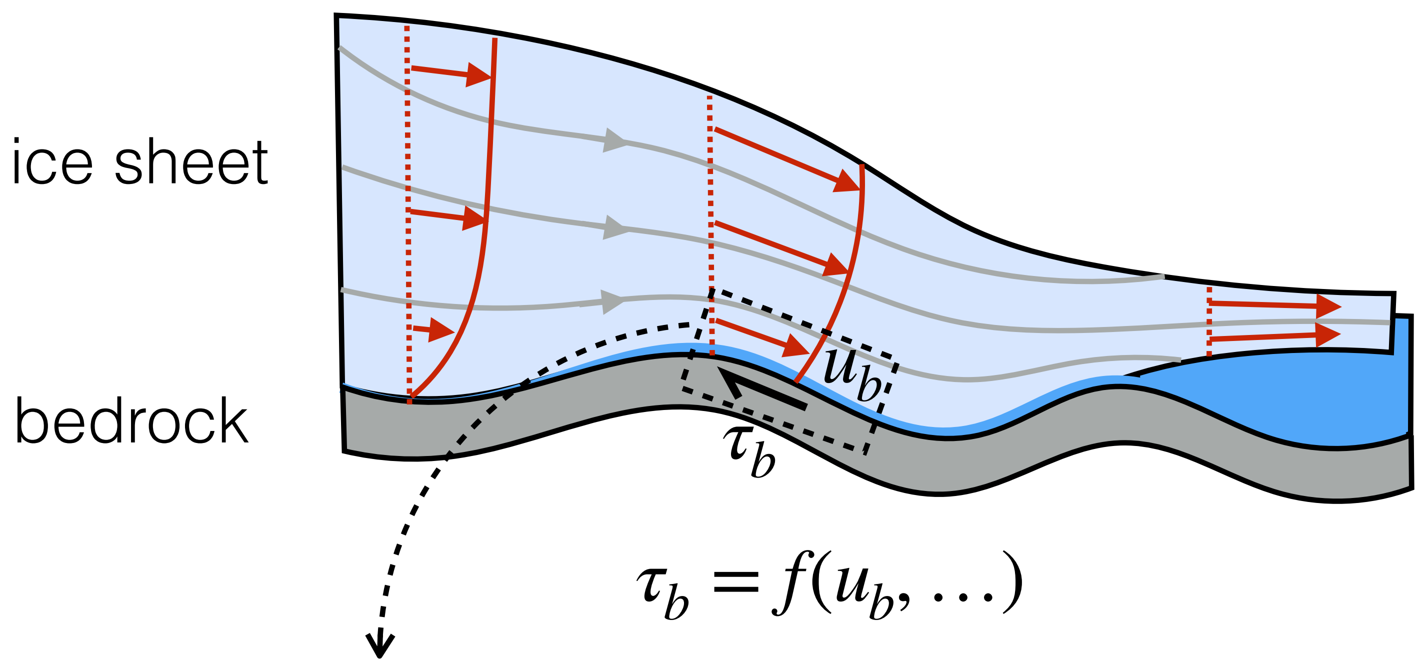




Hard-bed sliding

$$\tau_b = C u_b^m$$

Friction law



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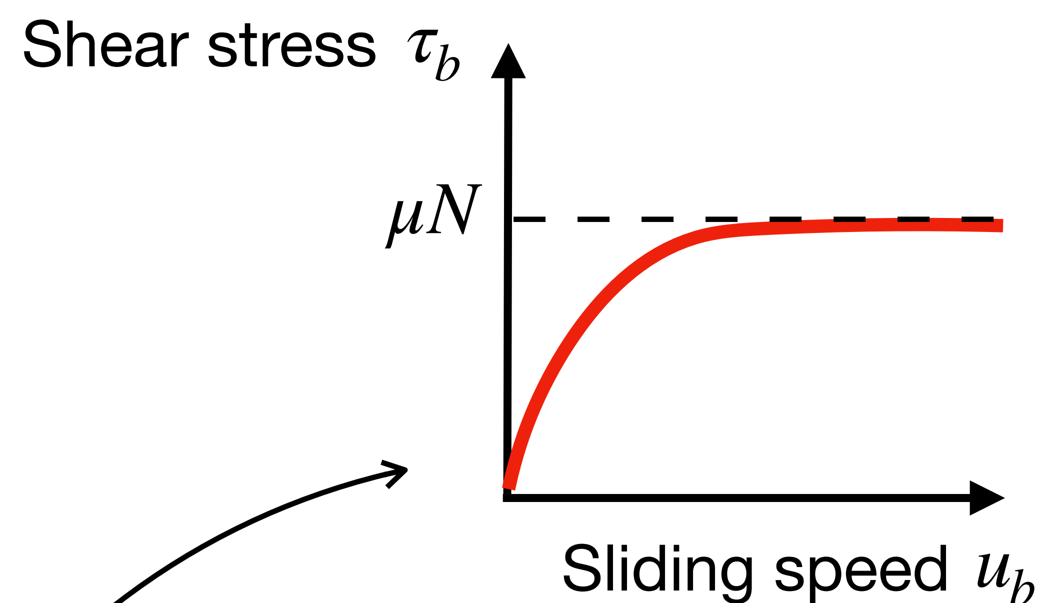

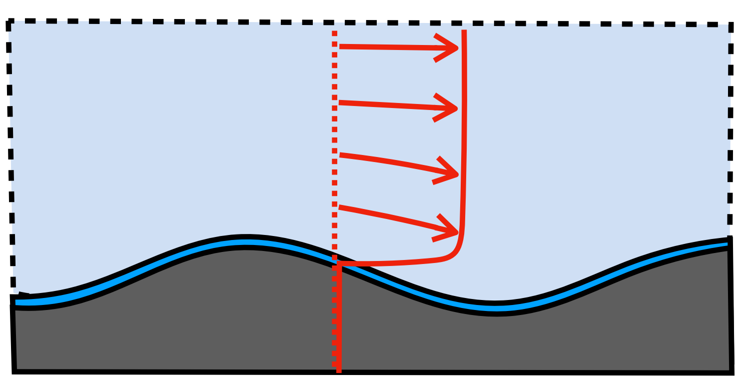

Hard-bed sliding

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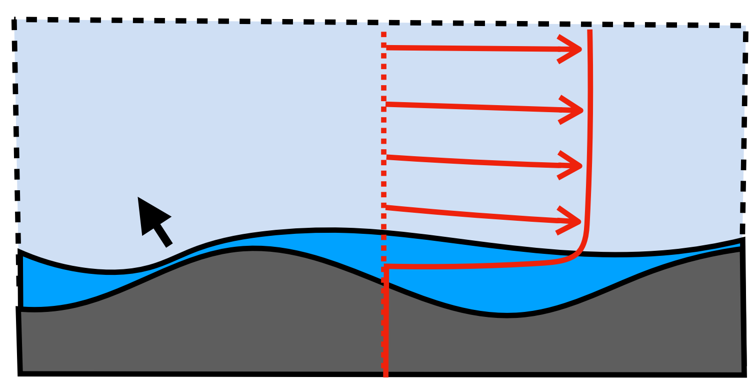
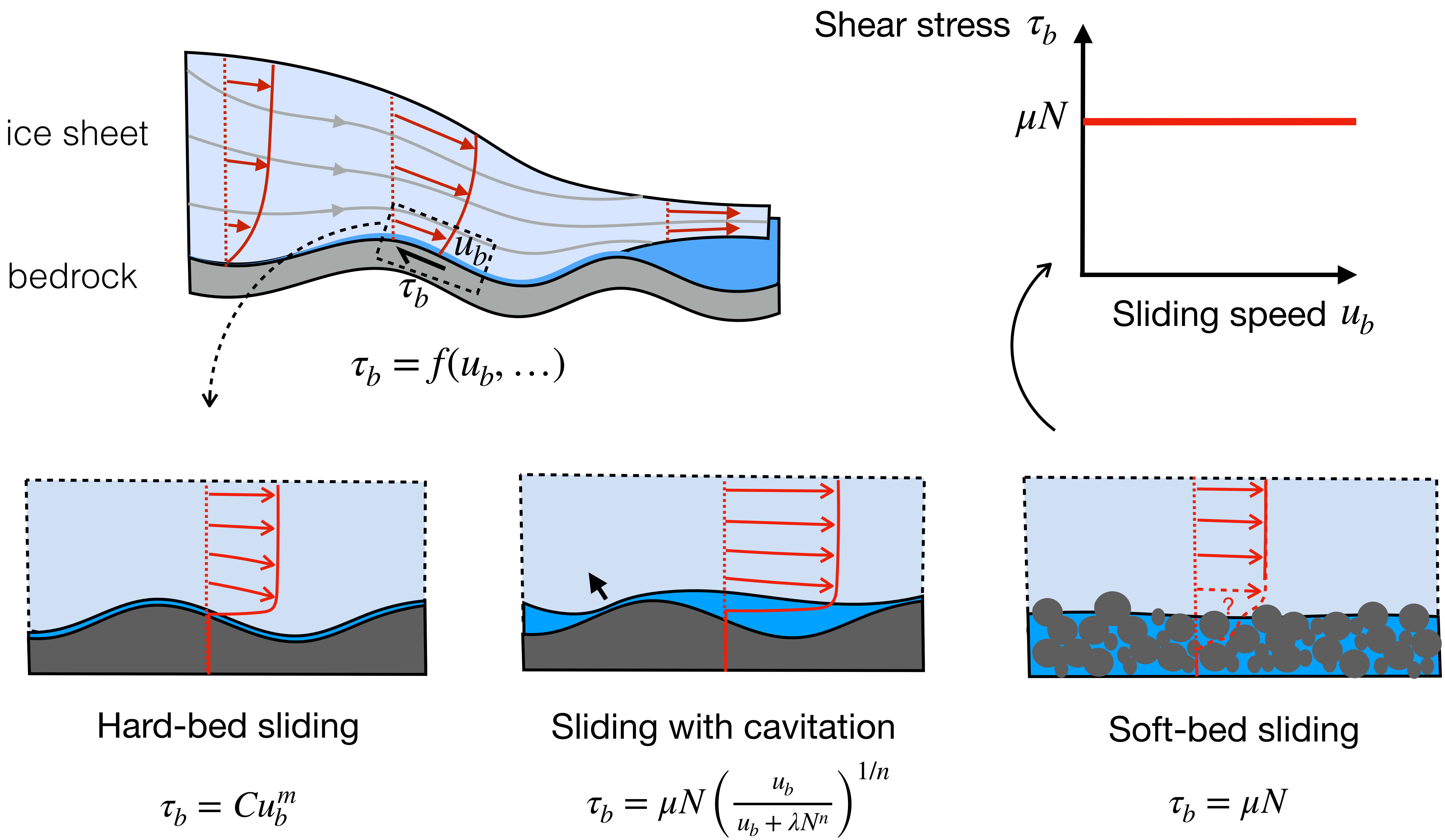


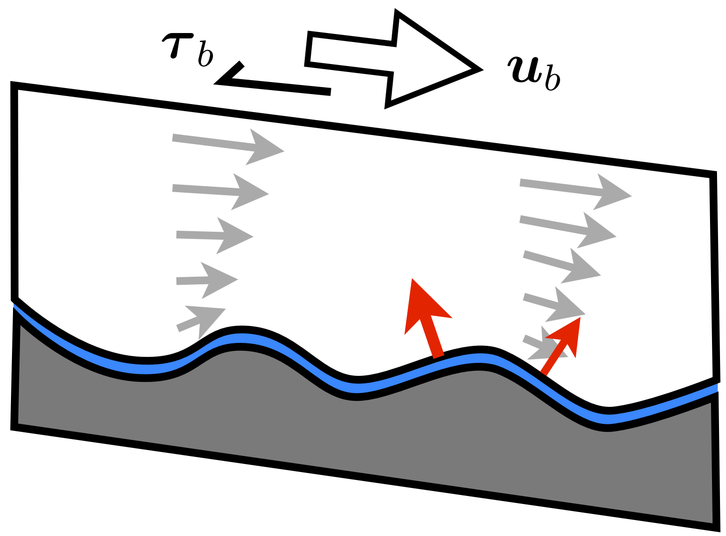
Sliding with cavitation

$$\tau_b = \mu N \left(\frac{u_b}{u_b + \lambda N^n} \right)^{1/n}$$

Friction law



Hard-bed sliding



A **film of water** exists between ice and the underlying bedrock (a few microns thick).

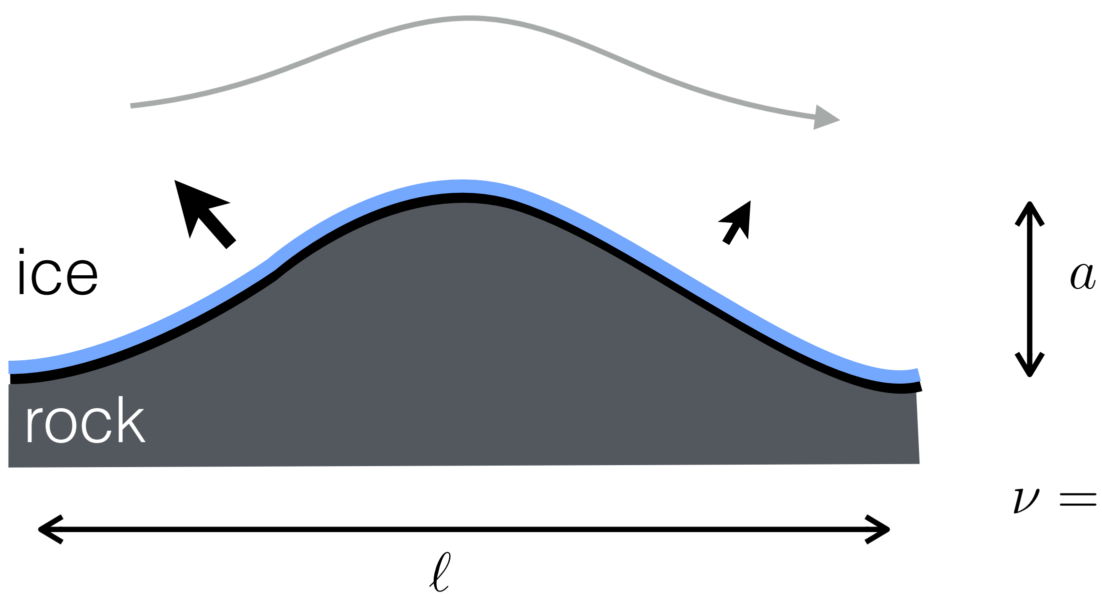
Microscopically, there is 'free slip'.

Macroscopic resistance comes from the **roughness** of the bedrock.

Flow over roughness occurs via **regelation** and **viscous (plastic) deformation**.

Viscous flow and regelation Weertman 1957

The ice deforms viscously around obstacles in the bed



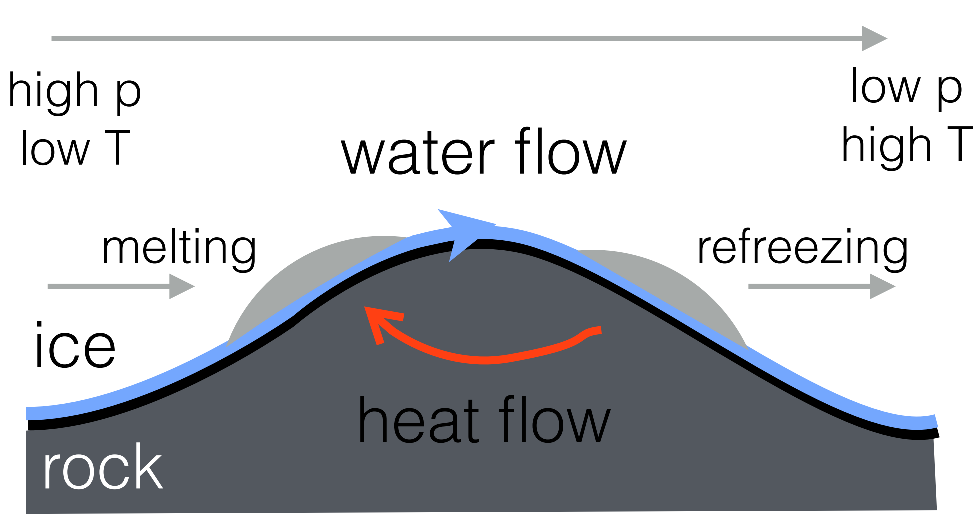
Dimensional analysis, using Glen's flow law

$$U_V \approx \left(\frac{aA}{2^n}\right)\frac{\tau_b^n}{\nu^{2n}}$$

$$\nu = \frac{a}{\ell}$$ 'roughness'

Regelation: pressure difference across obstacles causes a temperature difference

- results in upstream melting and downstream freezing

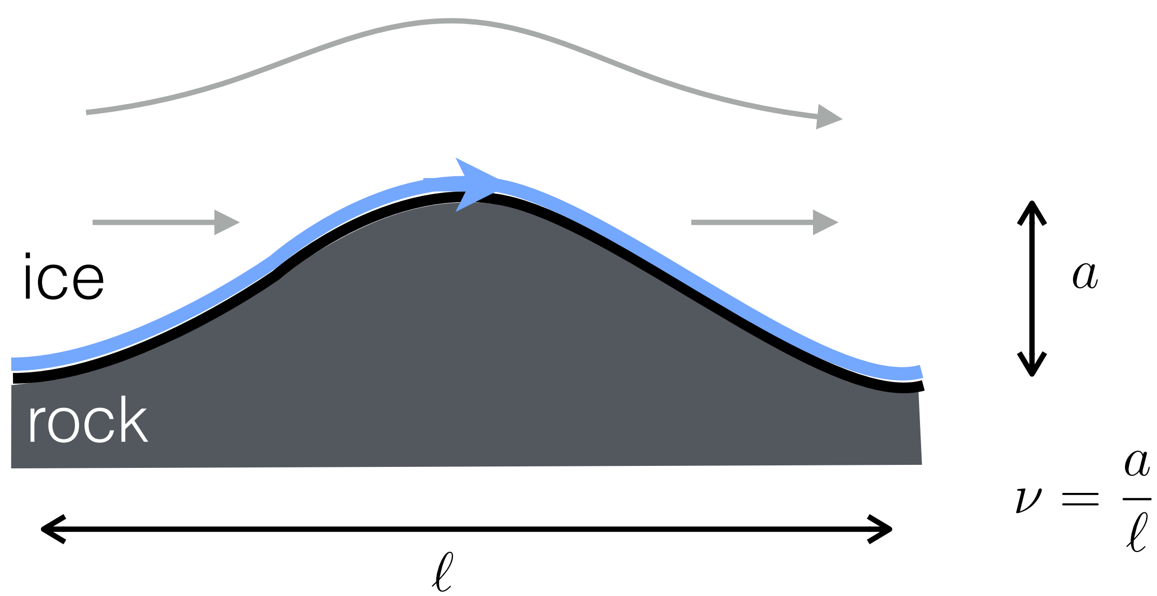


Balance of conductive / latent heat flow

$$U_R = \left(\frac{k\Gamma}{\rho_i L a}\right)\frac{\tau_b}{\nu^2}$$

Viscous flow and regelation Weertman 1957

Combining these two mechanisms:



$$U_V \approx \left(\frac{aA}{2^n}\right)\frac{\tau_b^n}{\nu^{2n}}$$ effective for LARGE bumps

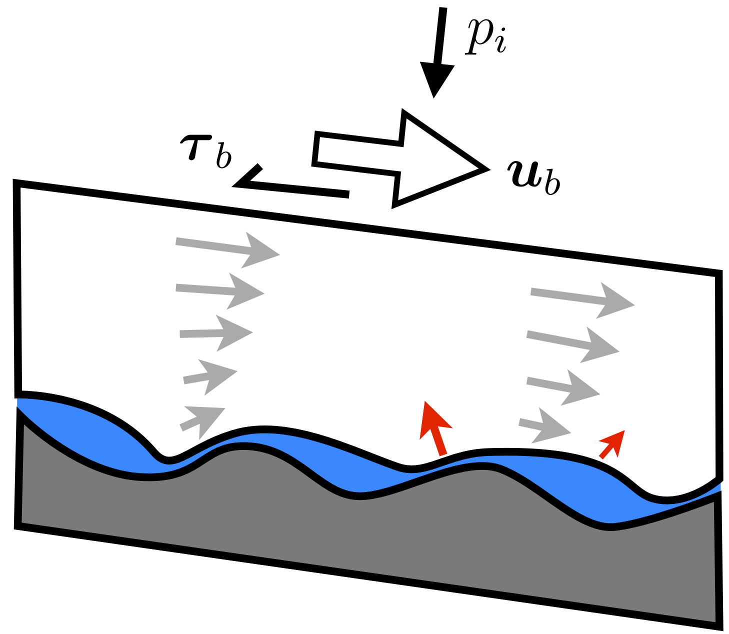
$$U_R = \left(\frac{k\Gamma}{\rho_i L a}\right)\frac{\tau_b}{\nu^2}$$ effective for SMALL bumps

There is a '**controlling obstacle size**' for which stress / speed cross over: $a \propto U_b^{-(n-1)/(n+1)}$

⇨ 'Weertman' sliding law $$\tau_b = \nu^2 \; R \; U_b^{2/(n+1)}$$

$$R = \left(\frac{\rho_i L}{2k\Gamma A}\right)^{1/(n+1)}$$

Sliding with cavitation Lliboutry 1968, Iken 1981, 1983



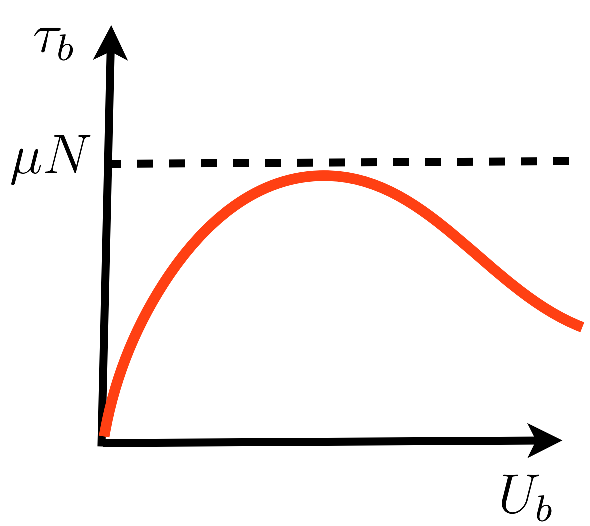
Cavitation occurs when pressure on downstream face of bumps reduces to critical level p_c

For steady-state cavities, friction law becomes dependent on **effective pressure** $N = p_i - p_c$

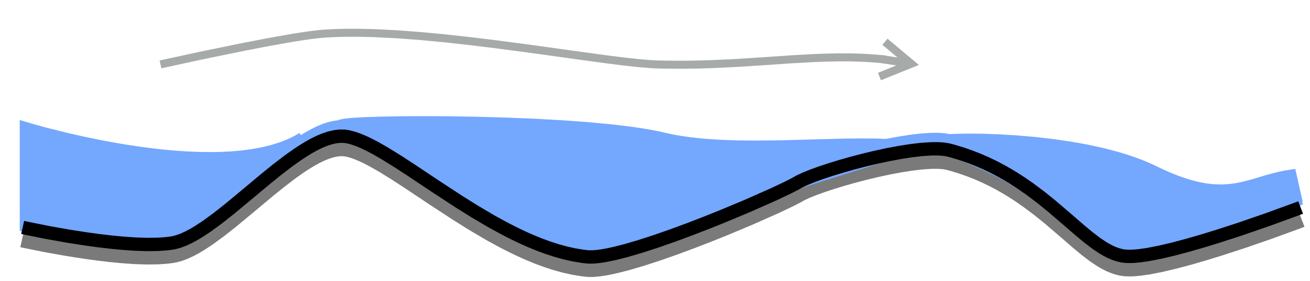
$$\Rightarrow \quad \tau_b = f(U_b, N)$$

p_i (macroscopic) ice normal stress

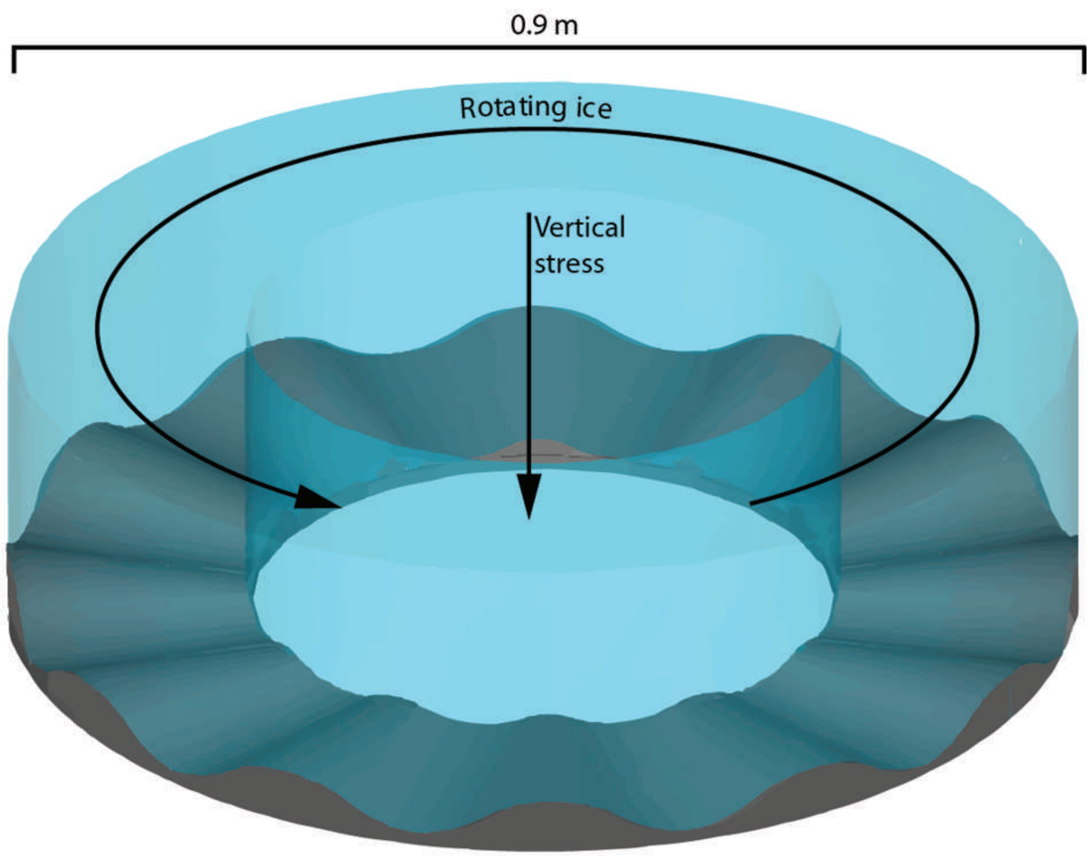
Iken suggested there should be a **maximum shear stress**

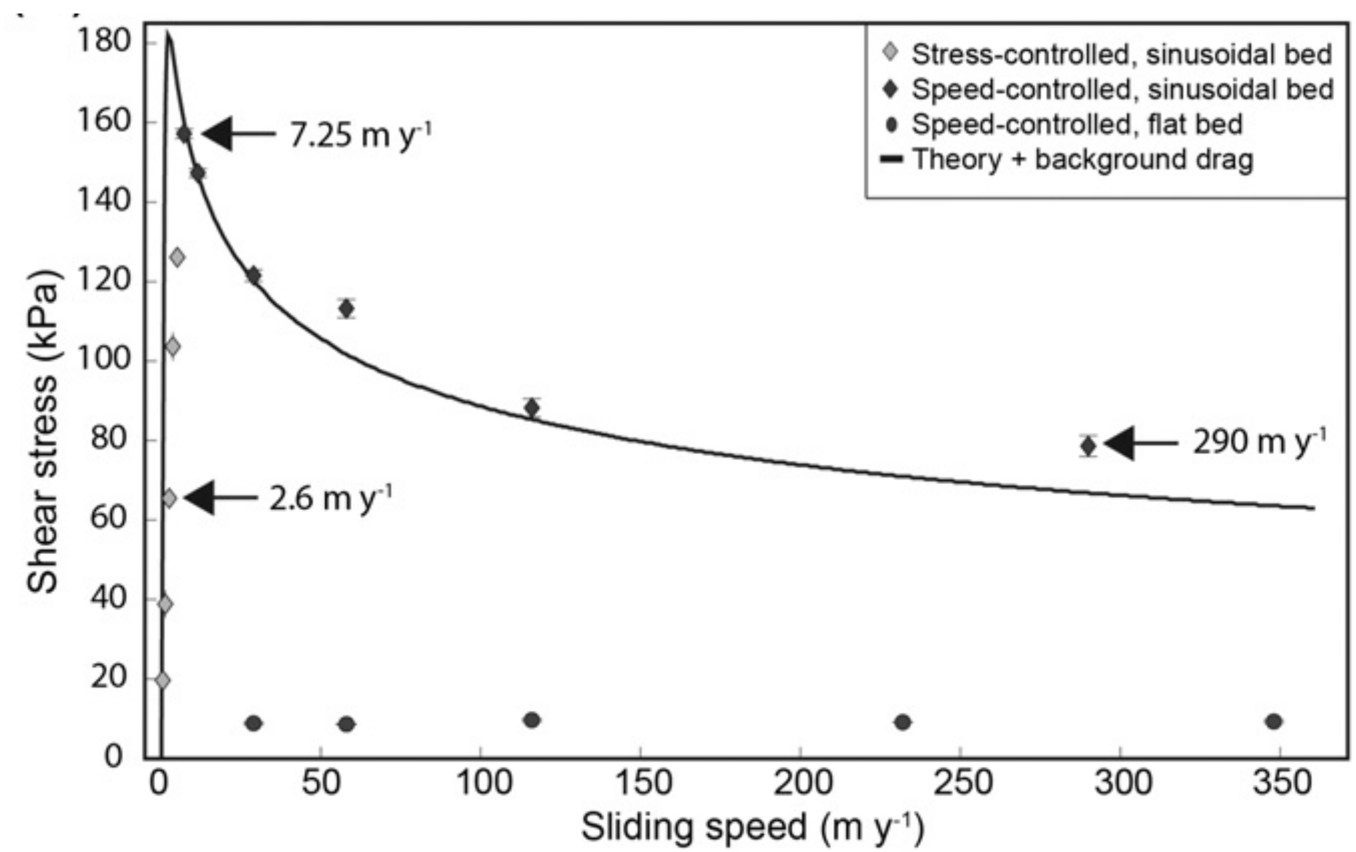


associated with cavities 'drowning' the bed roughness.



Laboratory experiments



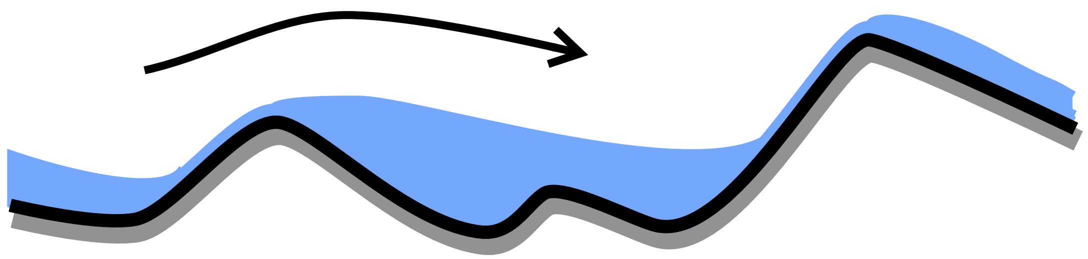


Iverson & Zoet 2015

Sliding with cavitation

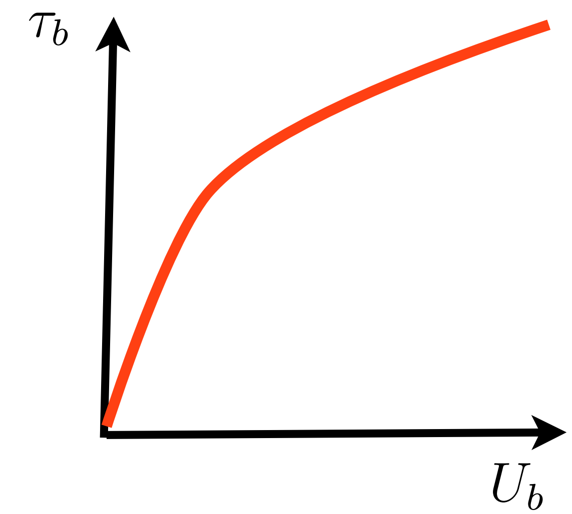
Budd et al 1979, Fowler 1986, Schoof 2005, Gagliardini et al 2007, Helanow et al 2019

Fowler suggested cavities never really ‘drown’ bed - stress is just transferred to larger bumps



⇨ ‘Generalized Weertman’ law / Budd law

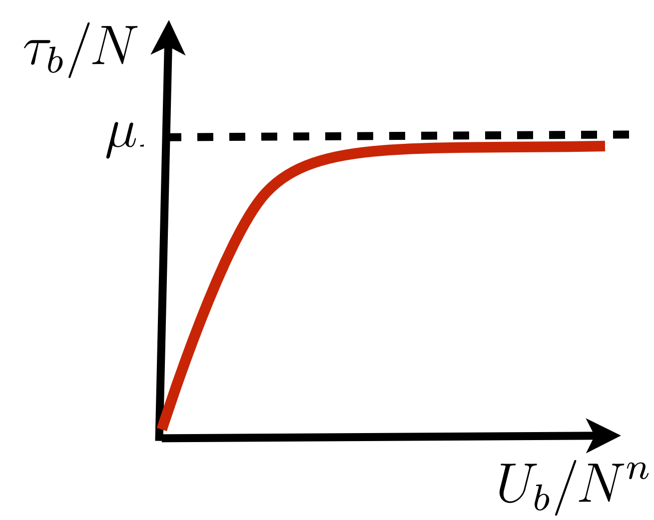
$$\tau_b = C U_b^p N^q$$



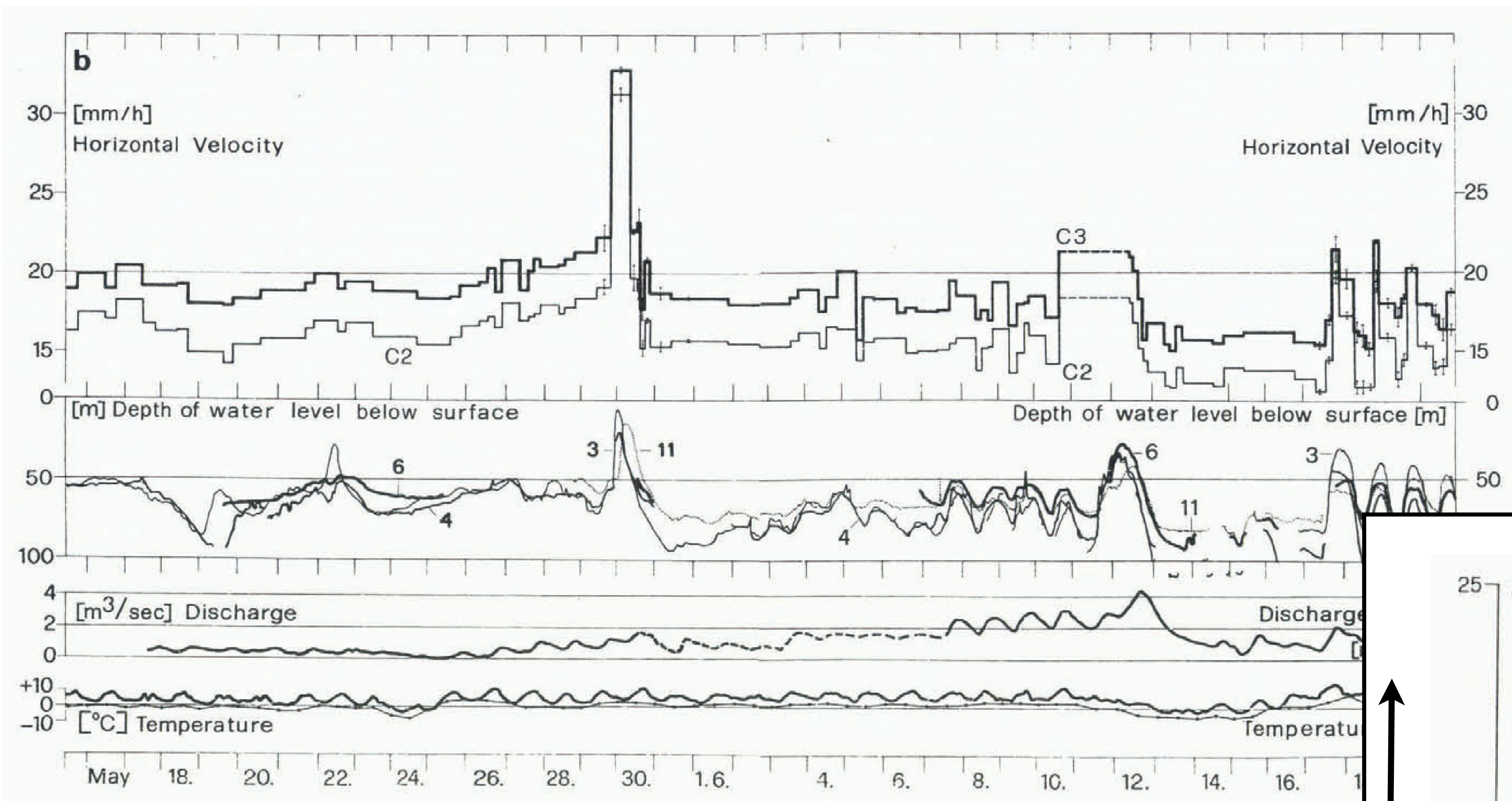
Schoof & Gagliardini et al. suggested an alternative with a maximum shear stress

⇨ ‘Regularised Coulomb’ law

$$\frac{\tau_b}{N} = \mu \left(\frac{U_b}{U_b + \lambda A N^n} \right)^{1/n}$$



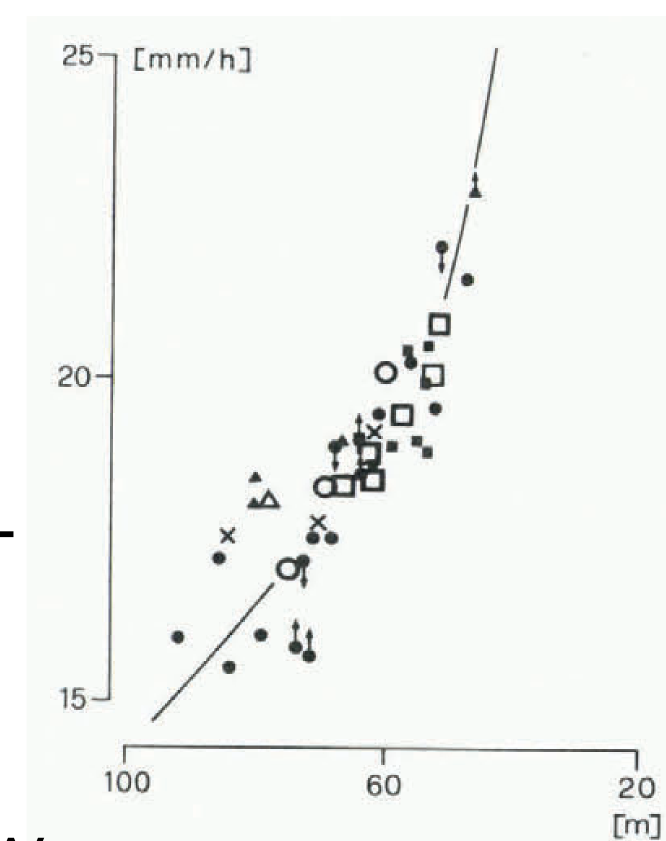
Field measurements



Speed

Water pressure

Ikem & Bindschadler 1986



Speed

Water pressure

Iken & Bindschadler 1986

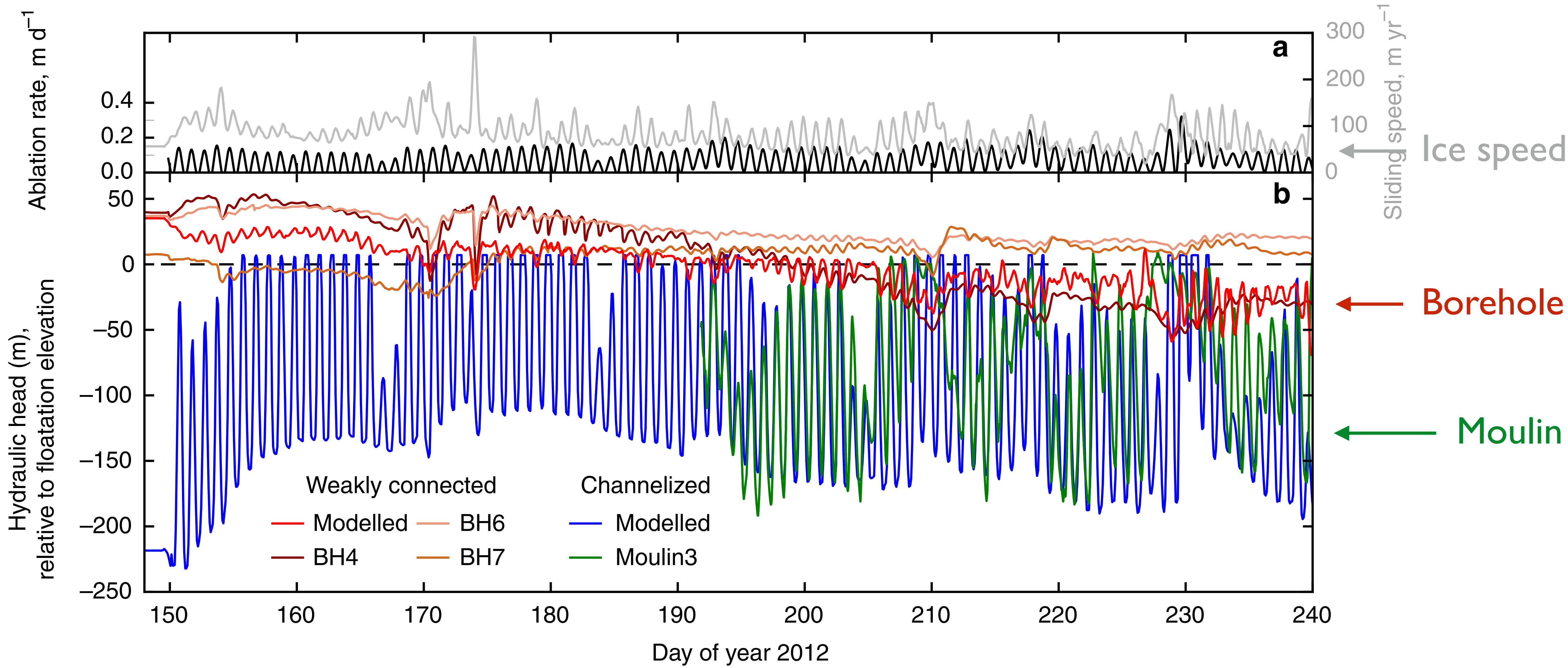
Some measurements show a robust relationship between ice speed and borehole water pressure eg. Bindschadler 1983, Iken & Bindschadler 1986

However, a consistent relationship is **not** always observed

eg. Sugiyama & Gudmundsson 2004, Harper et al 2007, Howat et al 2008, Fudge et al 2009

Field measurements

Measurements from **west Greenland** suggest diurnal variations in ice velocity correlate with water pressure in moulins, but are out of phase with pressure in boreholes.



Hoffman et al 2016

Soft-bed sliding

Basal sediments

Many glaciers and ice sheets have **a layer of sediments (till)** at the bed

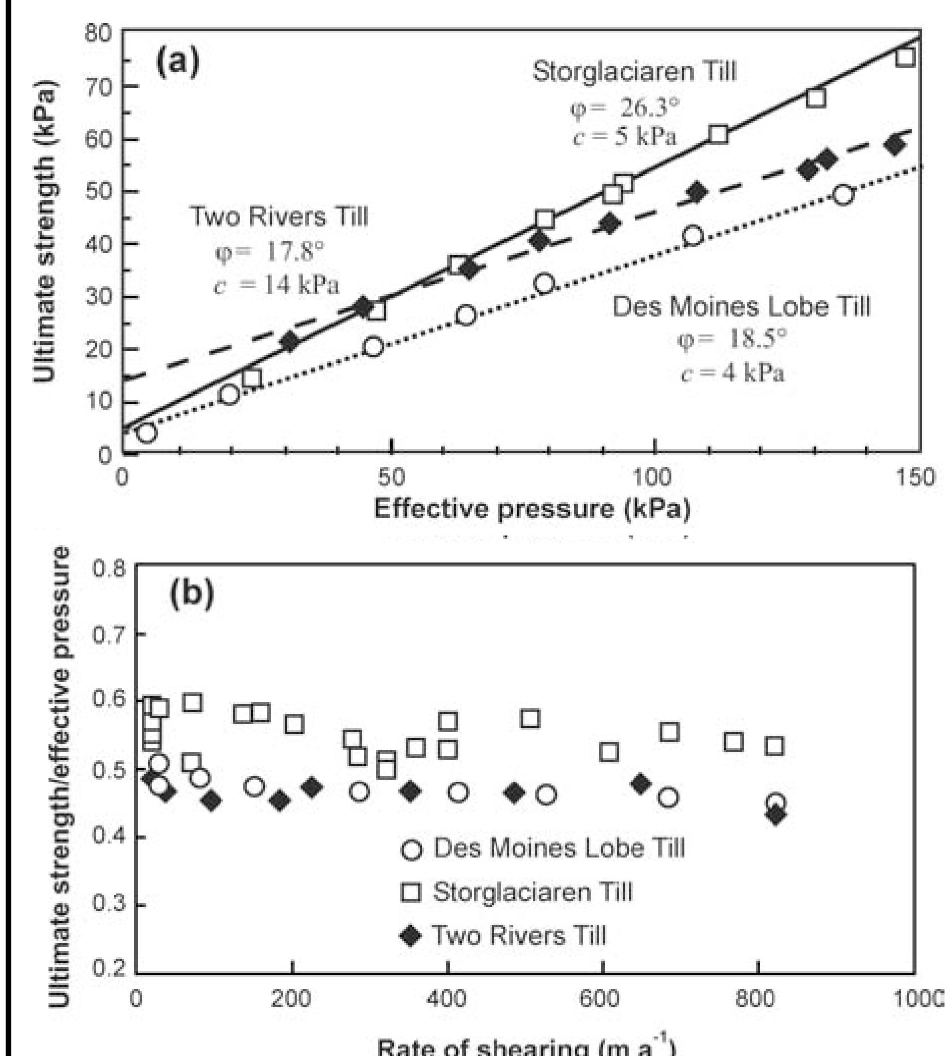
Sediments result from glacial erosion (eg. abrasion, plucking)

Laboratory experiments on till samples show that till has a **yield stress** $\tau_f = \mu \sigma_e$

Friction coefficient $\mu \approx 0.4$ Effective stress $\sigma_e = p - p_w$

Once yielded, **stress is almost independent of strain rate** ('perfect plasticity').

⇨ 'Coulomb' / 'plastic' friction law $\tau_b = \mu N$

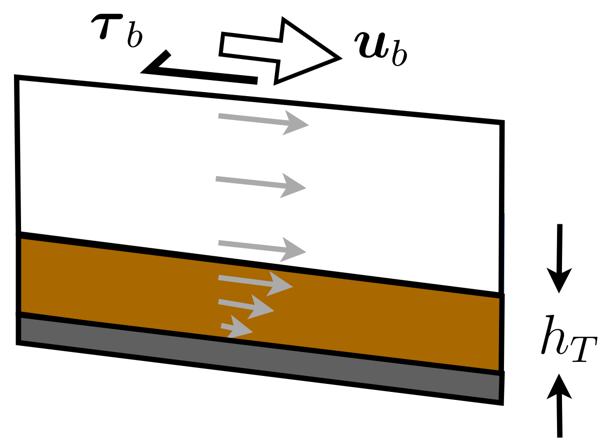


Iverson et al 1998, Clarke 2005

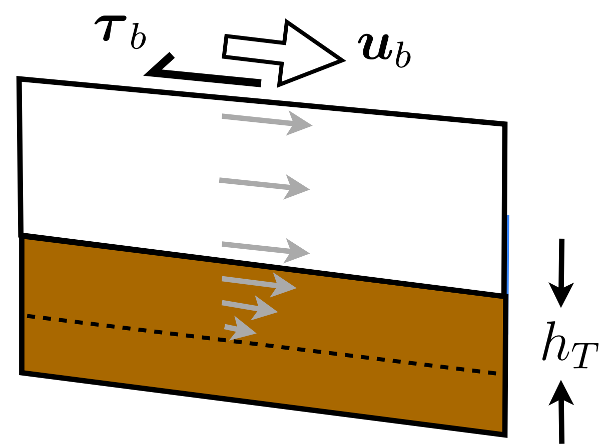
Sliding over sediments

'Sliding' could involve:

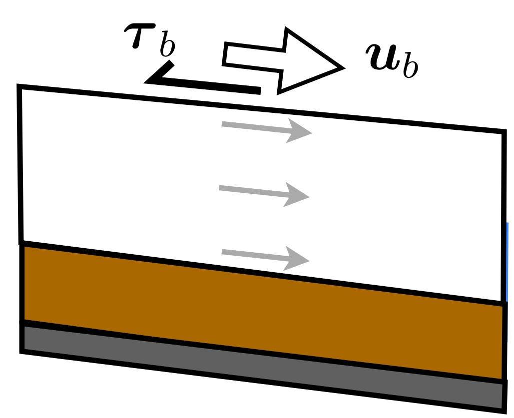
- Shear deformation of sediment layer



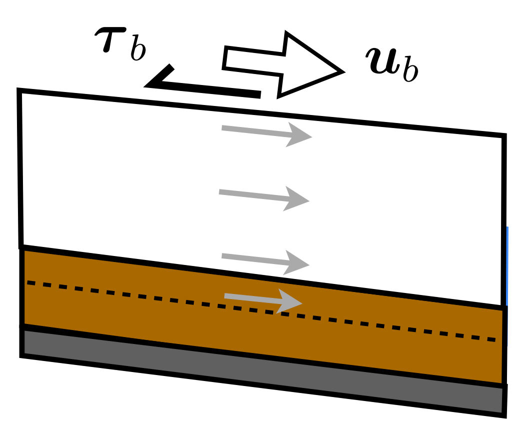
- Shear of a finite horizon of the sediment

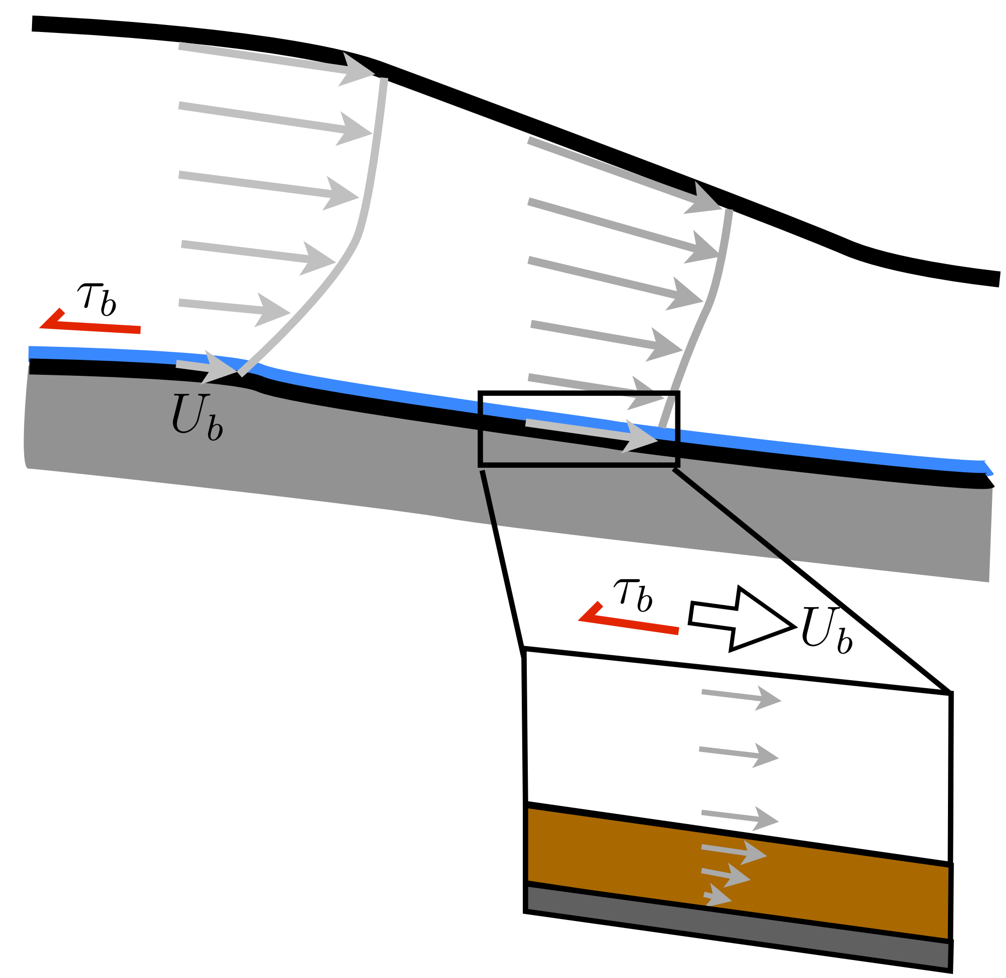


- Slip at the ice-till interface

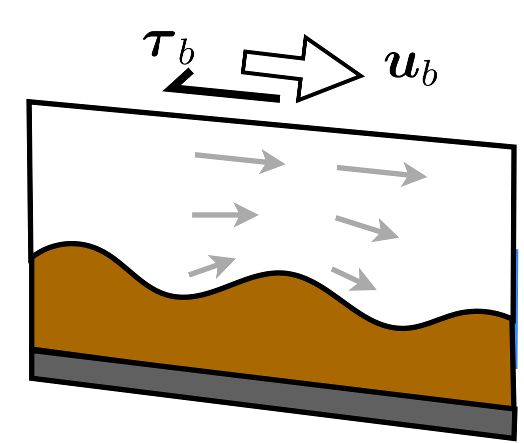


- Slip on slip-planes within the sediment layer

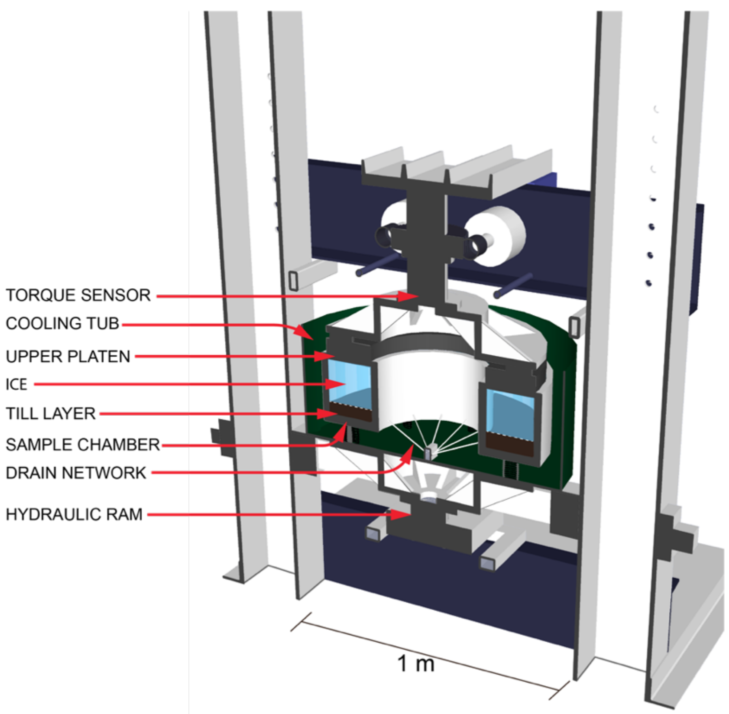




Macroscopic resistance may come from flow around sediment landforms

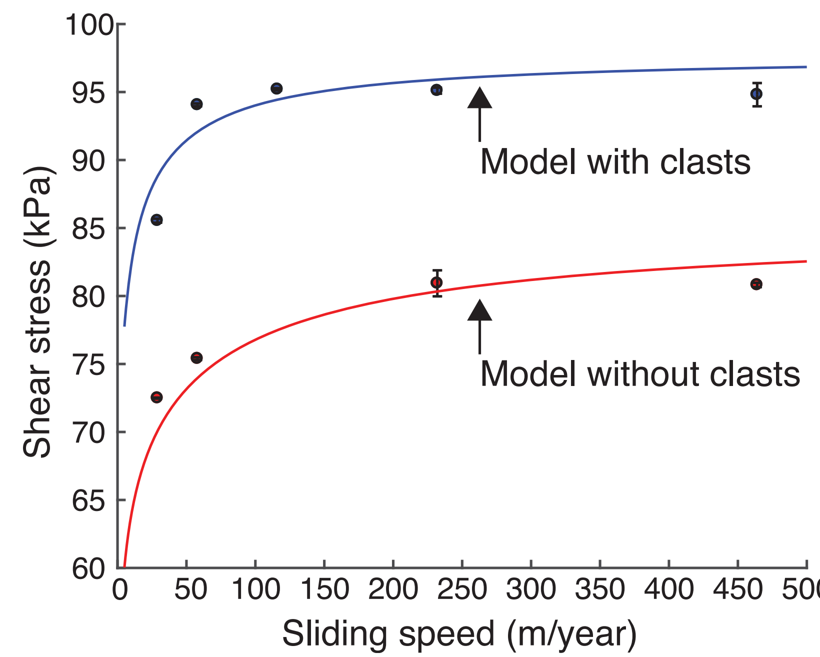


Laboratory experiments

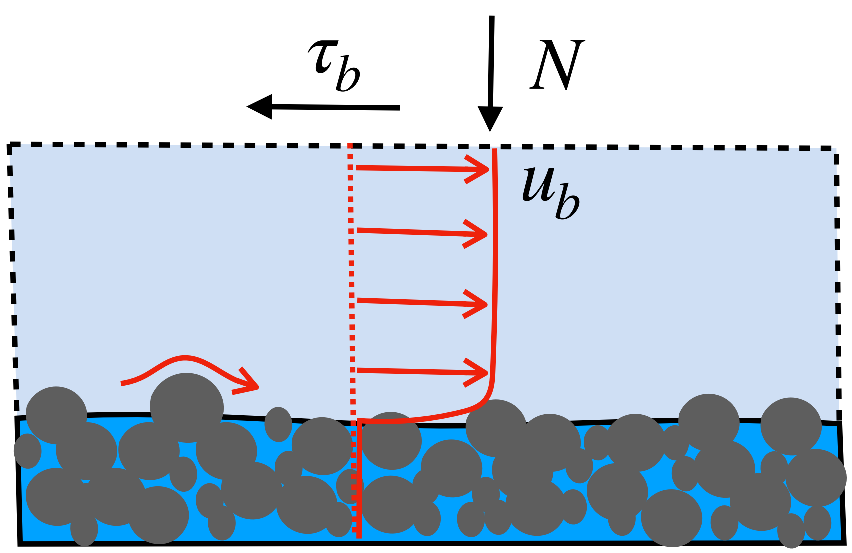


Zoet & Iverson 2020

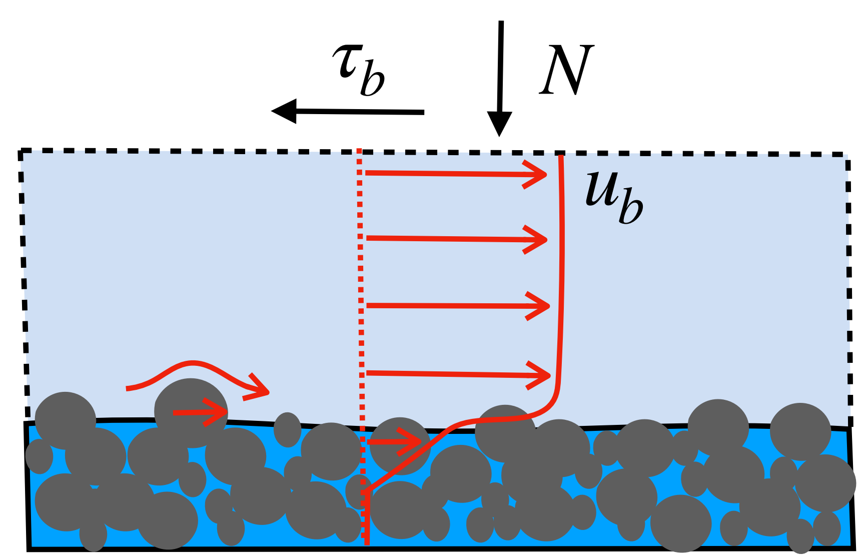
Shear stress τ_b



Sliding speed u_b



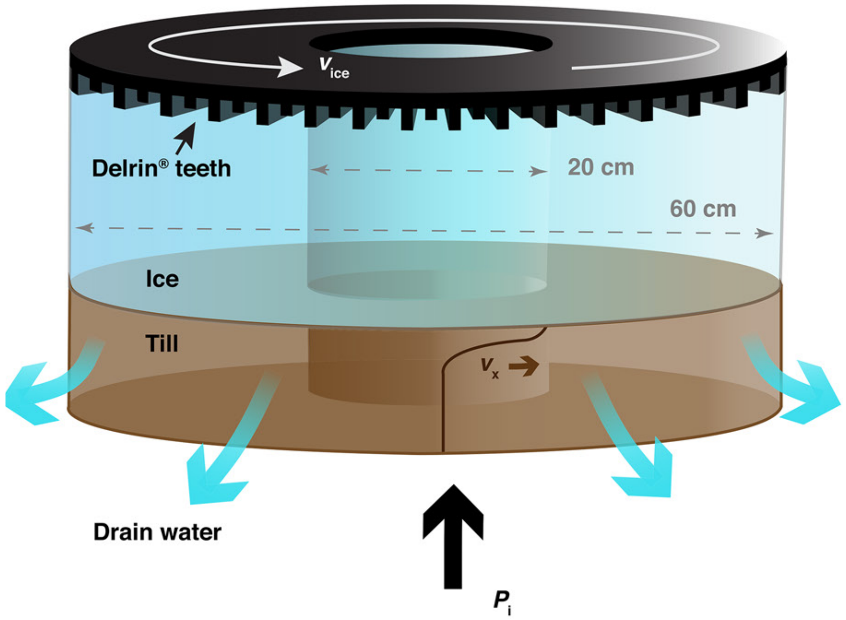
Unyielded till - slip at interface

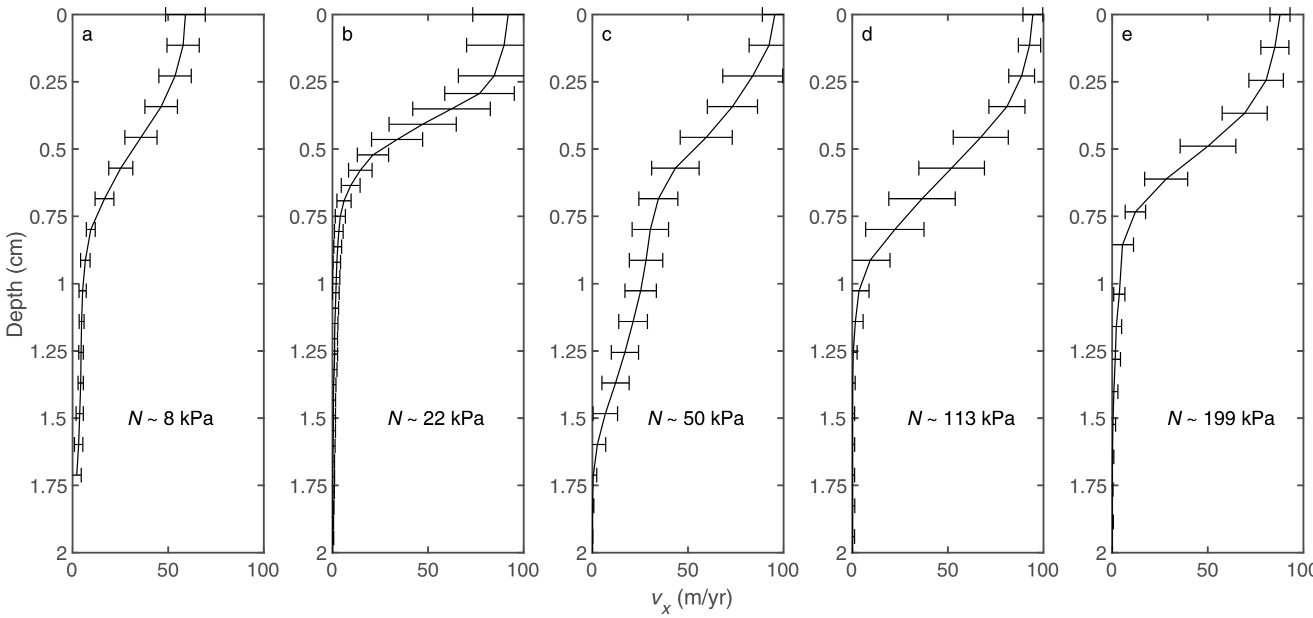


Yielded till - larger clasts plough through deforming till

Laboratory experiments

Laboratory ring shear experiments visualise till deformation, sediment flux, and ice-till slip



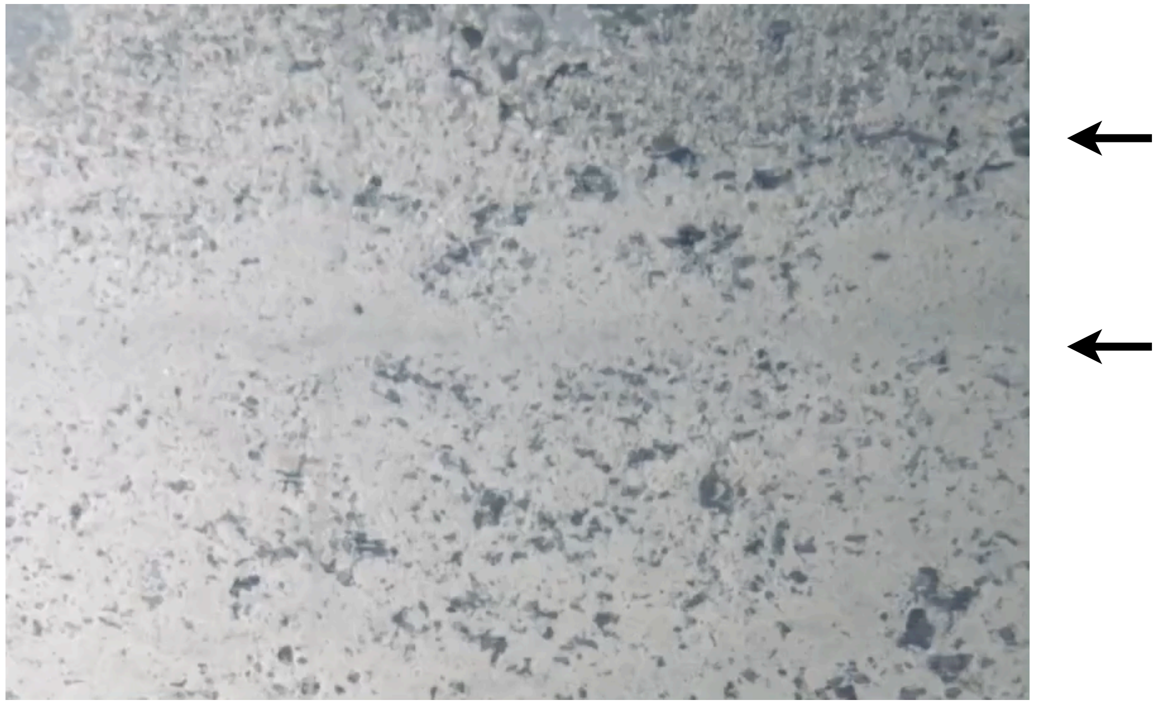


Hansen & Zoet 2022

Ice-till slip occurs at low effective pressure / low sliding speeds.

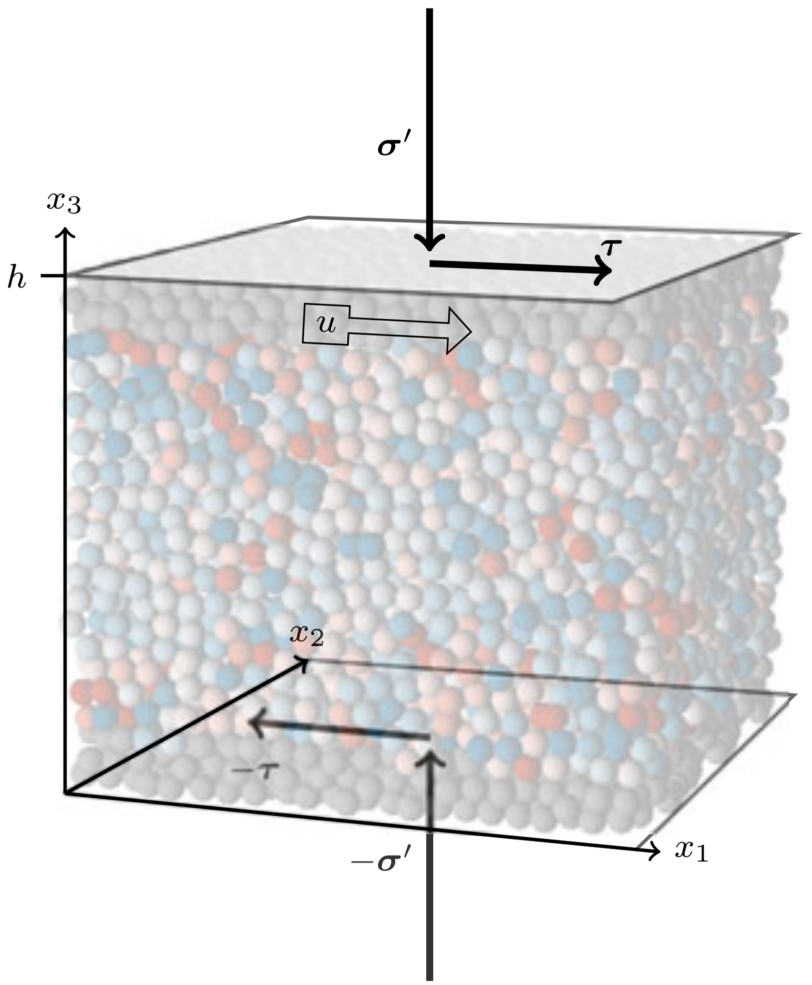
Depth of deformation increases then decreases with effective pressure.

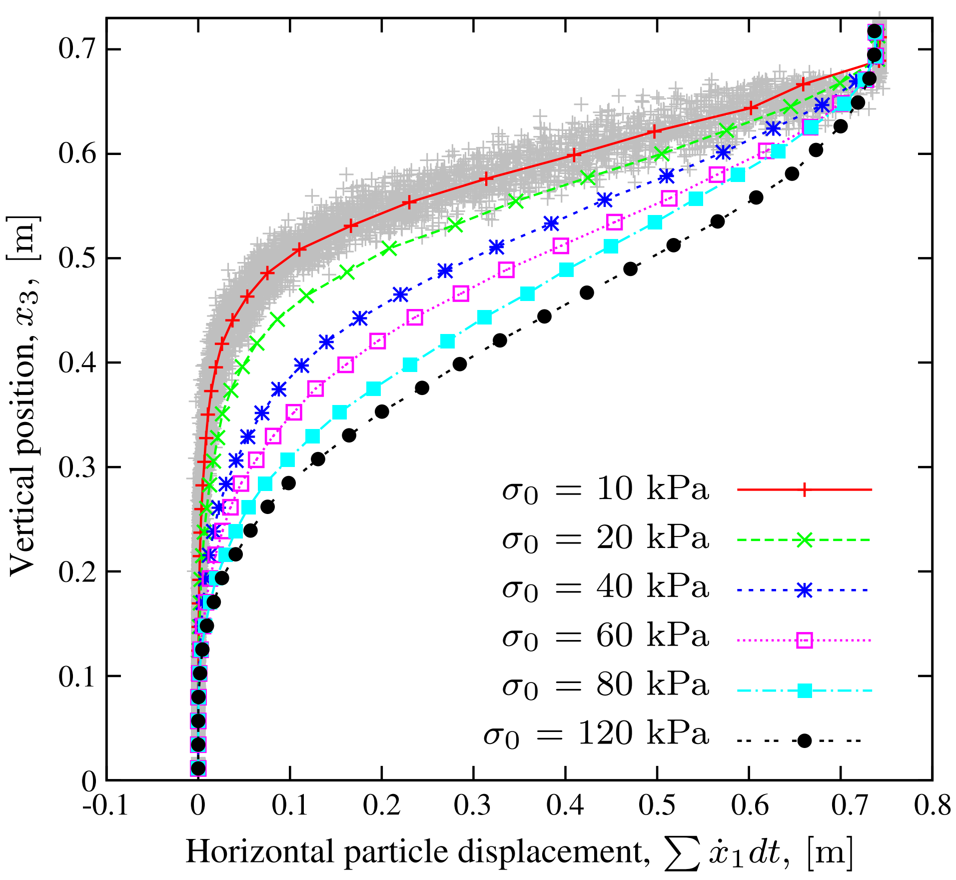
Sediment flux scales approximately **linearly with sliding speed**, and **non-monotonically** with effective pressure.



Computational experiments

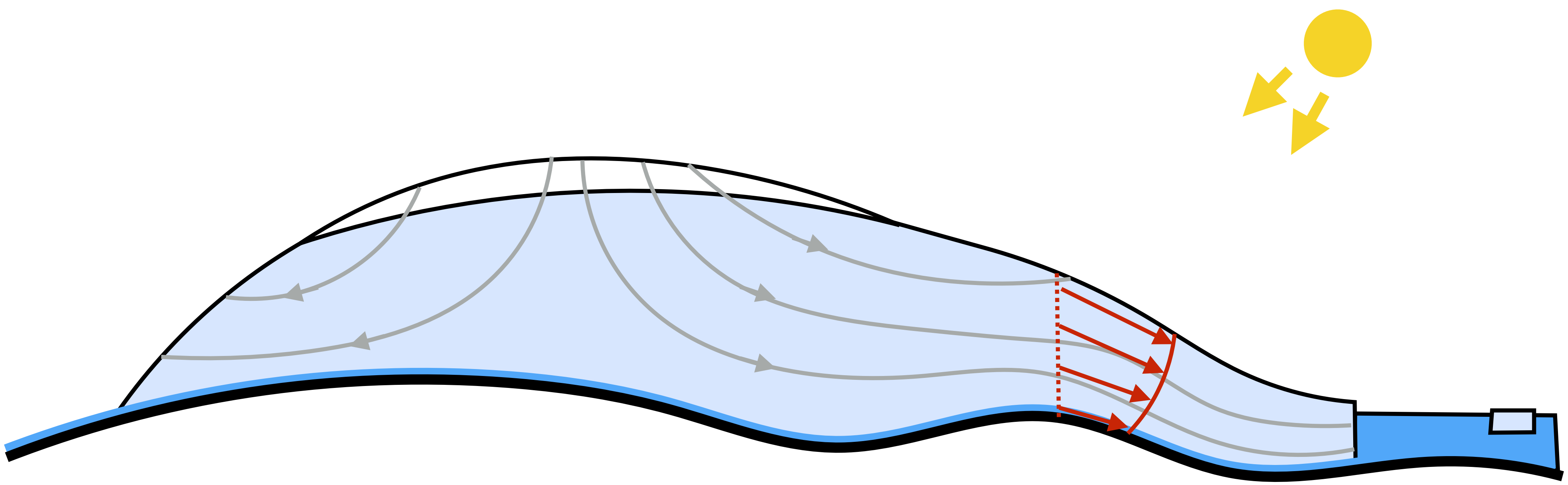
Discrete particle (DEM) experiments under imposed shearing velocity

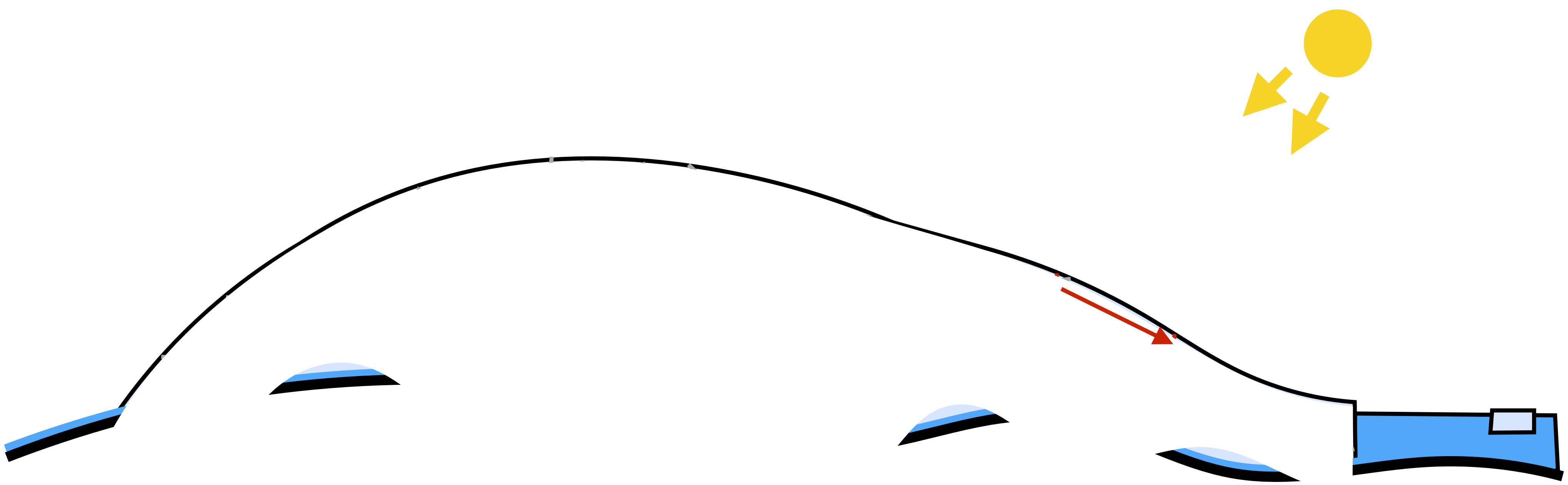




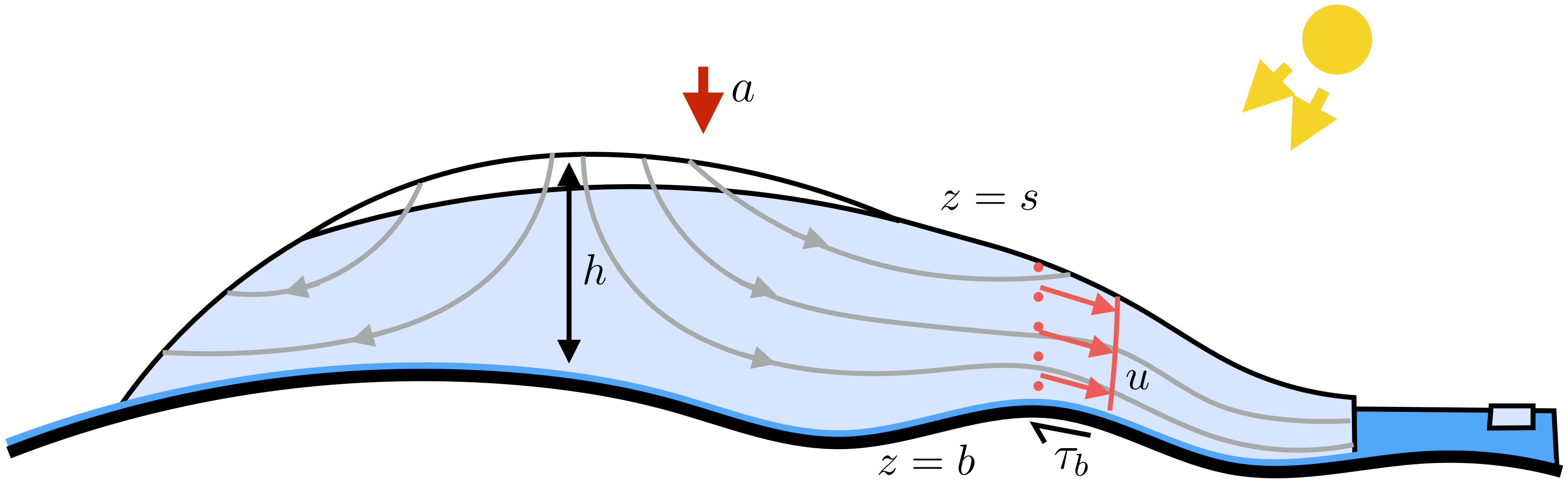
Damsgaard et al 2013

Ice-sheet modelling and basal inversions





Inverse methods



Forward model eg SSA

$$\frac{\partial h}{\partial t} + \nabla \cdot (h\mathbf{u}) = a$$

$$\mathbf{0} = -\rho g h \nabla s - C|\mathbf{u}|^{m-1}\mathbf{u} + \nabla \cdot (h\boldsymbol{T}) \qquad \boldsymbol{T} = \eta \begin{pmatrix} 4\frac{\partial u}{\partial x} + 2\frac{\partial v}{\partial y} & \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} & 2\frac{\partial u}{\partial x} + 4\frac{\partial v}{\partial y} \end{pmatrix}$$

Maps input parameters to outputs $\mathcal{F} : P \to Y$

Running the model gives $\mathbf{y} = \mathcal{F}(\mathbf{p})$ which we can compare with observations $\mathbf{y}_{\mathrm{obs}}$

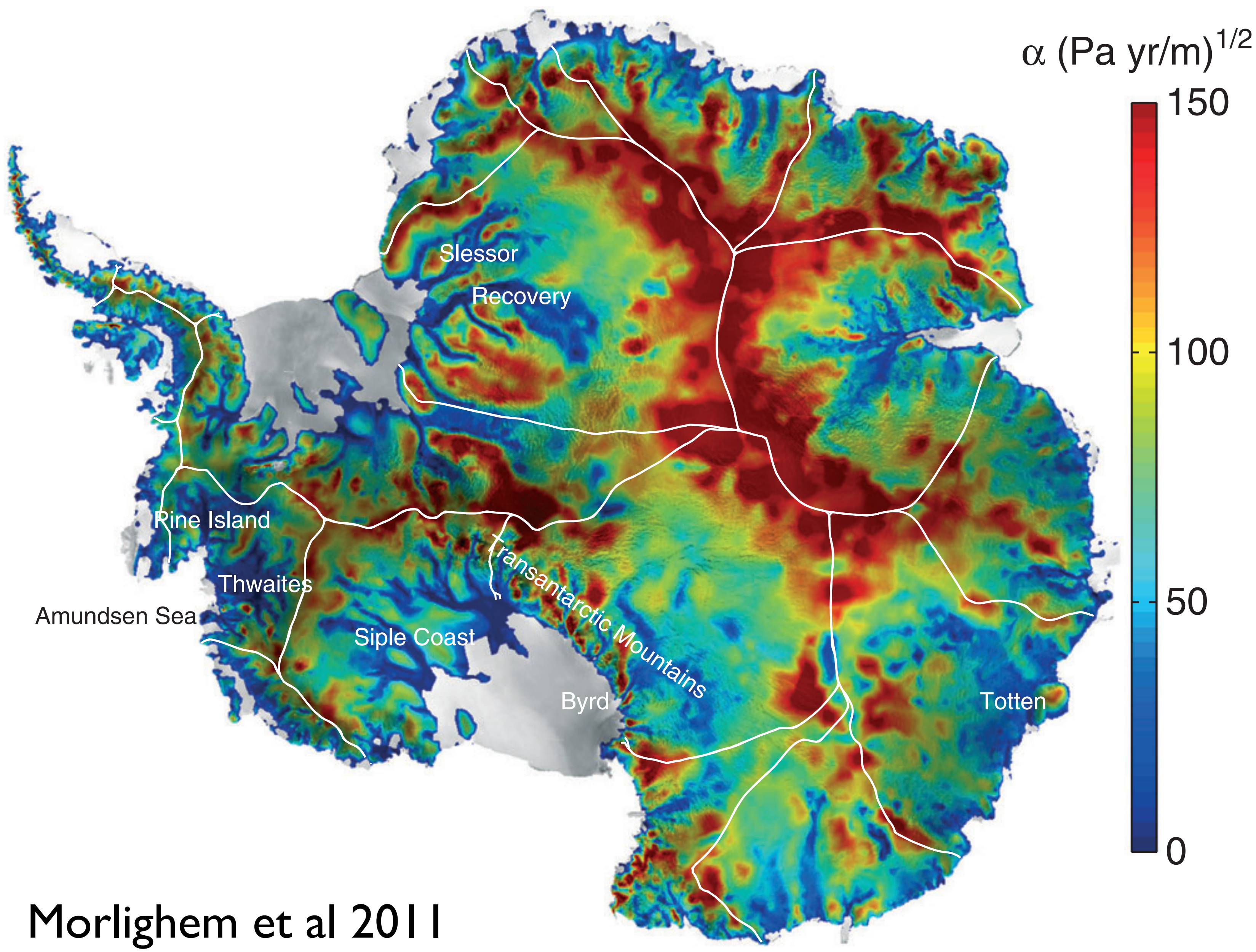
Inverse methods used to find input parameters that best fit observations

(or to find a 'posterior' probability distribution)

Minimise a cost function

$$\mathcal{J}(\mathbf{p}) = \frac{1}{2}\int_{\Omega} |\mathbf{y} - \mathbf{y}_{\mathrm{obs}}|^2 \; \mathrm{d}S + \mathcal{R}(\mathbf{p})$$

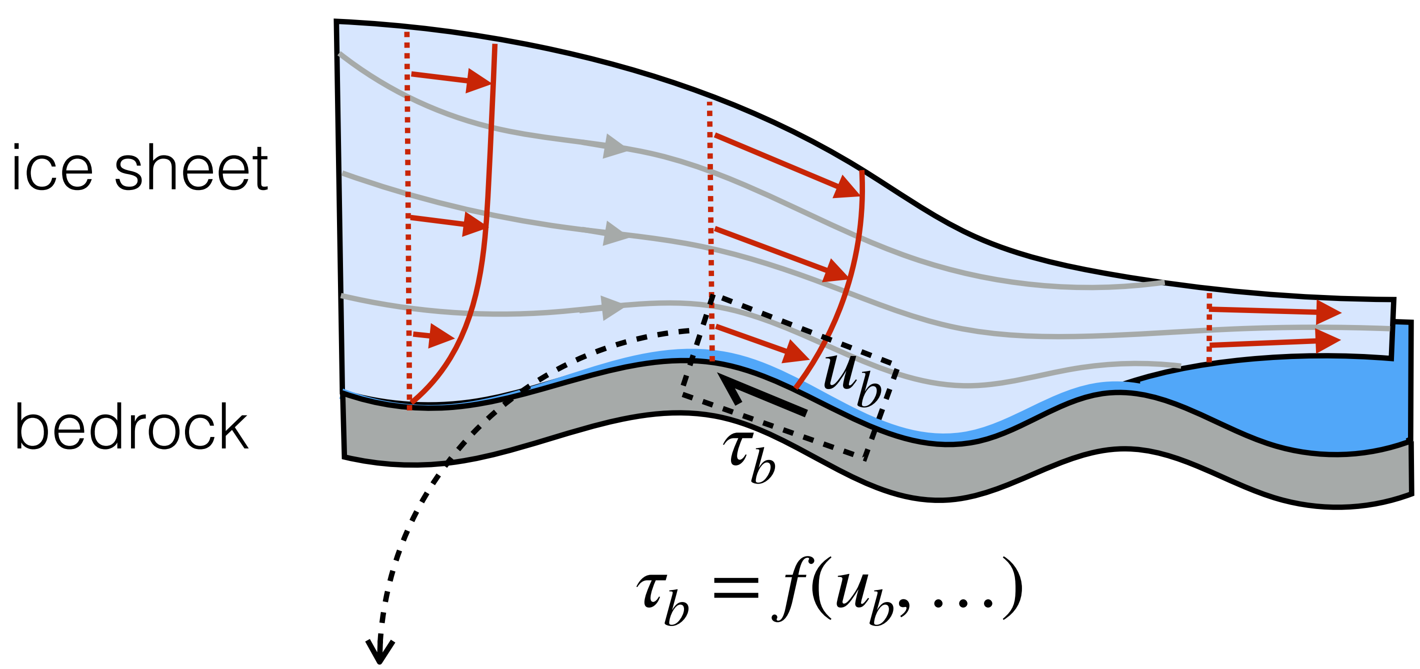
Inferred basal friction coefficient



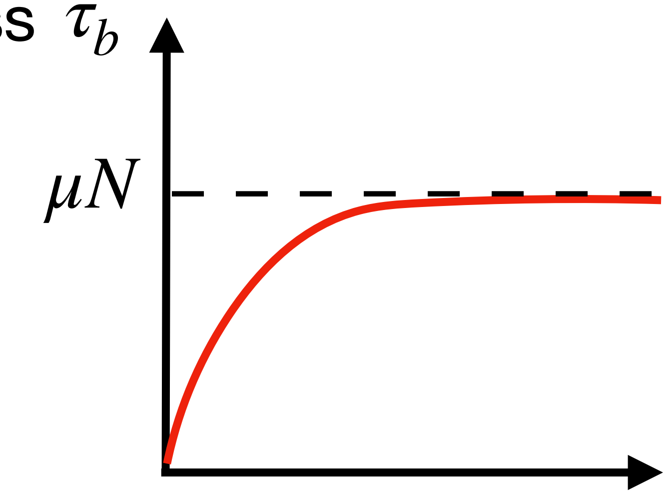
Morlighem et al 2011

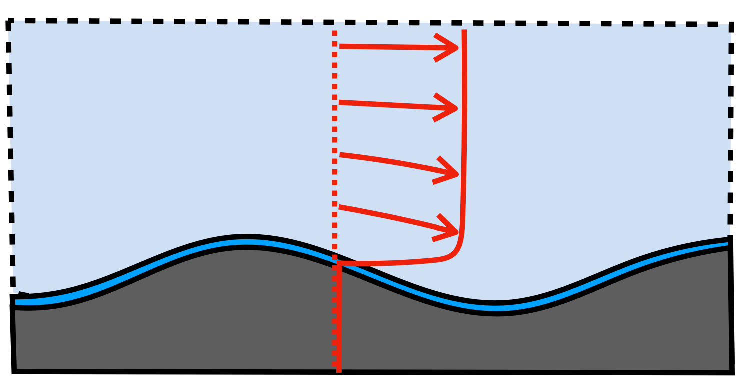
Note: the 'correct' friction law and value of coefficients depend on the **resolution** of your model (the friction law is to describe unresolved processes!)

Summary



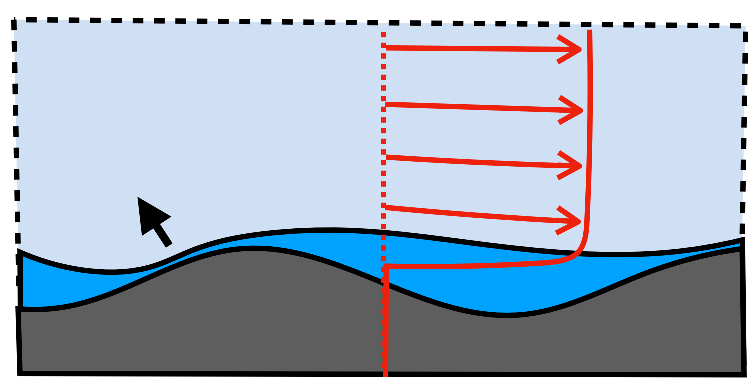
$$\tau_b = f(u_b, \ldots)$$





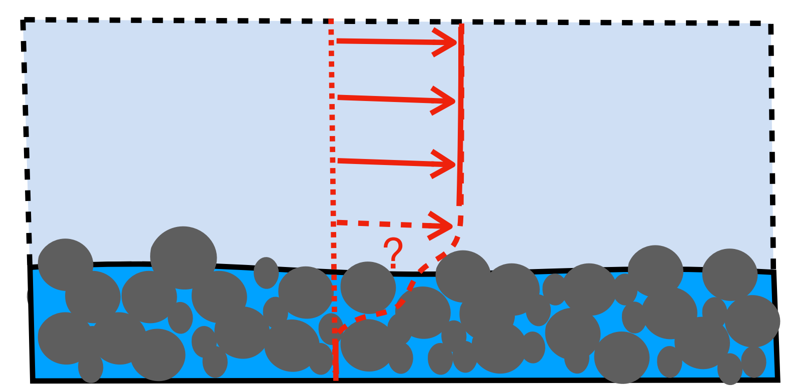
Hard-bed sliding

$$\tau_b = C u_b^m$$



Sliding with cavitation

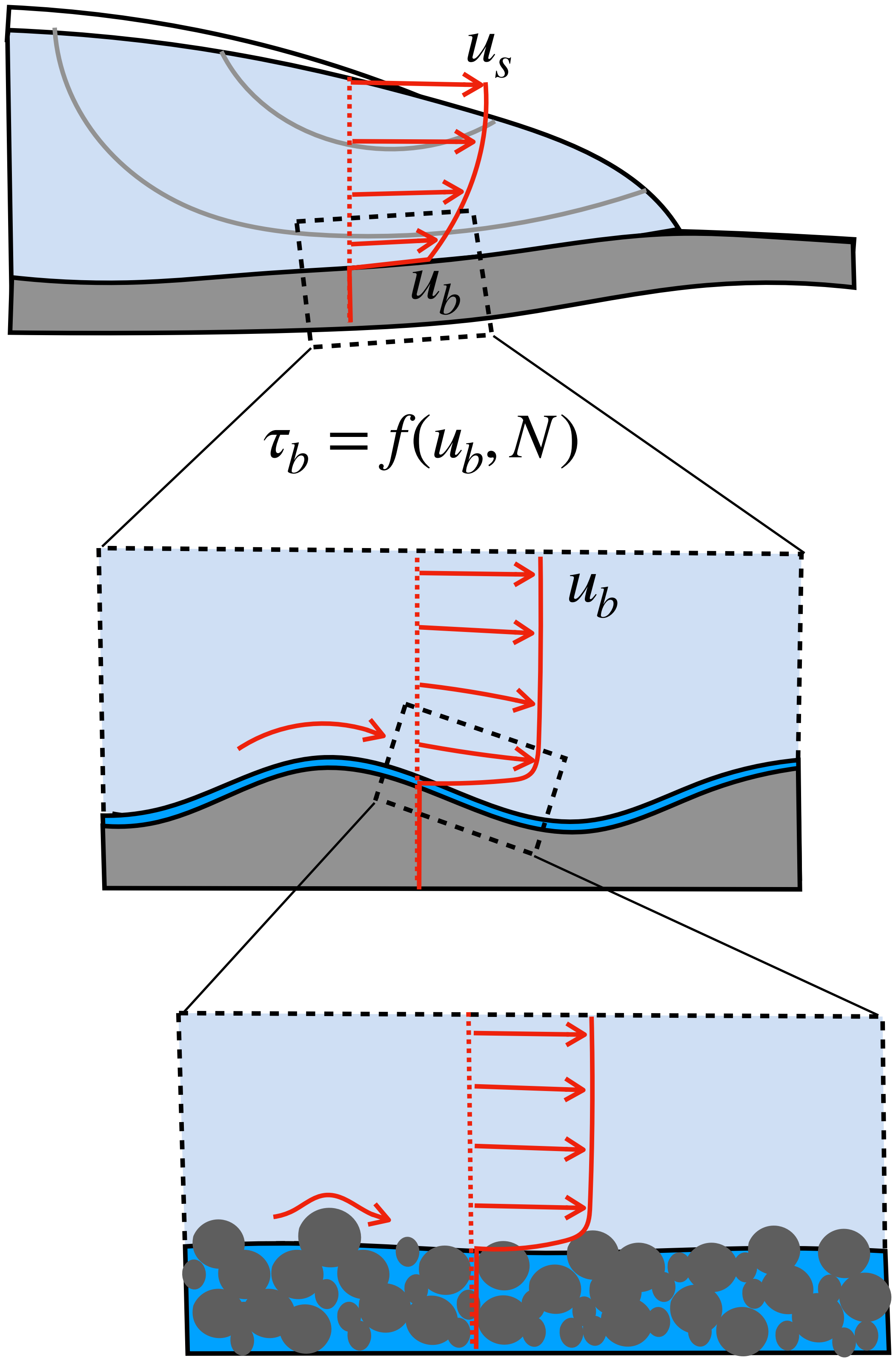
$$\tau_b = \mu N \left(\frac{u_b}{u_b + \lambda N^n} \right)^{1/n}$$



Soft-bed sliding

$$\tau_b = \mu N$$

The importance of 'form drag'



The sliding law needs to account for **all sub-grid scale 'roughness'**.

That often includes larger scales than those for which cavitation / bed deformation are relevant.

$$\tau_b = \mu N \left(\frac{u_b}{u_b + \lambda N^n} \right)^{1/n} + C u_b^{1/n}$$

Small-scale cavitation / bed deformation

Larger-scale form drag