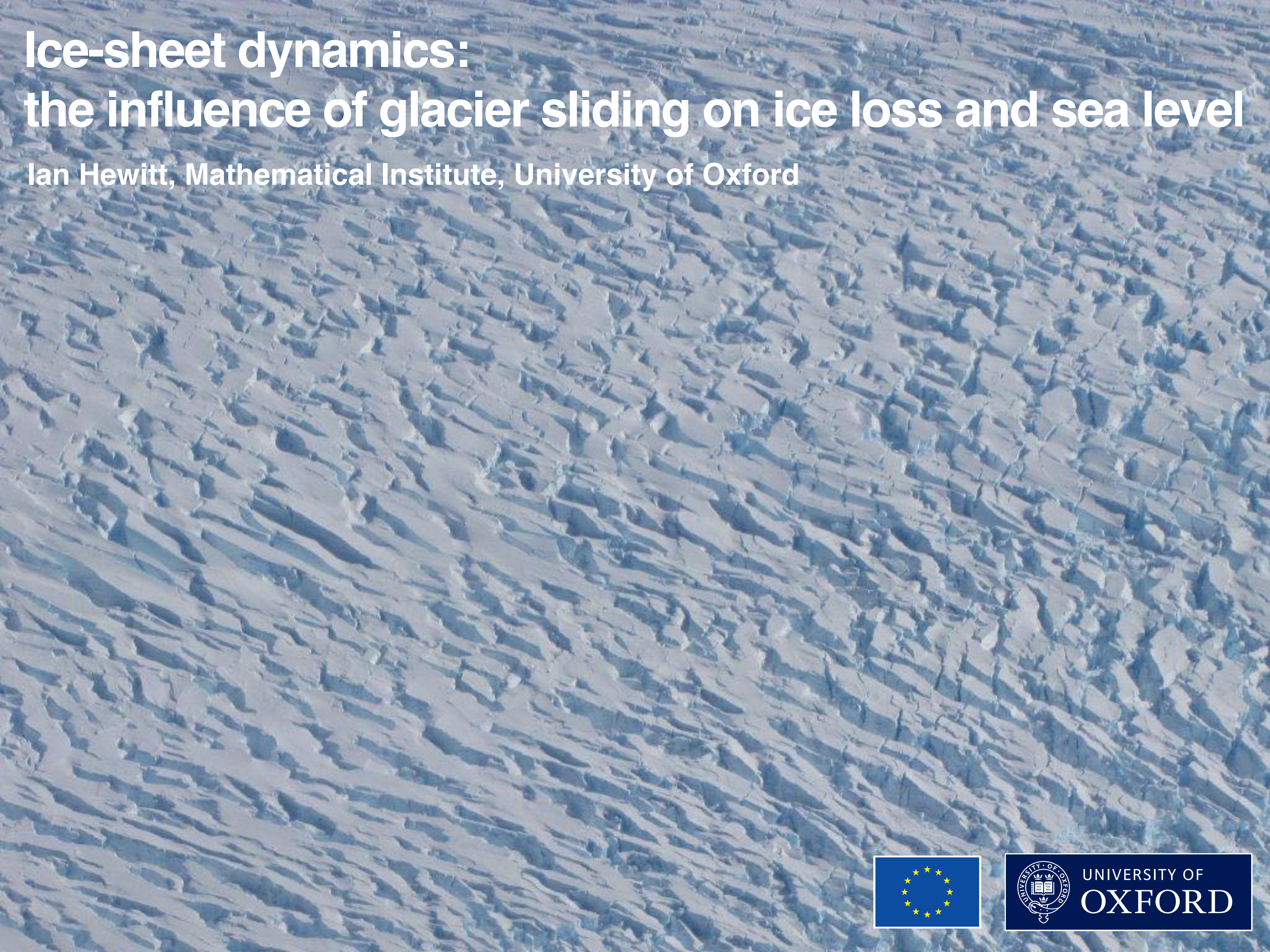
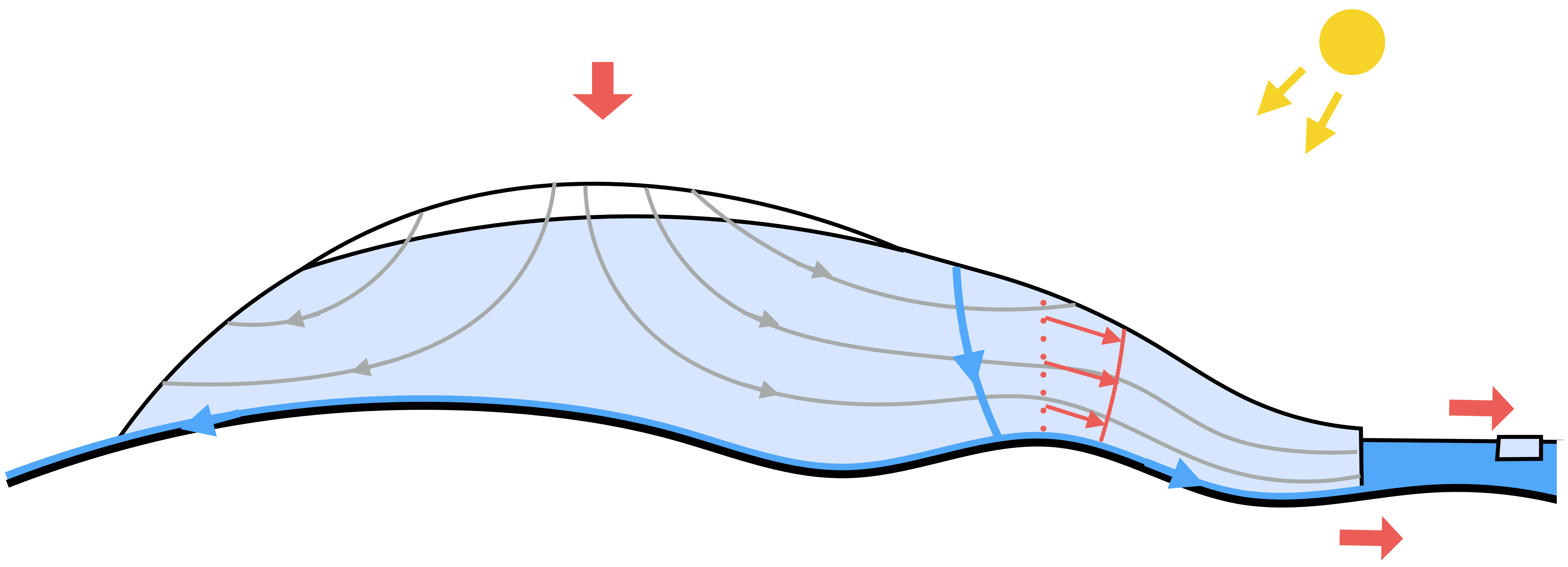


Ice-sheet dynamics: the influence of glacier sliding on ice loss and sea level

Ian Hewitt, Mathematical Institute, University of Oxford


Greenland

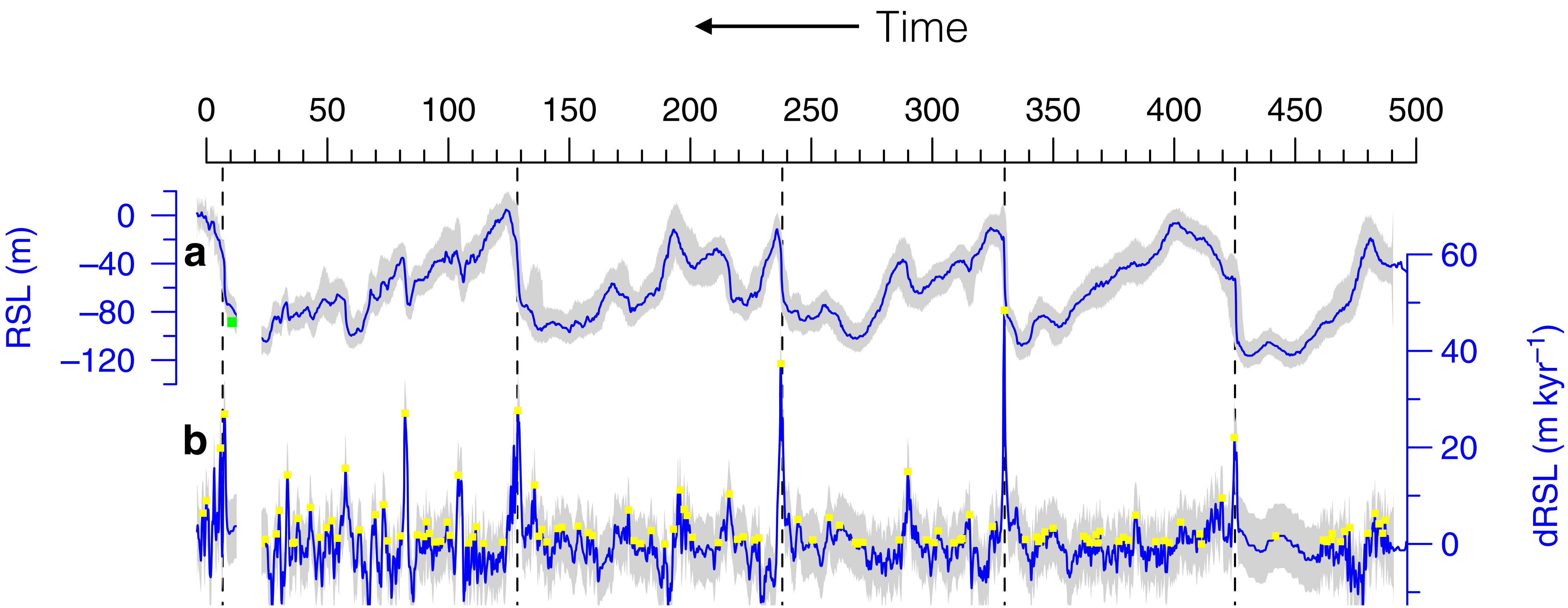


How does meltwater penetrating to the bed affect ice-sheet motion?

What implications does this have for ice loss (sea level)?

Sea level history

The glacial period is punctuated by several periods of rapid sea level rise (~1m/century)

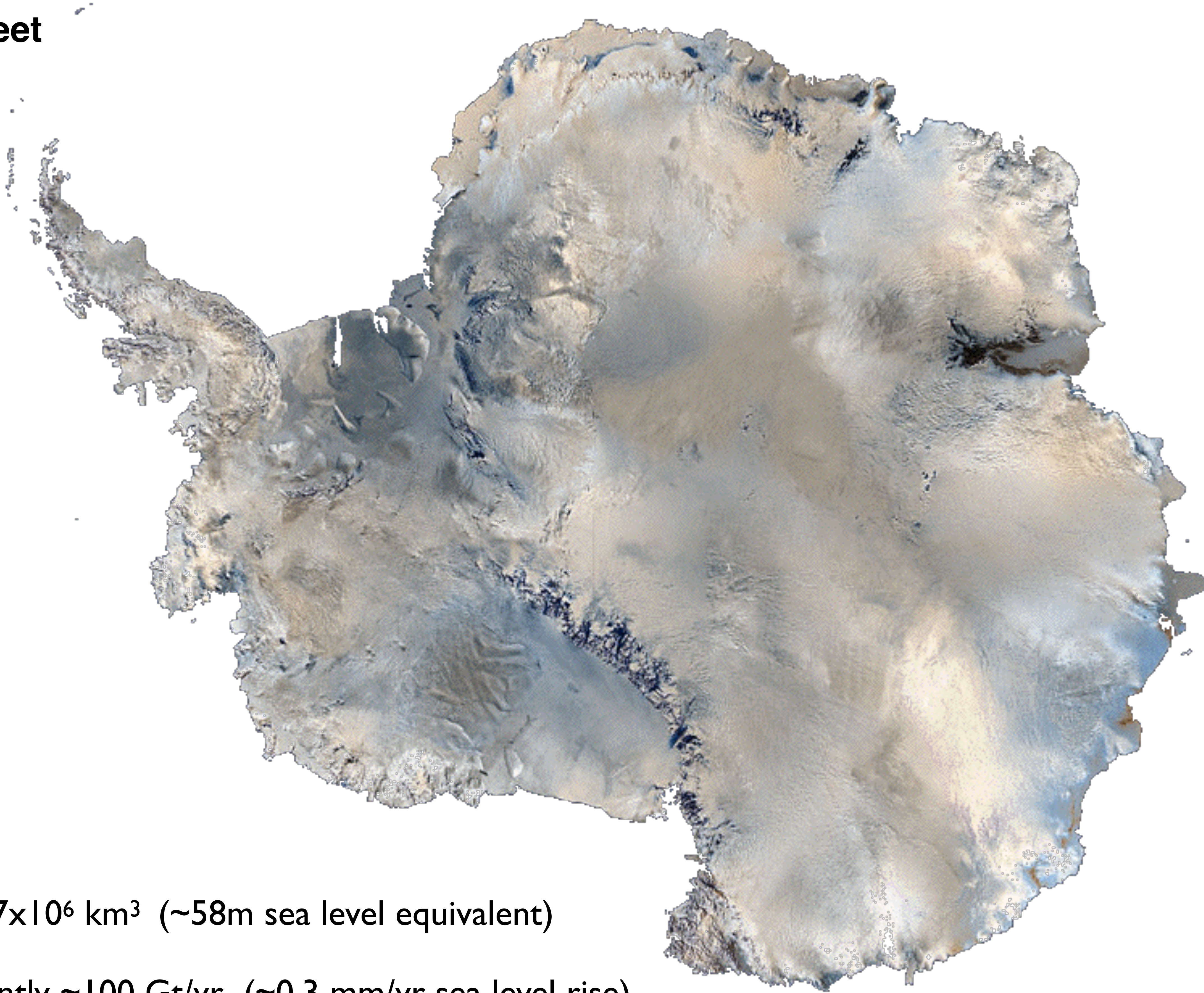


Grant et al 2014

Red Sea Relative Sea Level

Global sea level has been at least 6m higher in previous interglacials.

Antarctic Ice Sheet



Current volume ~27×10^6 km^3 (~58m sea level equivalent)

Net mass loss currently ~100 Gt/yr (~0.3 mm/yr sea level rise)

Greenland Ice Sheet

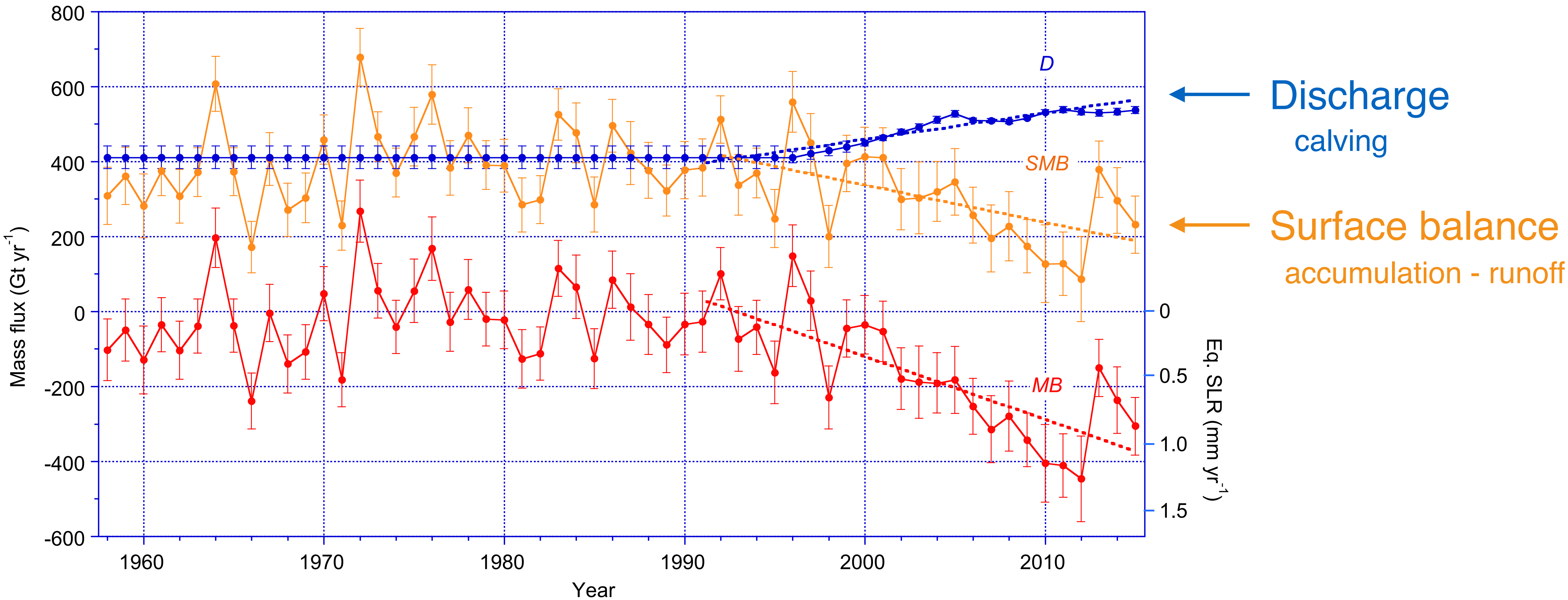
Current volume ~2.7×10^6 km^3 (~7m sea level equivalent)

Net mass loss currently ~200 Gt/yr (around 0.6 mm/yr sea level rise)

Timescale ~10,000 years

Laura Stevens

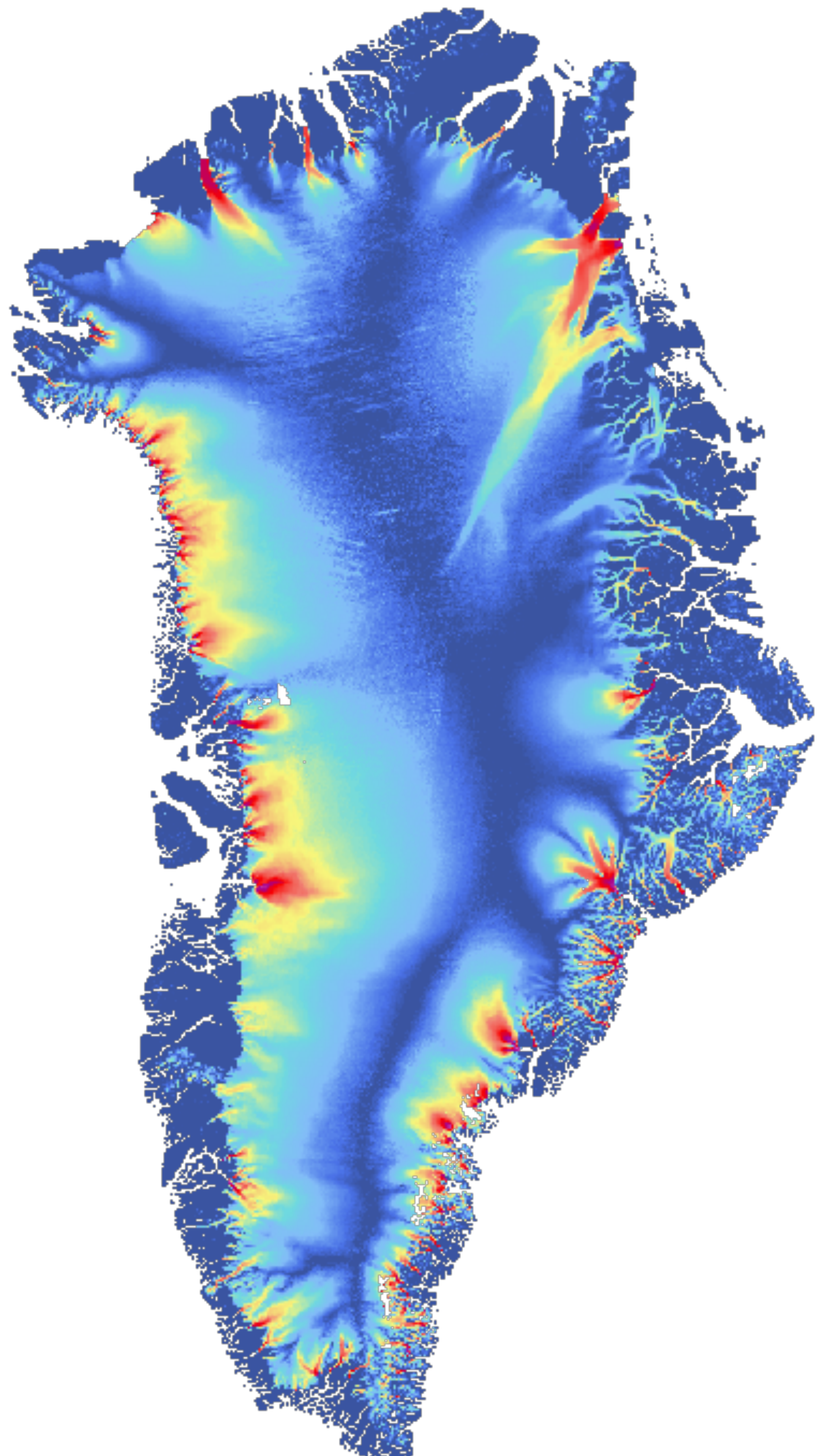
Greenland ice sheet mass balance



van den Broeke et al 2016

Greenland is losing mass due to **decreased** SMB and **increased** discharge

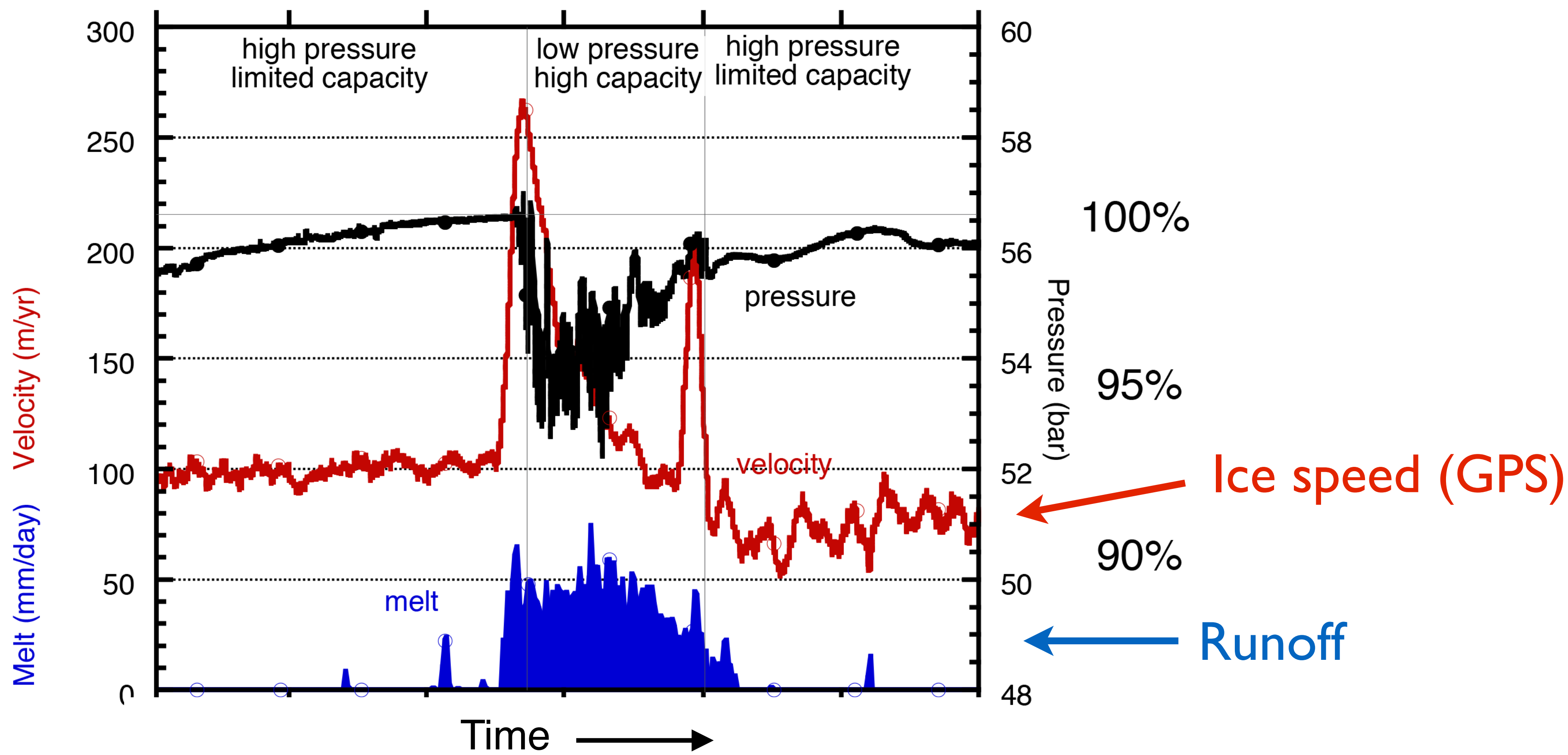
Satellite-derived ice surface speeds



Greenland ice sheet velocities

Summer drainage of surface meltwater to the bed causes large fluctuations in ice speed.

→ suggests potential for significant changes in ice velocity

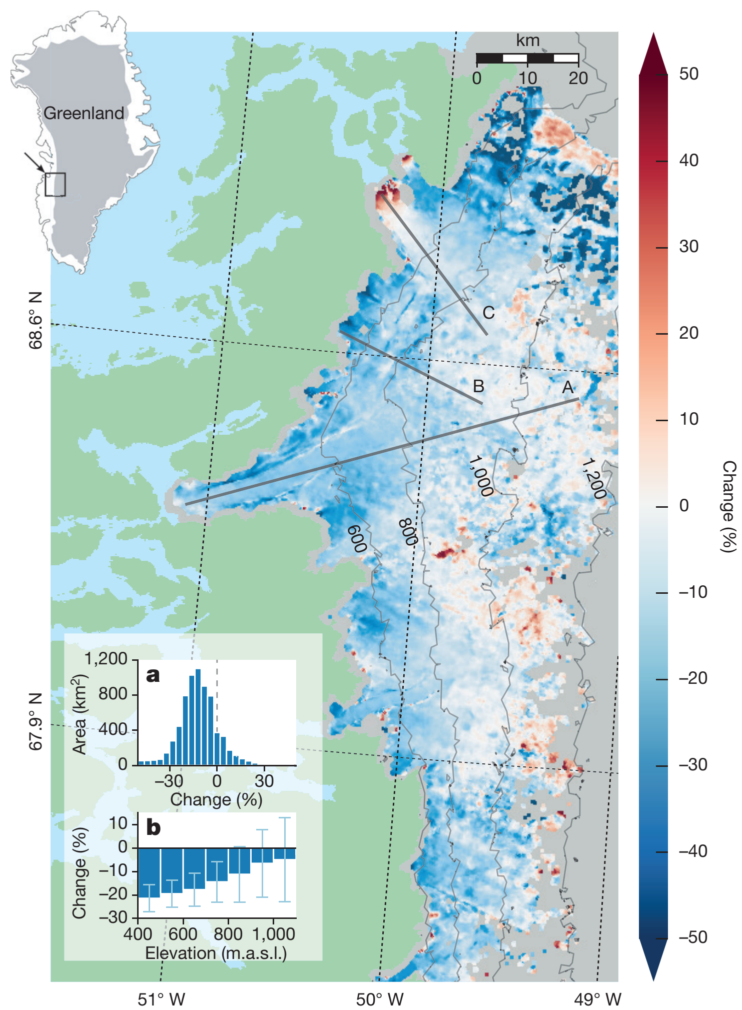


van de Wal et al 2015

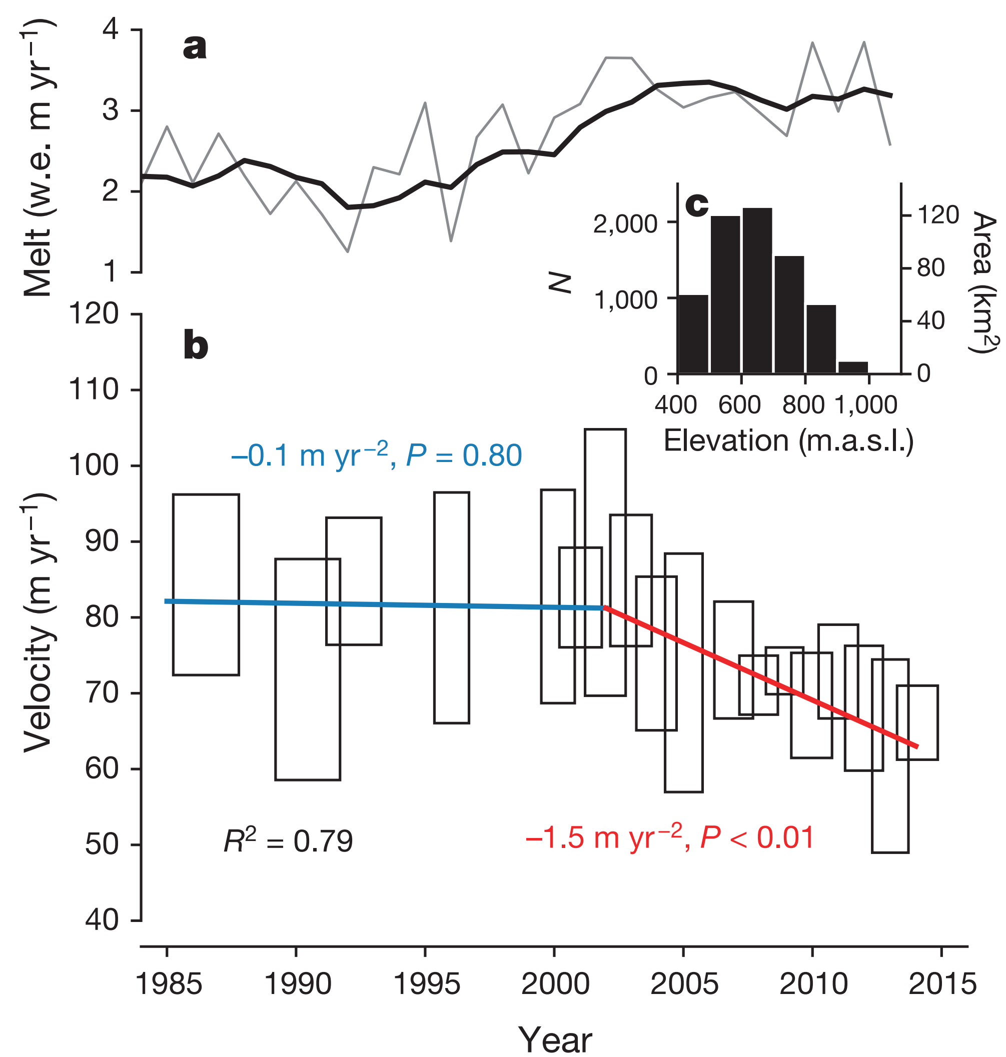
Greenland ice sheet velocities

Longer term measurements show a slight **decreasing** trend in average velocity, while runoff shows an increasing trend.

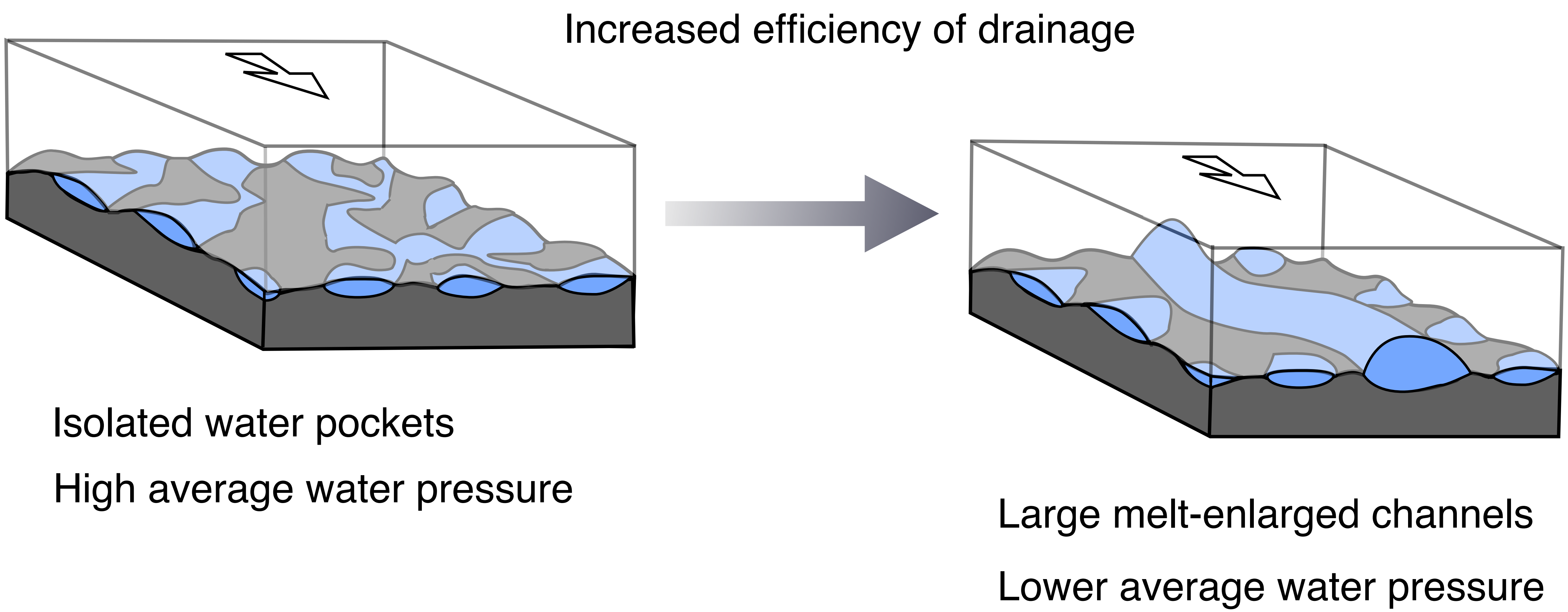
→ suggests possible negative relationship between runoff and average velocity?

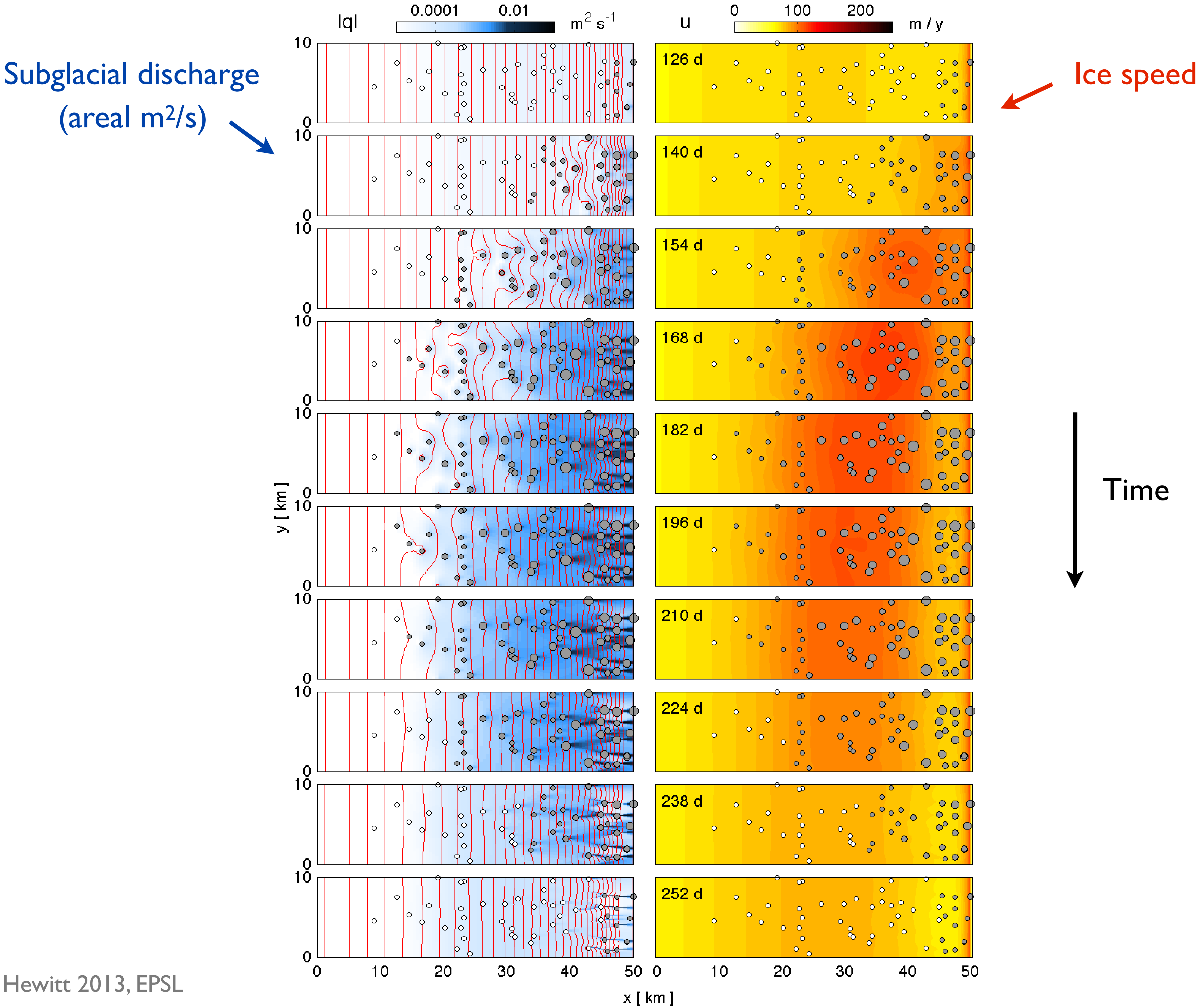


Tedstone et al 2015



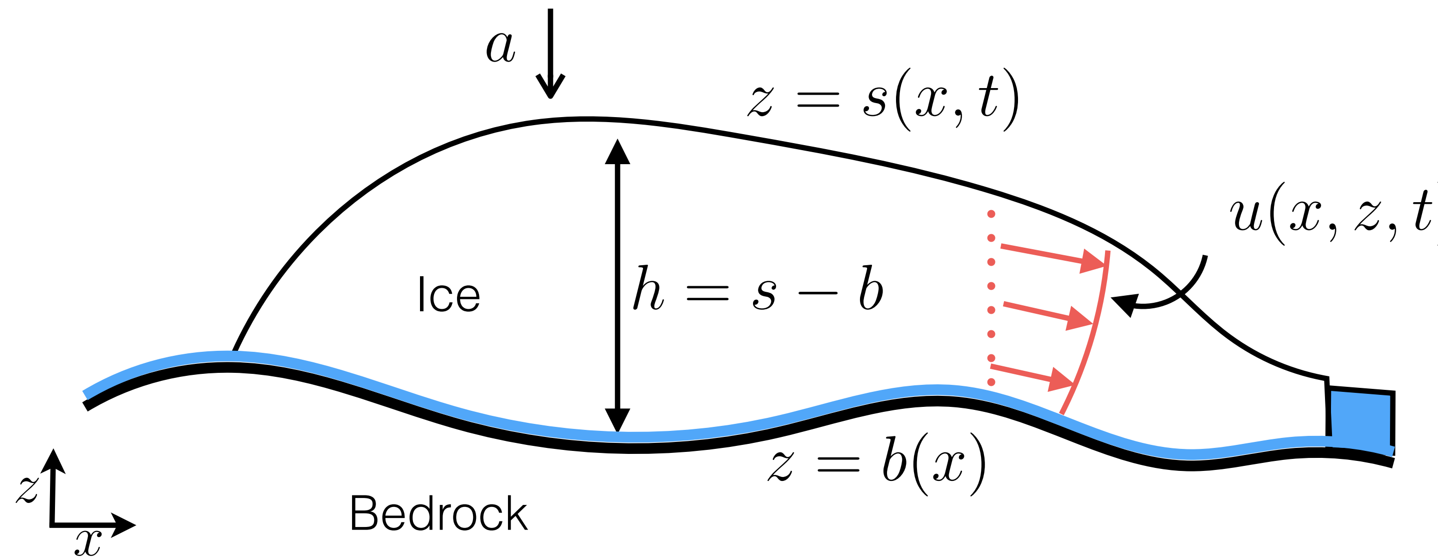
Evolution of the subglacial drainage system





Hewitt 2013, EPSL

Mathematical model



- Vertically-integrated mass conservation

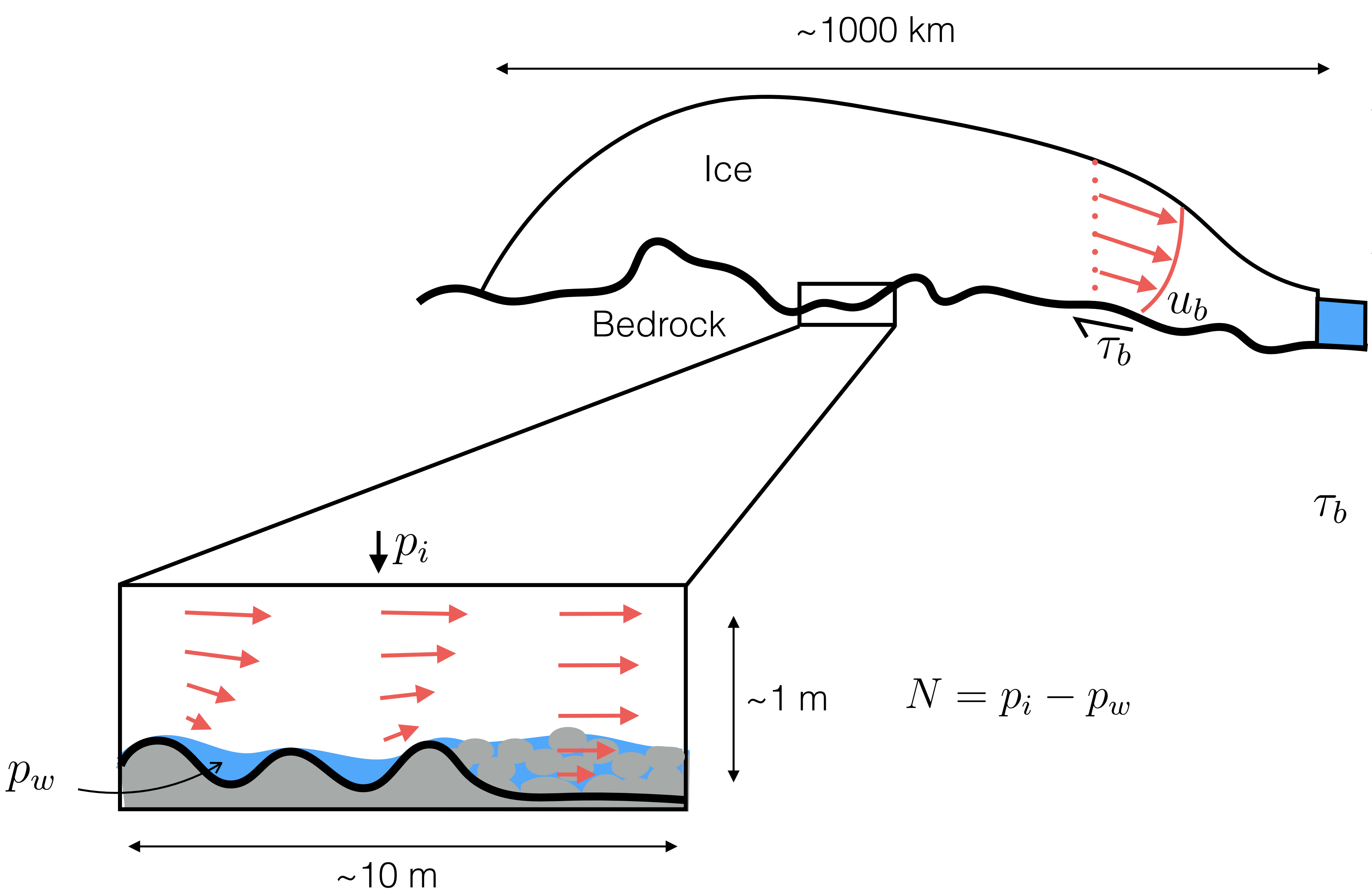
$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = a \qquad q(x,t) = h\overline{u} = \int_b^s u \, \mathrm{d}z$$

a = net accumulation - melting

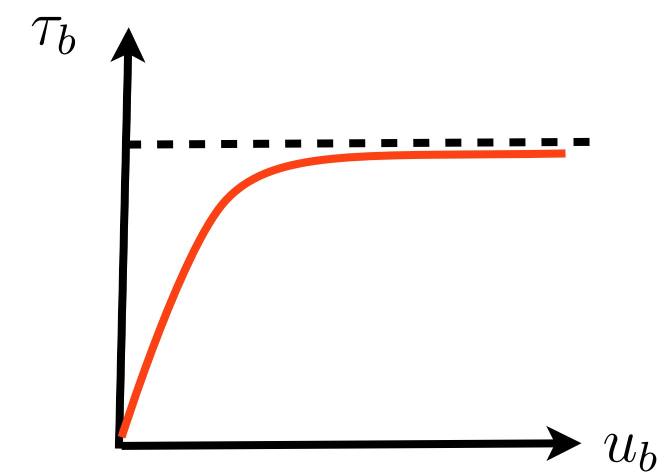
- Force balance $0 = \nabla \cdot \boldsymbol{\sigma} + \rho_i \mathbf{g}$

$$p_i = \rho_i g(s - z) \qquad \tau_b = -\rho_i g h \frac{\partial s}{\partial x} + \frac{\partial}{\partial x}\left(4h\eta_i \frac{\partial \overline{u}}{\partial x}\right) \qquad \tau_b = f(\overline{u}, N) \qquad N = p_i - p_w$$

Theoretical framework for modelling sliding



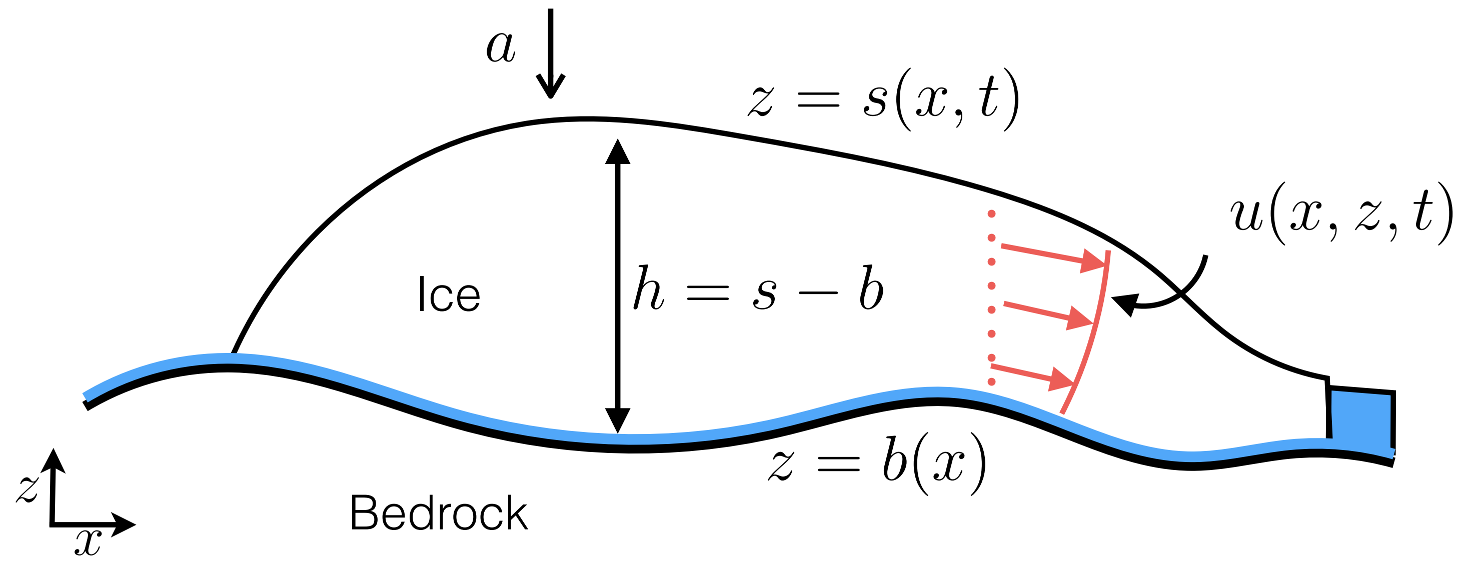
$$N = p_i - p_w$$



Theory and *some* measurements suggest a **friction / slip law** of the form

$\tau_b = f(u_b, N)$ to be applied to the large-scale ice flow

Mathematical model



- Boundary conditions

land-terminating $\quad h = 0, \quad q = 0 \quad$ at $\quad x = x_m$

marine-terminating $\quad h\dot{x}_m = q - q_c, \quad 4h\eta_i \dfrac{\partial \overline{u}}{\partial x} = \dfrac{1}{2}\left(\rho_i g h^2 - \rho_o g b^2\right) \quad$ at $\quad x = x_m$

+ calving condition $\quad h = f h_f \quad$ at $\quad x = x_m$

$$h_f = -\frac{\rho_o}{\rho_i} b$$ flotation thickness

Glacier flow

Extreme Ice Survey - Time-lapse camera
Khumbu glacier, Nepal

~10,000,000 x real time

Glacier flow

Extreme Ice Survey - Time-lapse camera
Khumbu glacier, Nepal

~10,000,000 x real time

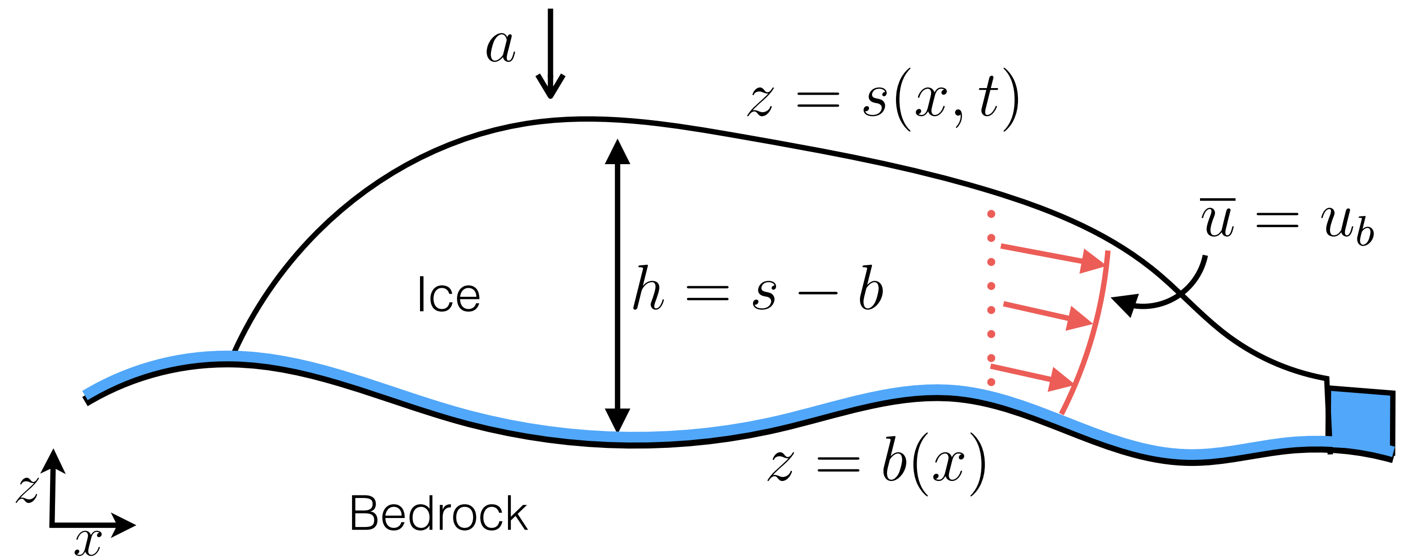
Marine-terminating / tidewater glaciers

Extreme Ice Survey - Time-lapse camera
Columbia Glacier, Alaska

Marine-terminating / tidewater glaciers

Extreme Ice Survey - Time-lapse camera
Columbia Glacier, Alaska

Mathematical model

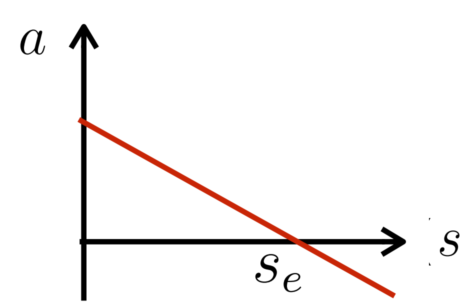


- If friction law is invertible, $\overline{u} = F(\tau_b, N)$, and bed topography relatively flat, the problem is seemingly **diffusive**

$$\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(K\frac{\partial h}{\partial x}\right) + a \qquad K = \rho_i g h F_\tau$$

However, the diffusion coefficient may be highly non-linear, and accumulation rate varies with ice thickness.

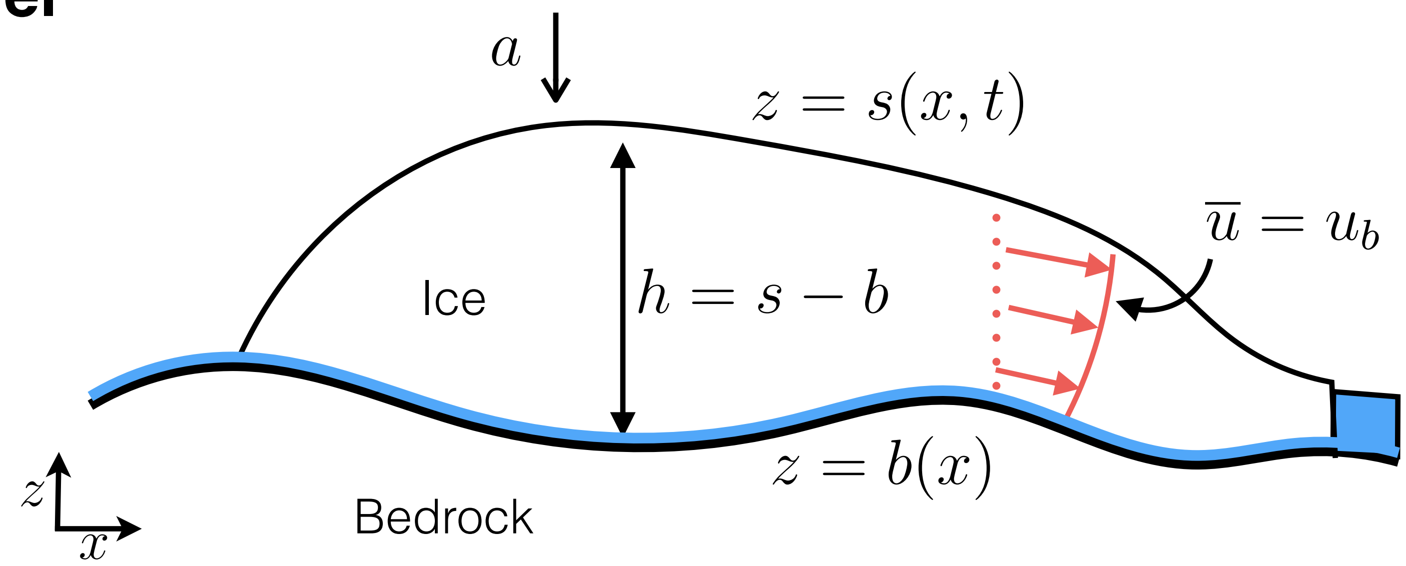
- e.g. $a = \lambda(s - s_e)$



s_e equilibrium line altitude (ELA)

→ generic behaviour is ‘blow-up’ (cf. reaction-diffusion problems)

A reduced 'plastic bed' model



- Vertically-integrated mass conservation

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = a \qquad q = h\overline{u} \qquad a = \lambda(s - s_e)$$

net accumulation - melting

s_e equilibrium line altitude (ELA)

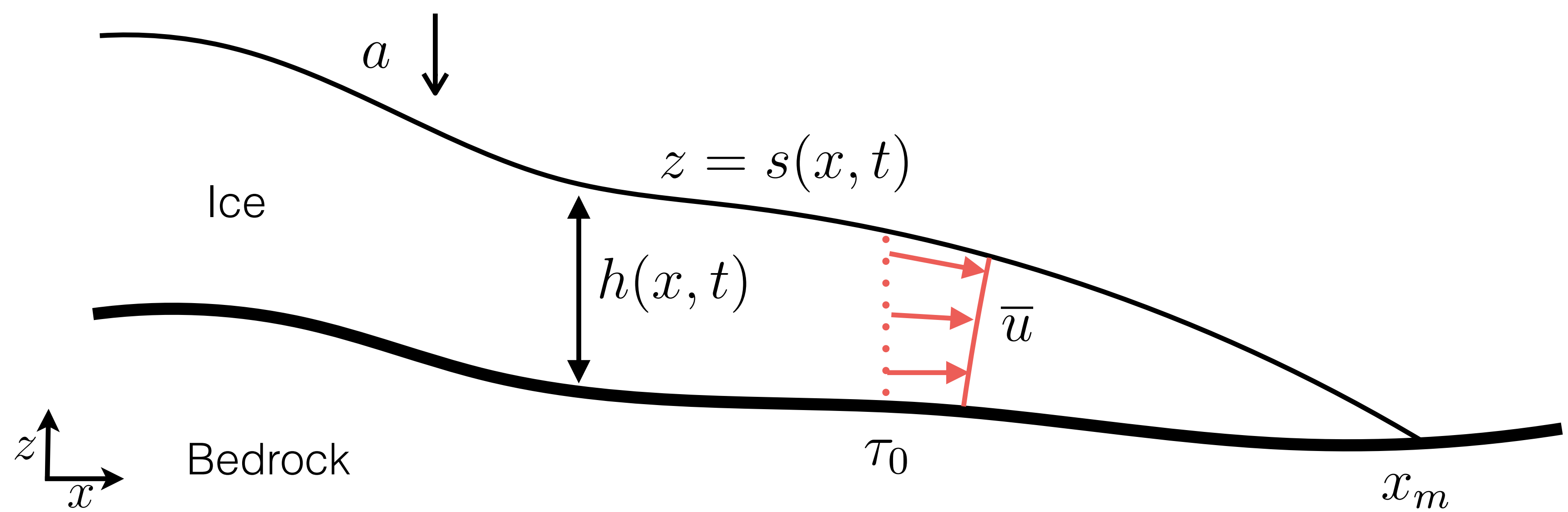
- Force balance + friction parameterisation

$$\tau_b = -\rho_i g h \frac{\partial s}{\partial x} + \frac{\partial}{\partial x}\left(4h\eta_i \frac{\partial \overline{u}}{\partial x}\right) \qquad \tau_b = \mu\langle N\rangle = \tau_0$$

bed 'strength'

Goal: Consider effect of a long-term changes in τ_0 and s_e

Land-terminating glacier



- Boundary conditions $\quad h = 0, \quad q = 0 \quad \text{at} \quad x = x_m(t)$

Force balance $\quad \tau_0 = -\rho_i g h \dfrac{\partial s}{\partial x} \quad \longrightarrow$ ice thickness & volume $\quad V = \displaystyle\int_0^{x_m} h \, \mathrm{d}x$

e.g. for a flat bed profile $\quad h = \sqrt{\dfrac{2\tau_0}{\rho_i g}} (x_m - x)^{1/2}$

Mass conservation $\longrightarrow$

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \int_0^{x_m} a \, \mathrm{d}x$$

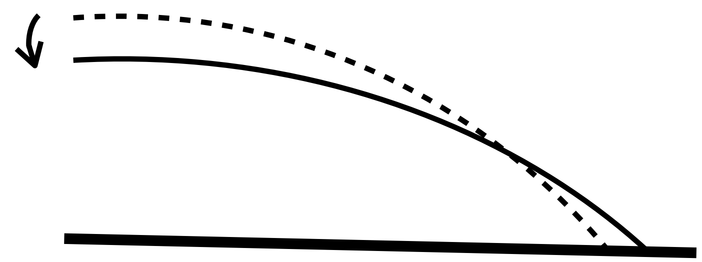
an ODE for the evolution of ice volume

e.g. $\quad \dfrac{\mathrm{d}V}{\mathrm{d}t} = \lambda \left[V - V^{2/3} \left(\dfrac{9\rho_i g}{8\tau_0} \right)^{1/3} s_e \right]$

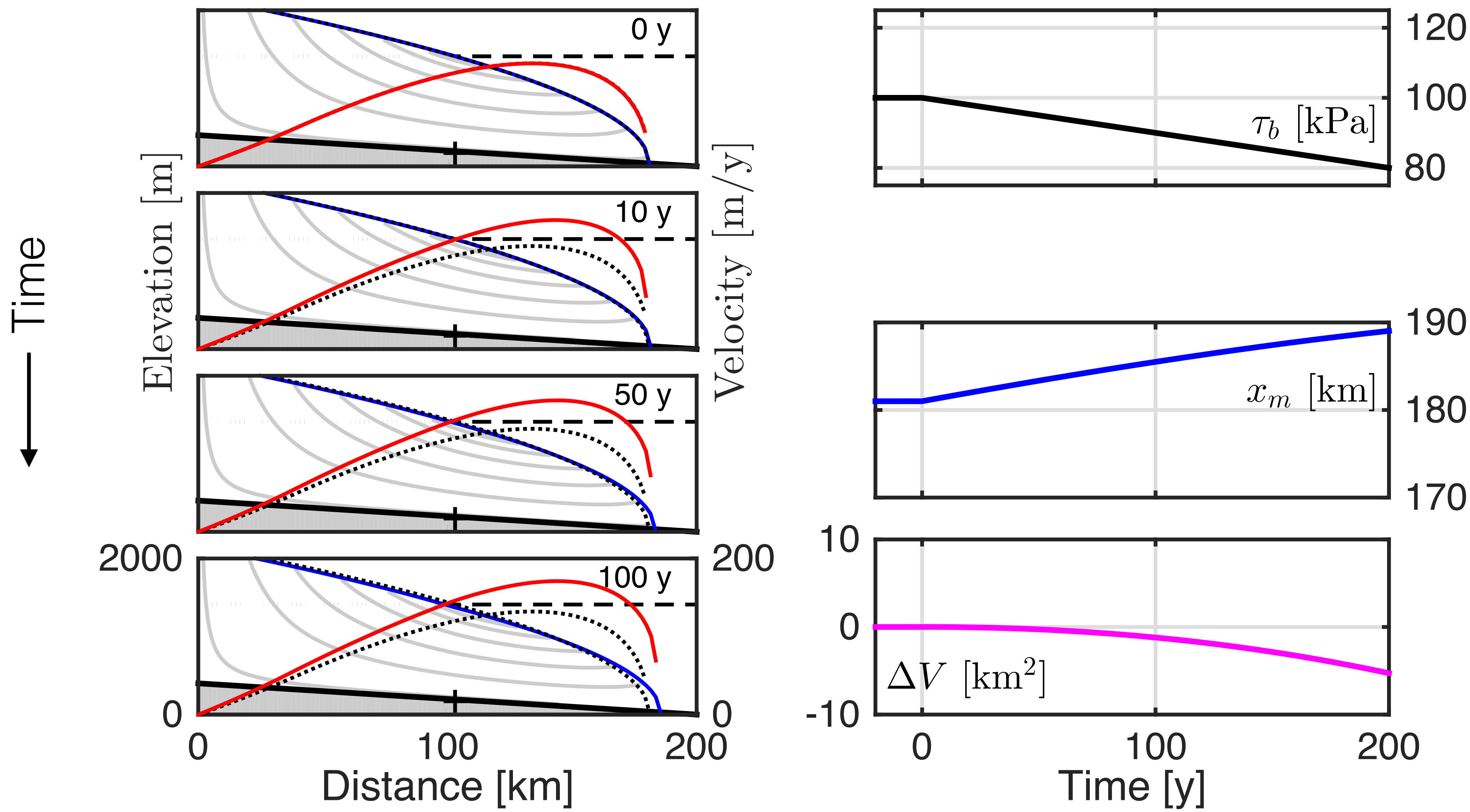
cf. Nye 1951, Weertman 1961, 1976

Land terminating glacier

A gradual **decrease** in bed strength results in **increased velocities** and **mass loss**

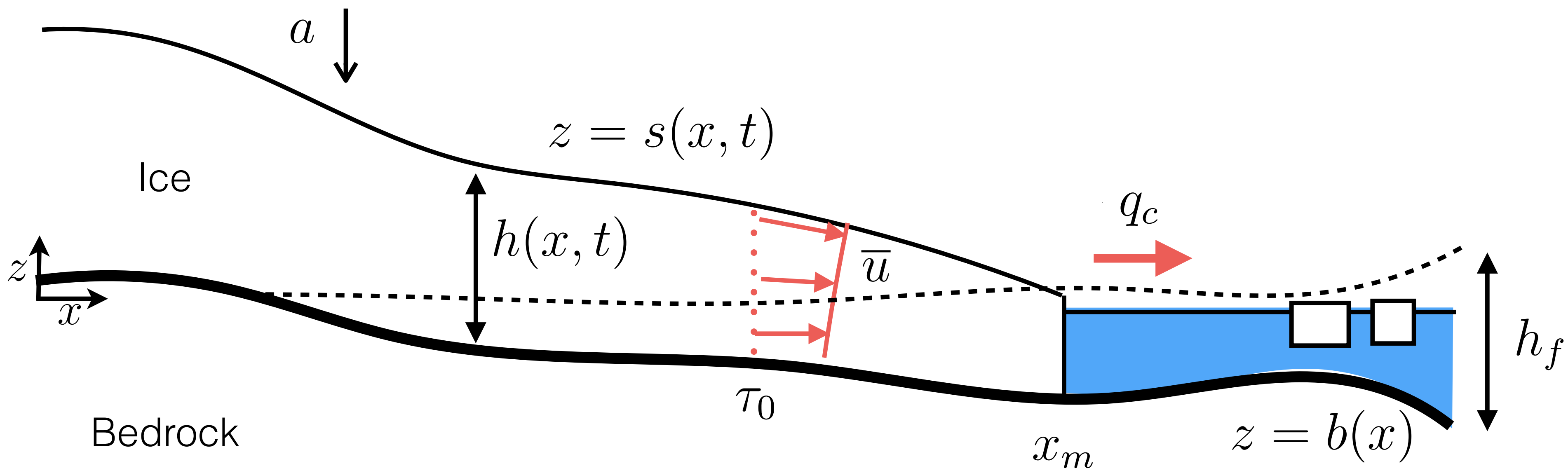


(an increase in bed strength
causes the opposite)



Rate of ice loss controlled by SMB

Marine-terminating glacier



- Boundary conditions $h = f h_f, \quad h\dot{x}_m = q - q_c, \quad 4h\eta_i \frac{\partial \overline{u}}{\partial x} = \frac{\rho_i g}{2}\left(h - \frac{\rho_i}{\rho_o} h_f^2\right) \quad \text{at} \quad x = x_m(t)$

Force balance $\tau_0 = -\rho_i g h \frac{\partial s}{\partial x}$ → thickness + volume $V = \int_0^{x_m} h \, \mathrm{d}x$

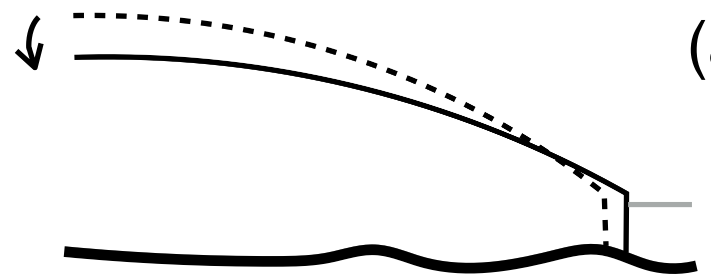
Mass conservation → $\frac{\mathrm{d}V}{\mathrm{d}t} = \int_0^{x_m} a \, \mathrm{d}x - q_c$ Calving flux $q_c = \frac{\rho_i g}{\eta_i \mu} \hat{Q}(f) h_f^3$

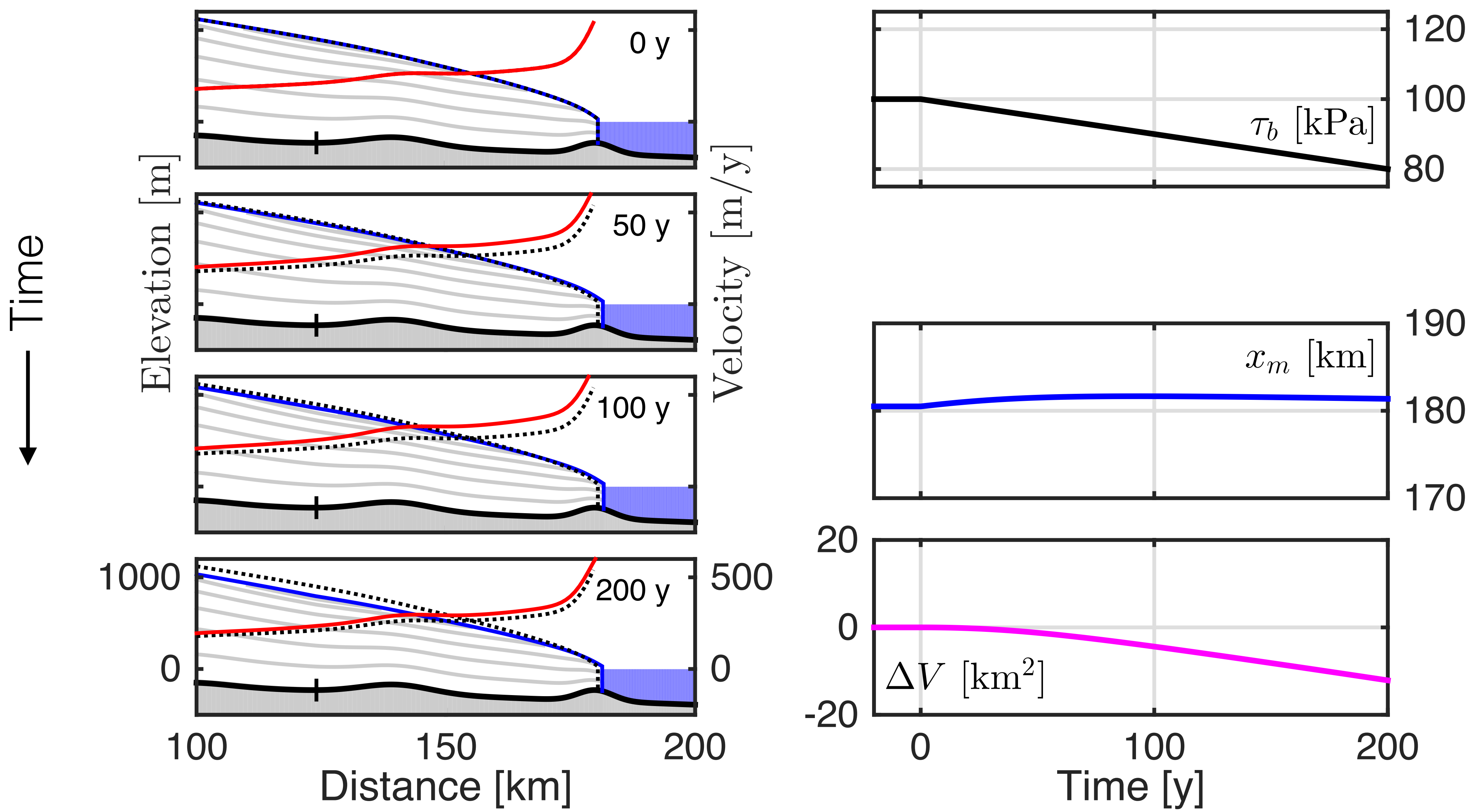
an ODE for the evolution of ice volume

Marine terminating glacier

A gradual **decrease** in bed strength results in **increased velocities** and **mass loss**

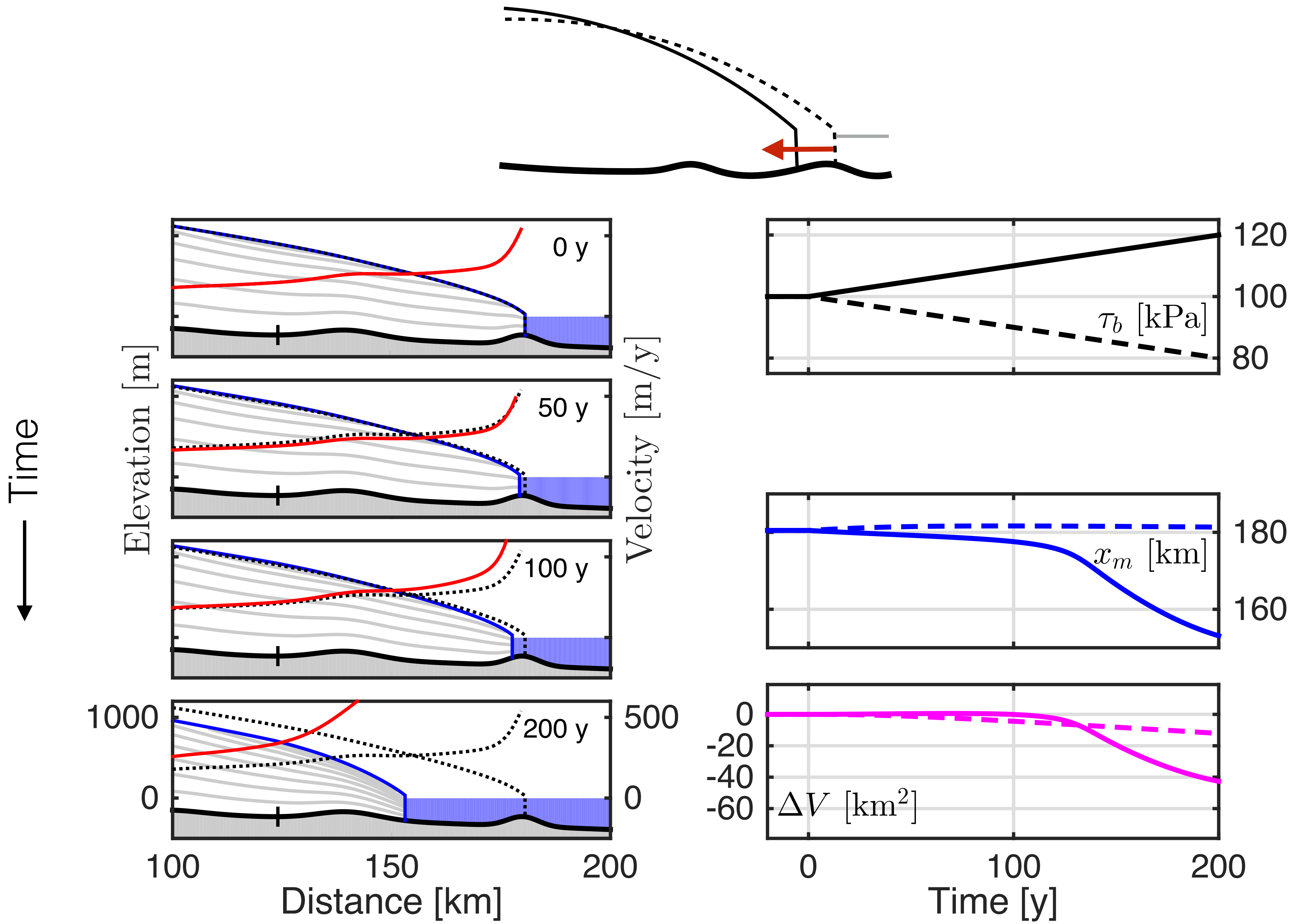
(as for land terminating glaciers)





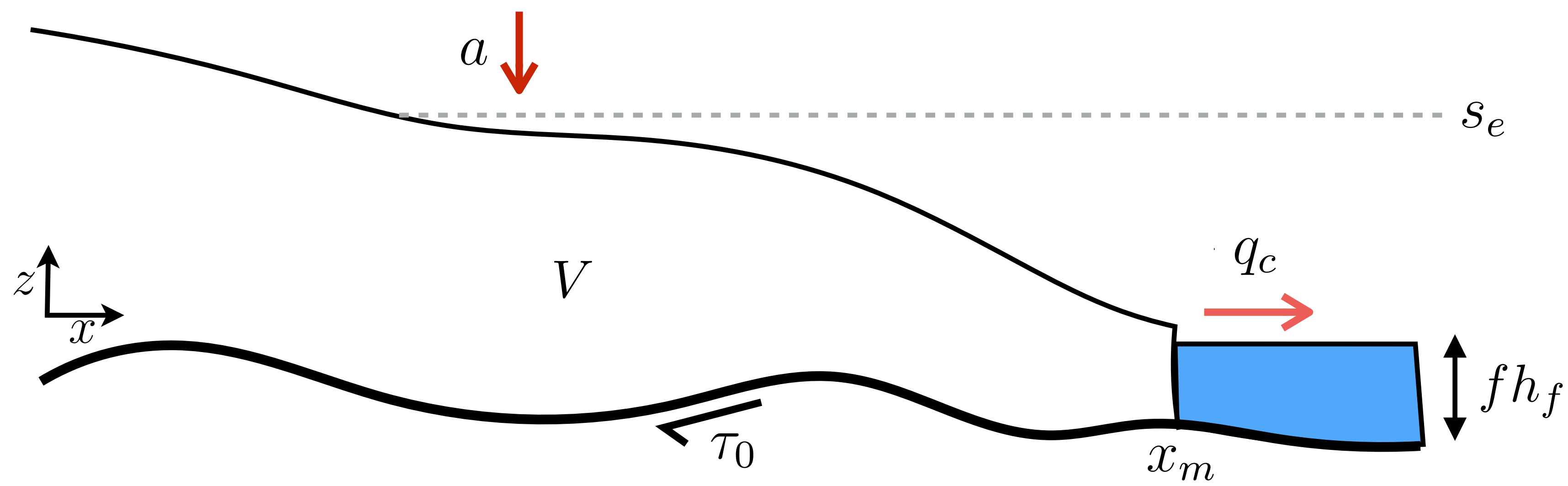
Marine terminating glacier

An **increase** in bed strength results in **initially decreased velocities**
… but this initiates terminus retreat and acceleration**.**



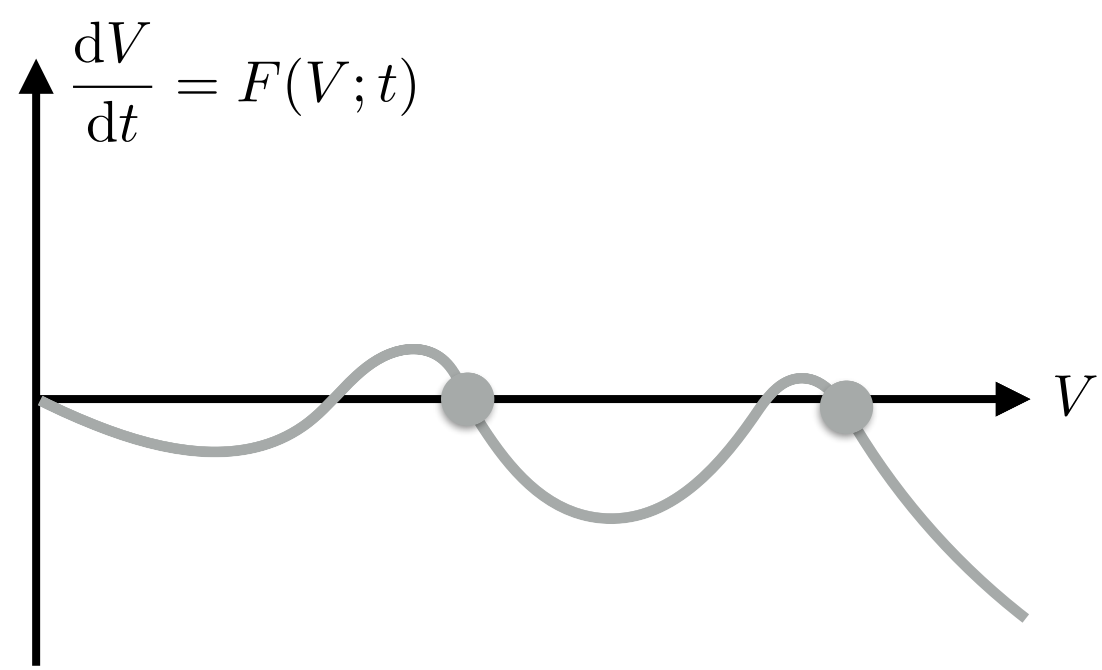
Rate of ice loss controlled by ice mechanics (& topography)

Marine terminating glacier

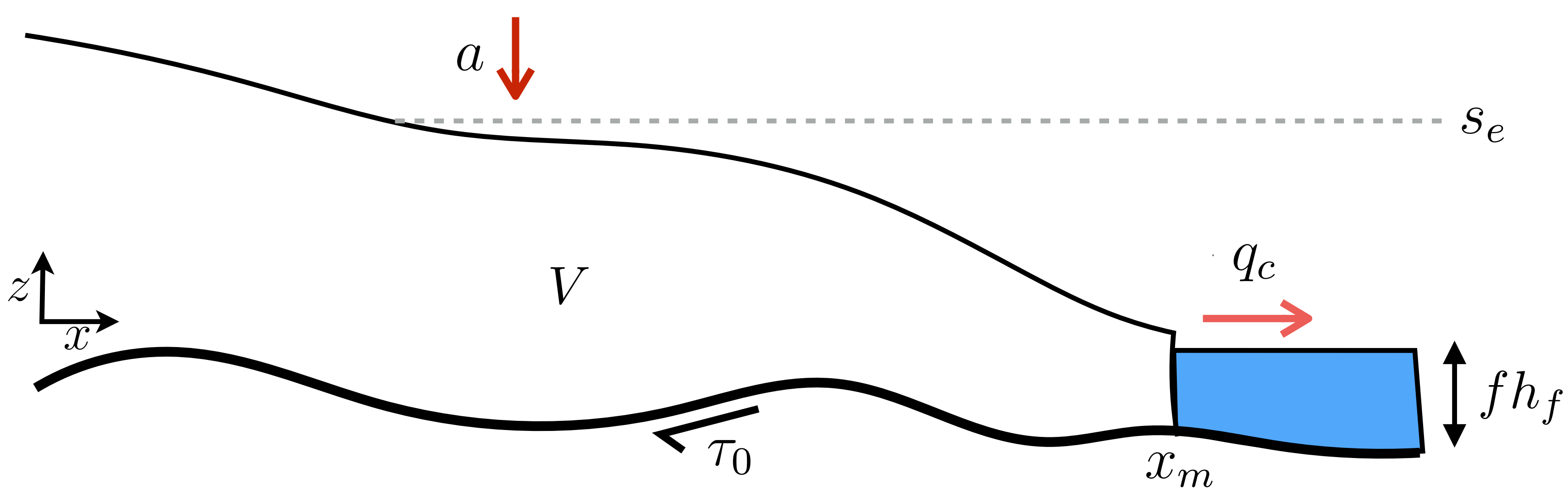


$$\frac{\mathrm{d}V}{\mathrm{d}t} = \int_0^{x_m} a \ \mathrm{d}x - q_c$$

$$q_c = \frac{\rho_i g}{\eta_i \mu} \hat{\mathcal{Q}}(f) h_f^3$$

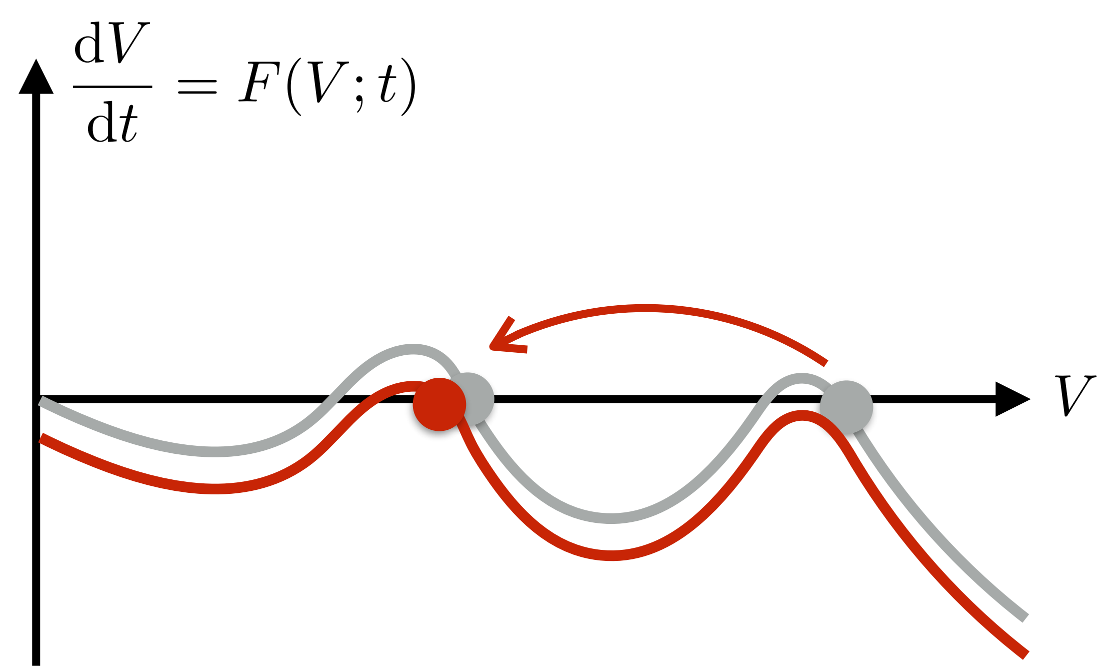


Marine terminating glacier

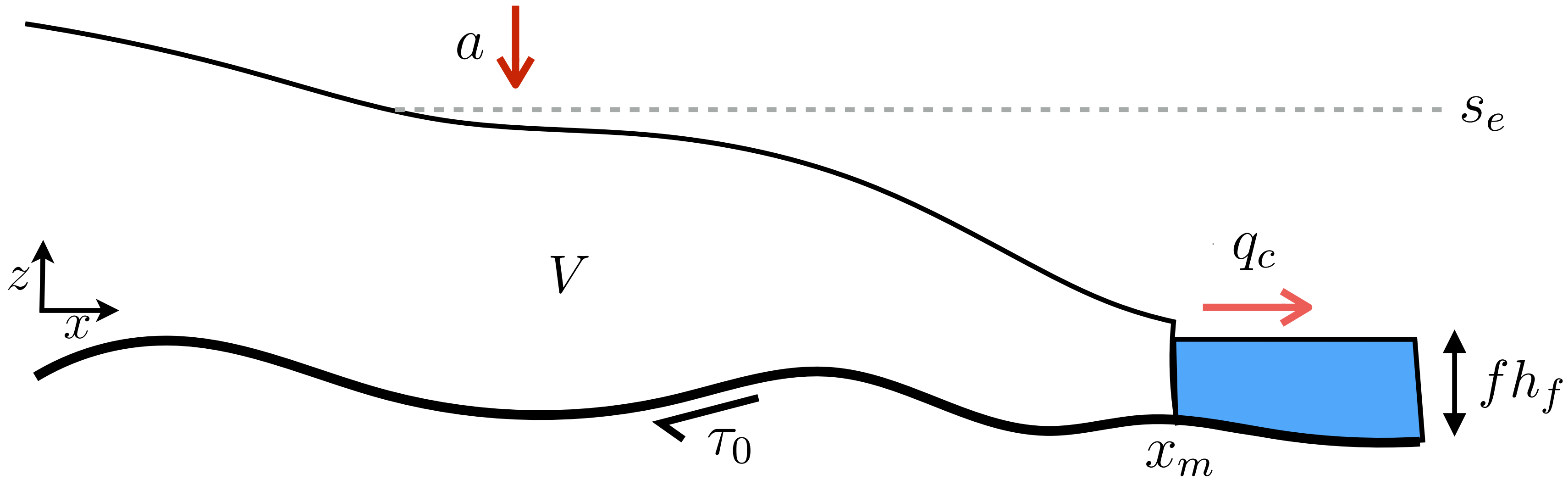


$$\frac{\mathrm{d}V}{\mathrm{d}t} = \int_0^{x_m} a \ \mathrm{d}x - q_c$$

$$q_c = \frac{\rho_i g}{\eta_i \mu} \hat{\mathcal{Q}}(f) h_f^3$$

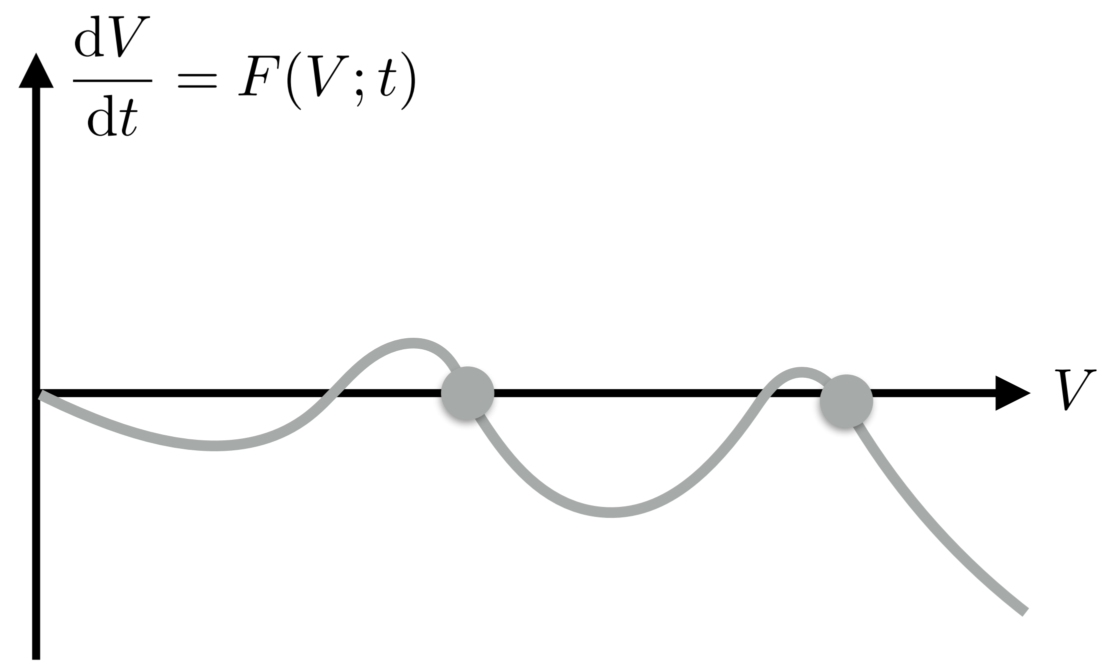


Marine terminating glacier

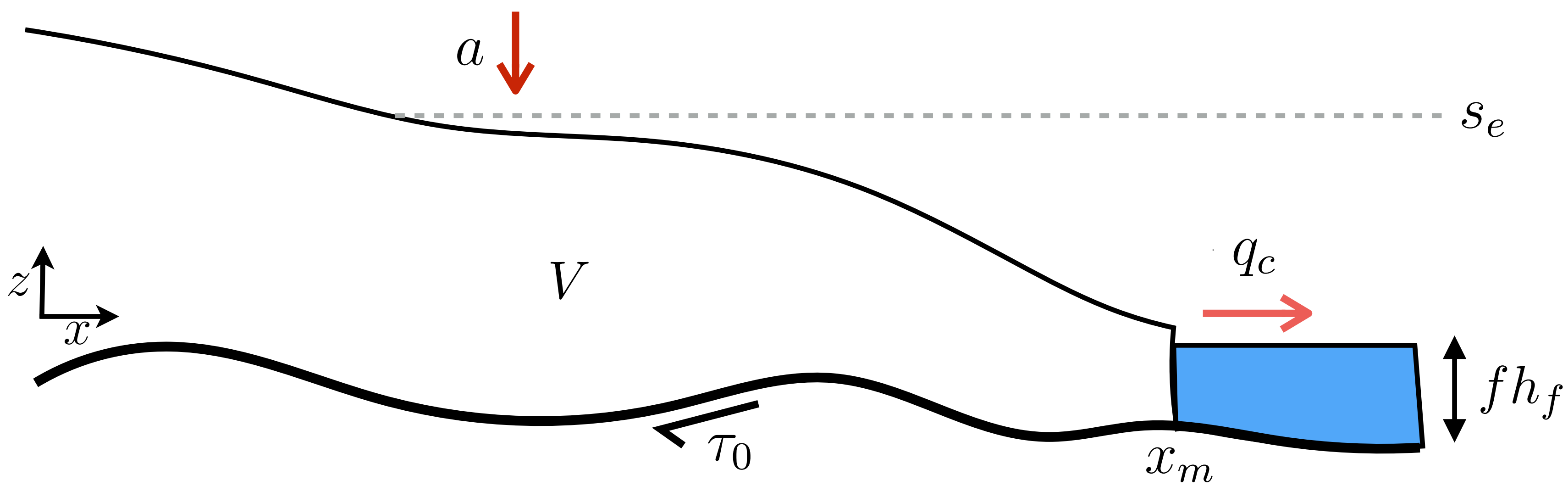


$$\frac{\mathrm{d}V}{\mathrm{d}t} = \int_0^{x_m} a \ \mathrm{d}x - q_c$$

$$q_c = \frac{\rho_i g}{\eta_i \mu} \hat{\mathcal{Q}}(f) h_f^3$$

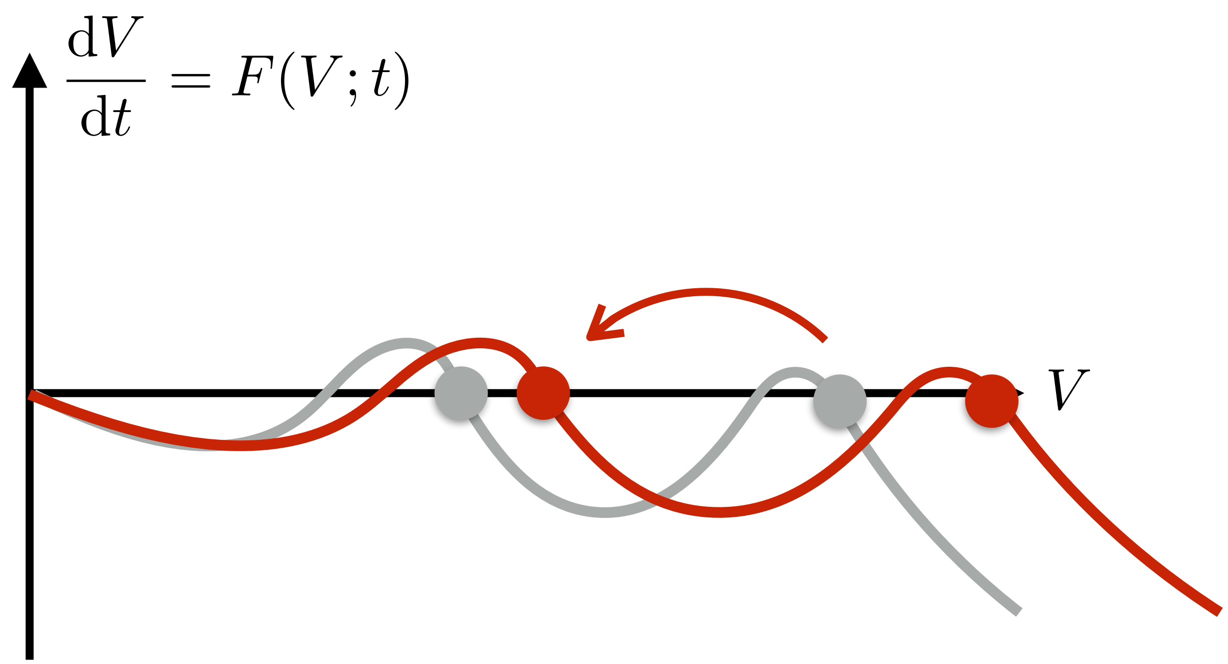


Marine terminating glacier



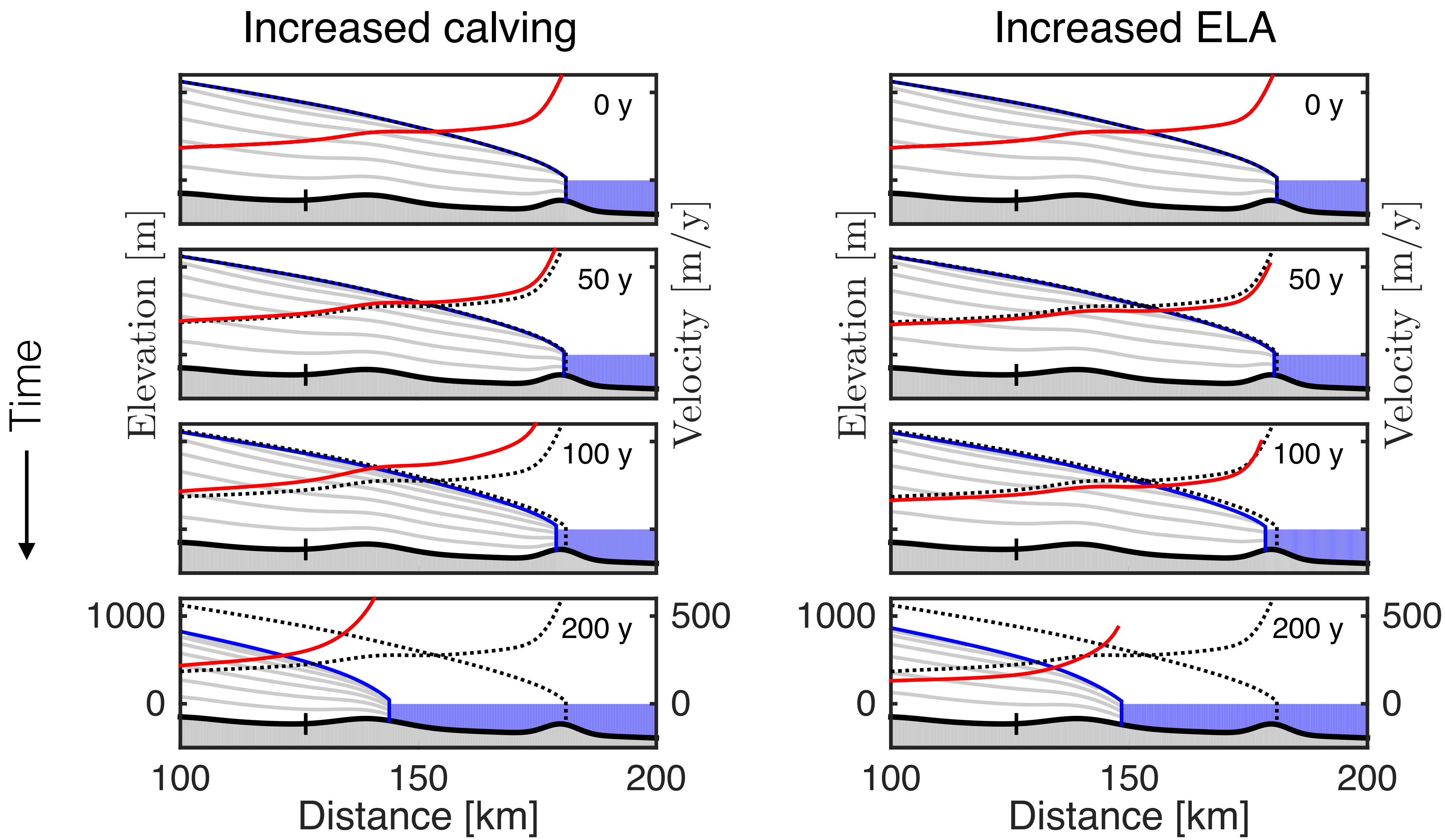
$$\frac{\mathrm{d}V}{\mathrm{d}t} = \int_0^{x_m} a \ \mathrm{d}x - q_c$$

$$q_c = \frac{\rho_i g}{\eta_i \mu} \hat{\mathcal{Q}}(f) h_f^3$$



Marine terminating glacier

An essentially indistinguishable response occurs to an **increase in calving** or an **increase in ELA**



Summary

Subglacial meltwater can both **increase** and **decrease** average ice speeds. Changes in **either direction** have potential to influence ice loss.

A simplified model suggests ice-sheet **slow down** can help induce **tidewater-glacier retreat**, and hence may facilitate rapid ice loss.

Recent retreat and speed-up of tidewater glaciers in Greenland may be as much a response to **inland** forcing as **ocean** forcing.