Two-phase flow dynamics in ice sheets

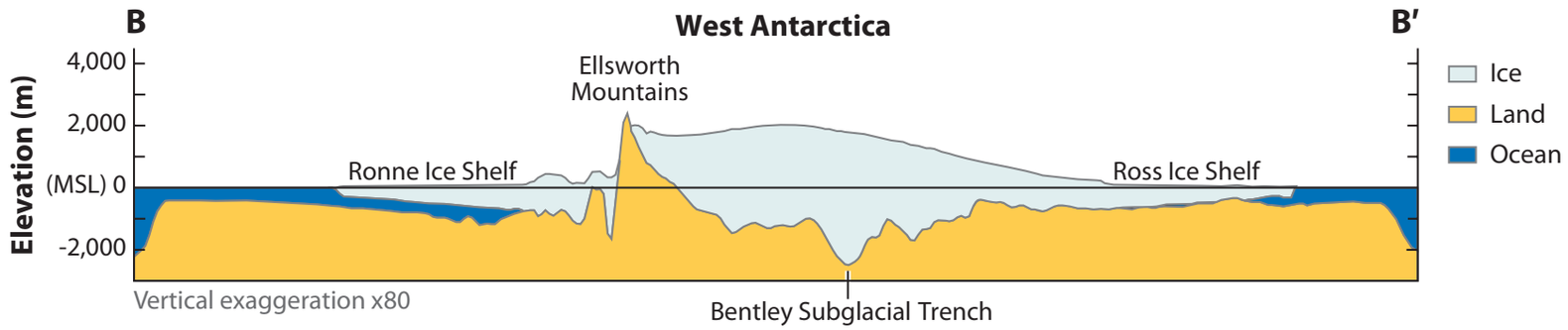
Ian Hewitt, University of Oxford

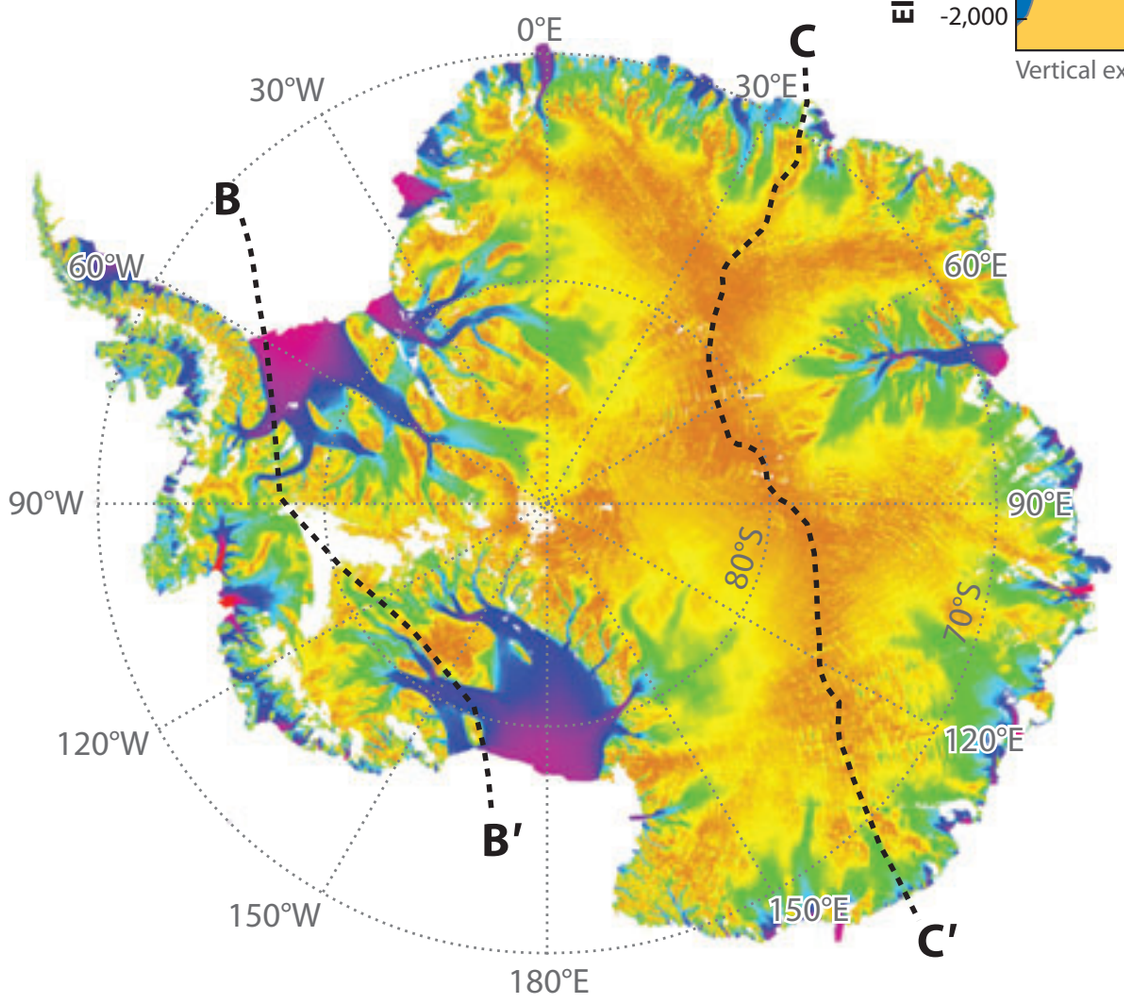
Thanks to: Christian Schoof, Richard Katz, ERC

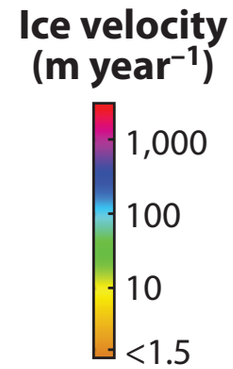
1. Subglacial drainage systems as a compacting/reacting porous medium

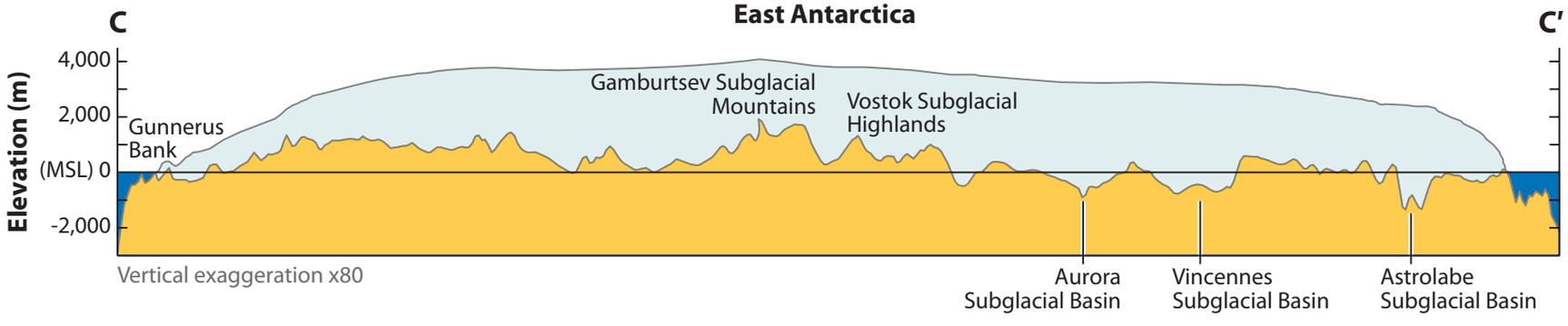
2. Two-phase mechanics of temperate ice

Scene setting









Introduction

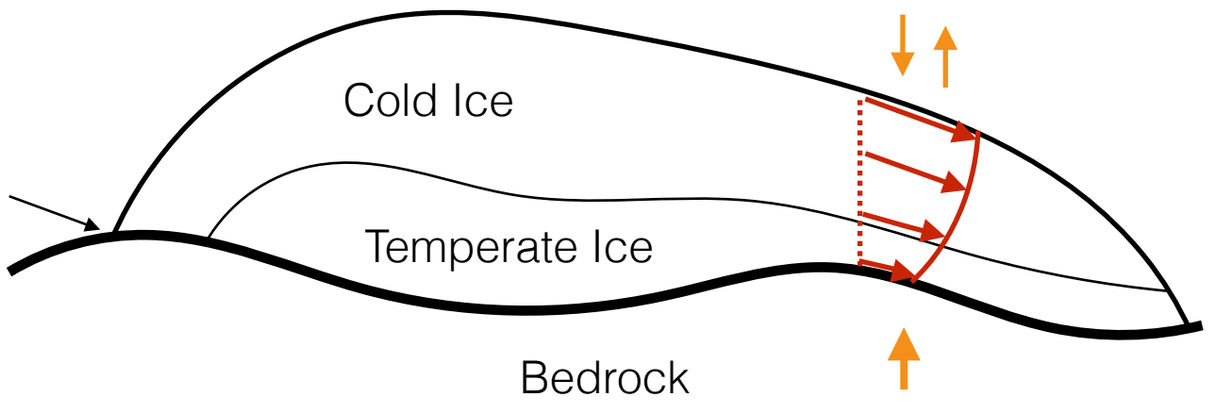
Ice is a polycrystalline material. It deforms through a combination of diffusion and dislocation creep.

Models usually treat ice sheets as a shear-thinning viscous fluid, with effective viscosity dependent on temperature.

$$\tau_{ij} = 2\eta\dot{\varepsilon}_{ij} \qquad \eta = \eta(T, \phi)$$

Basal boundary condition is crucial

$$\boldsymbol{\tau}_b = \beta(\boldsymbol{x}, U_b, \ldots)\mathbf{u}_b$$



Ice temperature controlled by: radiation, advection, conduction, shear heating, basal friction and geothermal heat flux.

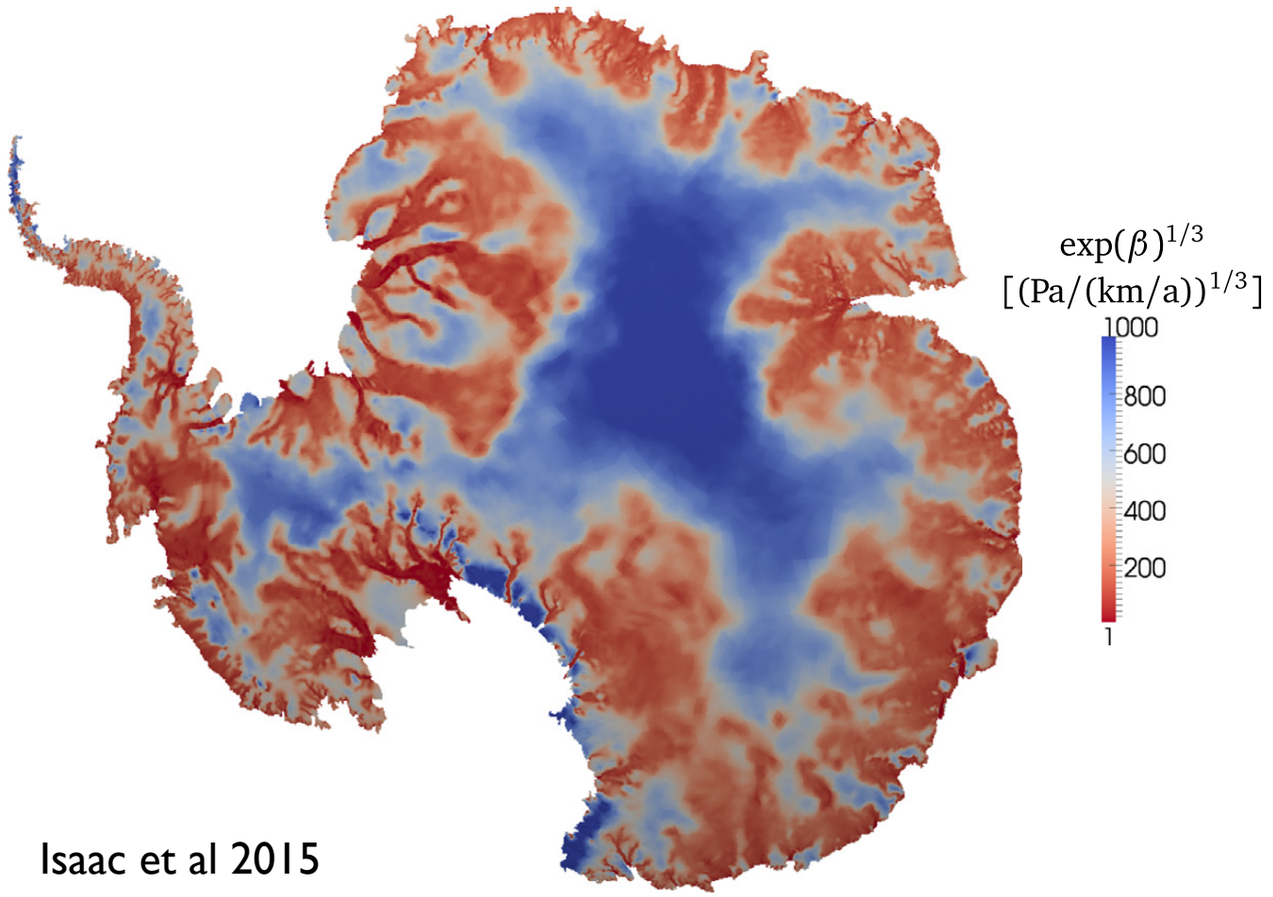
Subglacial effective pressure

Evidence suggests that basal slipperiness depends on subglacial water pressure

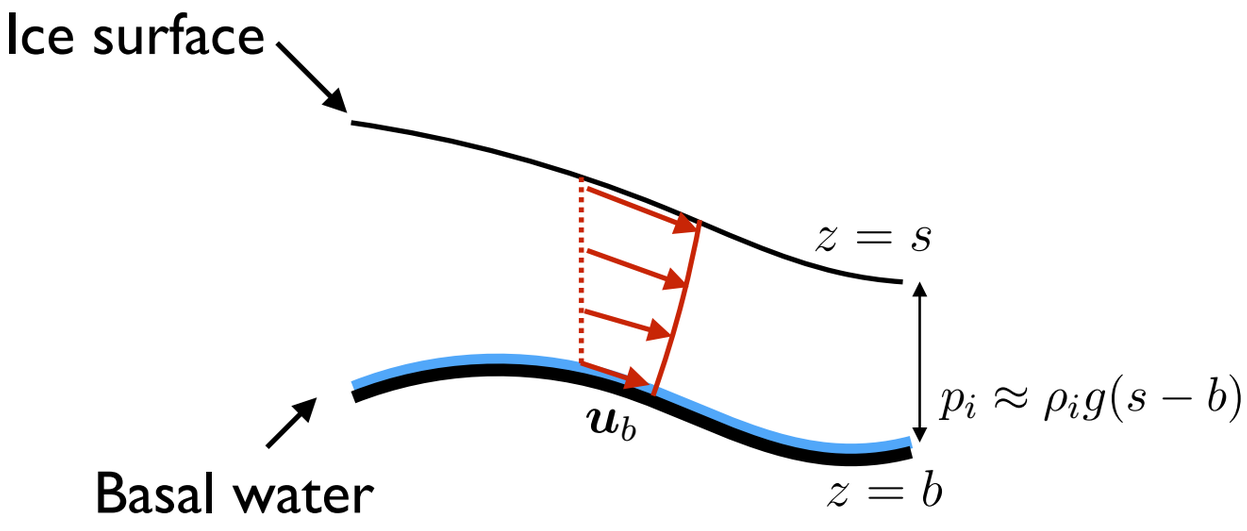
$$\boldsymbol{\tau}_b = \beta(\boldsymbol{x}, U_b, N)\mathbf{u}_b$$

Effective pressure $N = p_i - p_w$

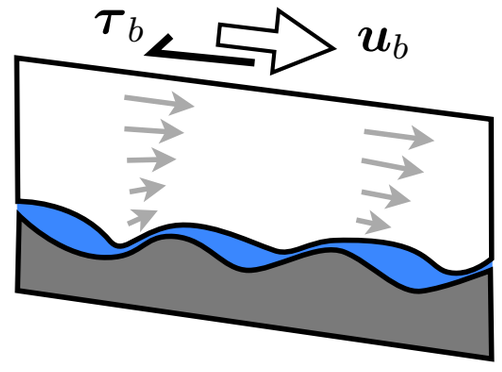
Inferred basal slipperiness



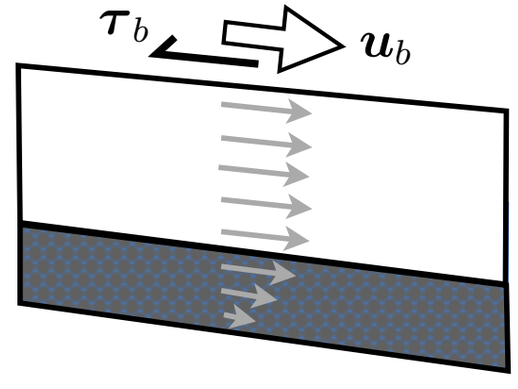
Isaac et al 2015



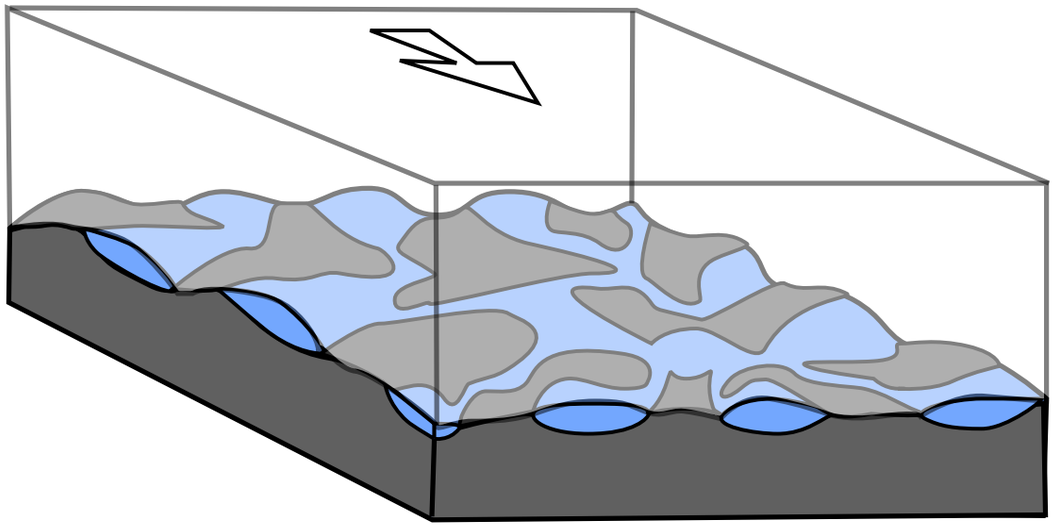
Water-filled cavities

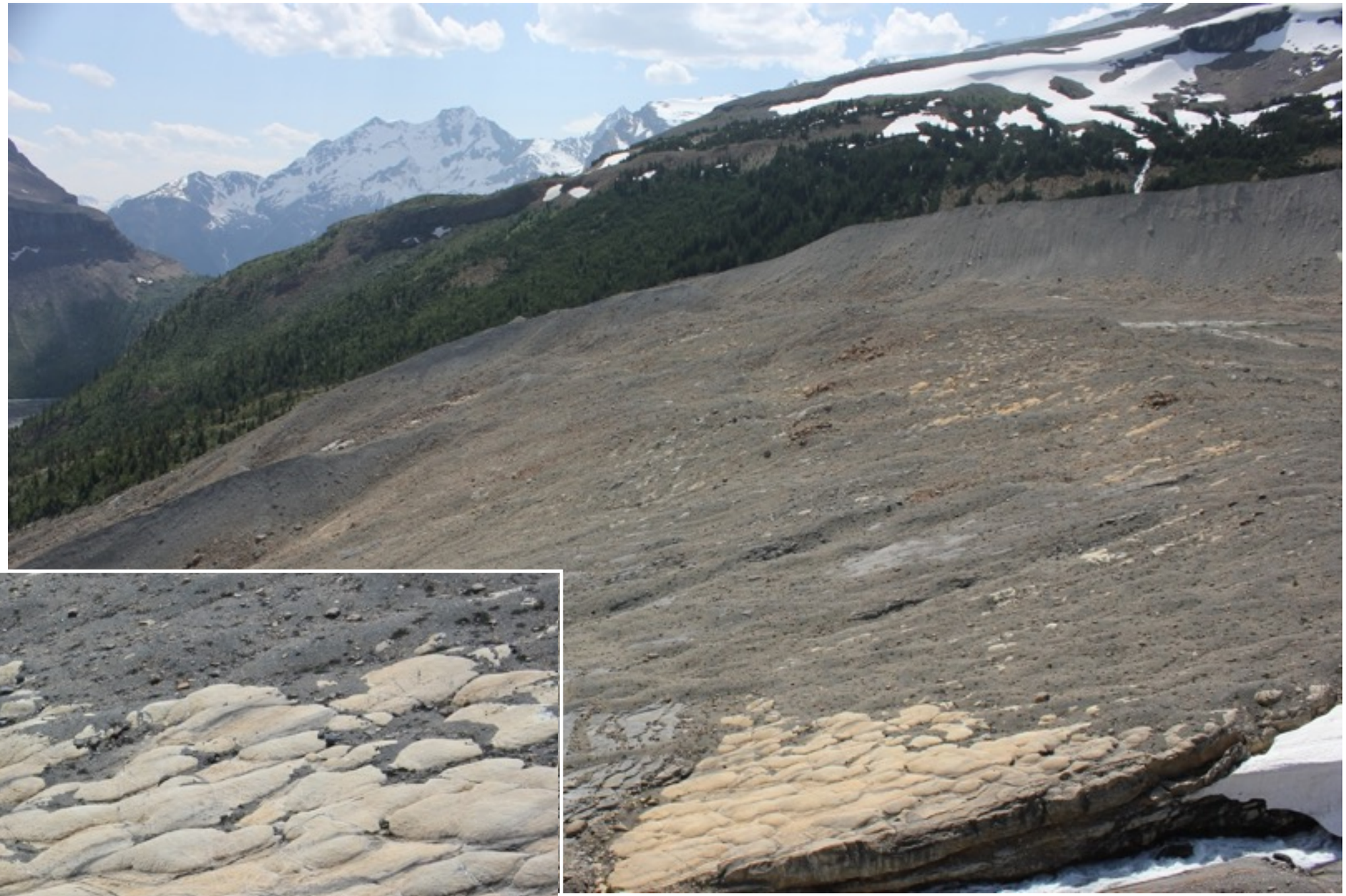


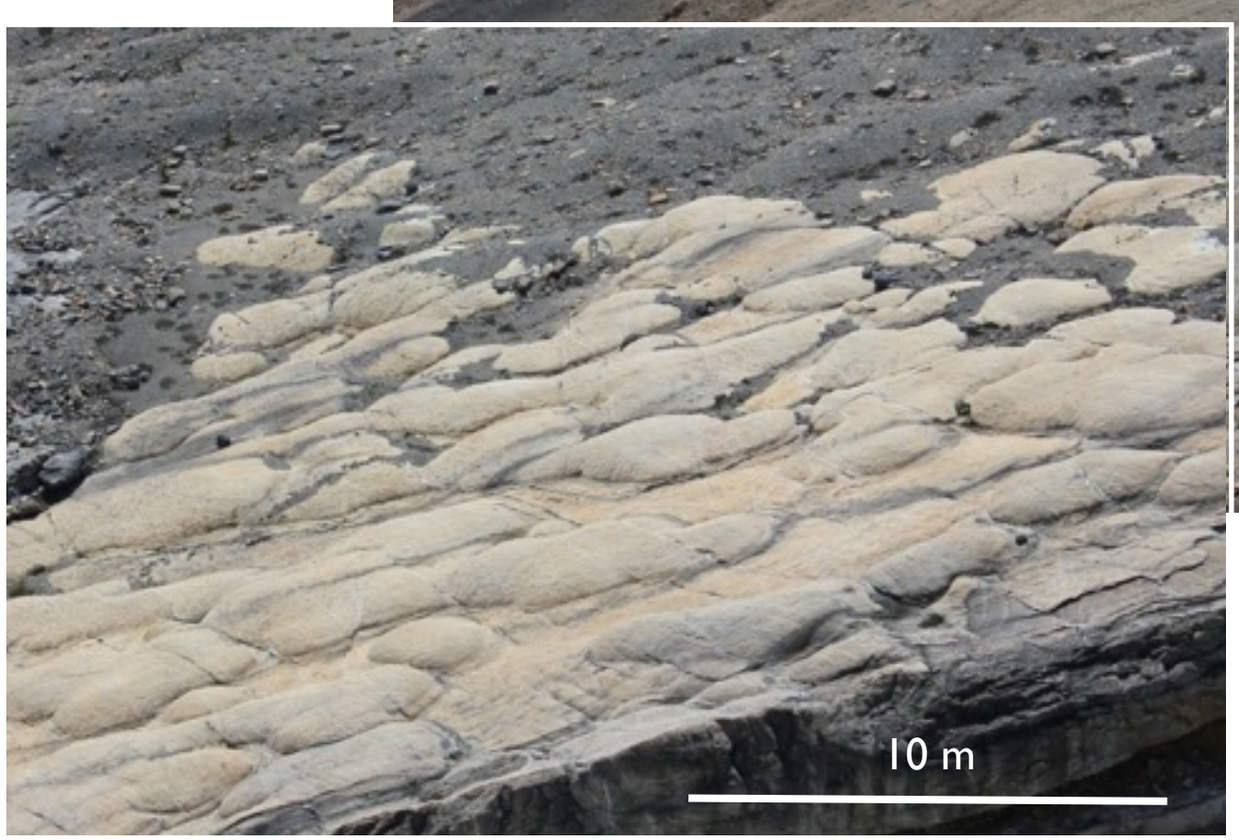
Deformation of wet till



Subglacial drainage systems



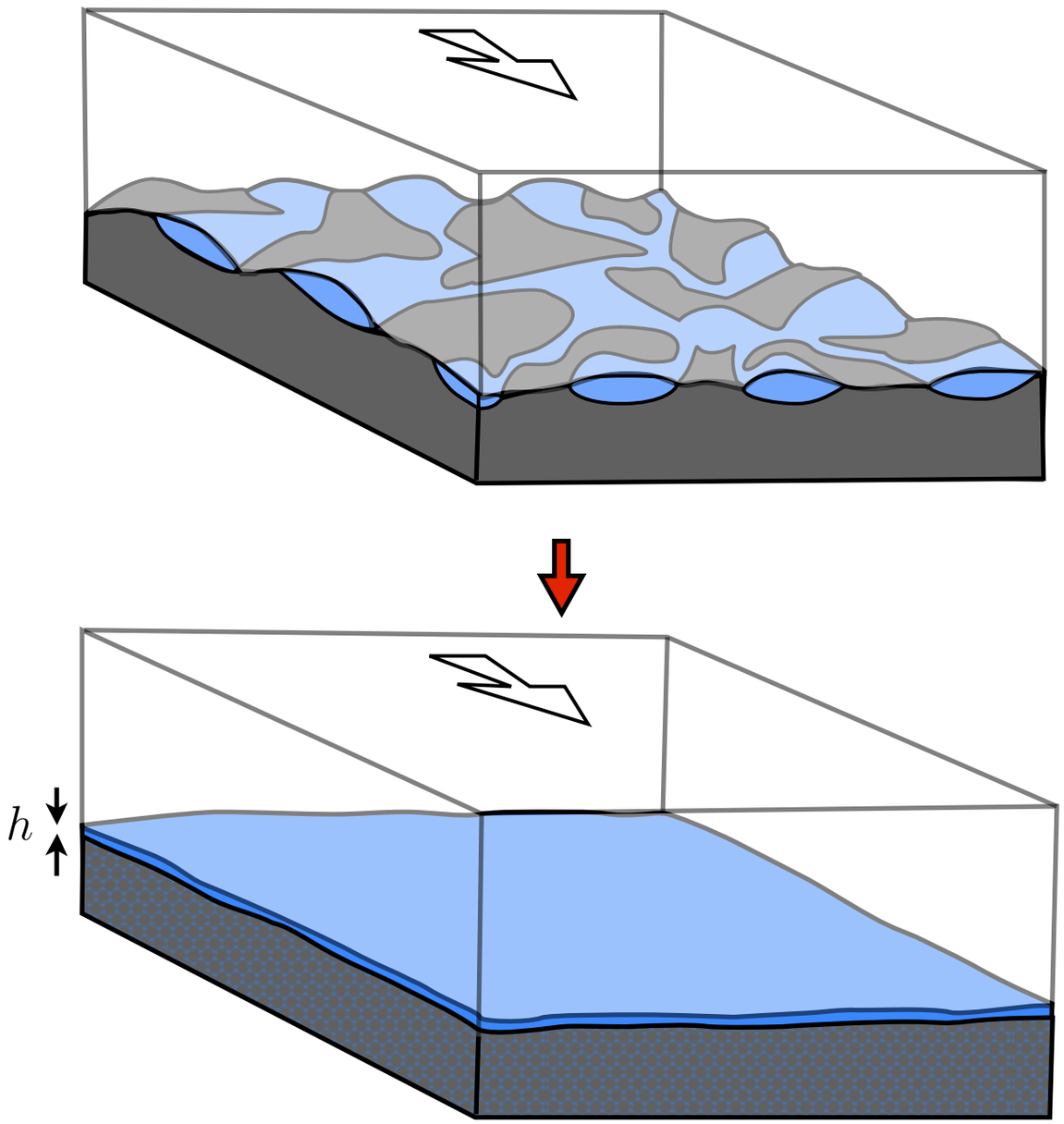




Mount Robson, Canada

Cordillera Blanca, Peru

A distributed drainage model



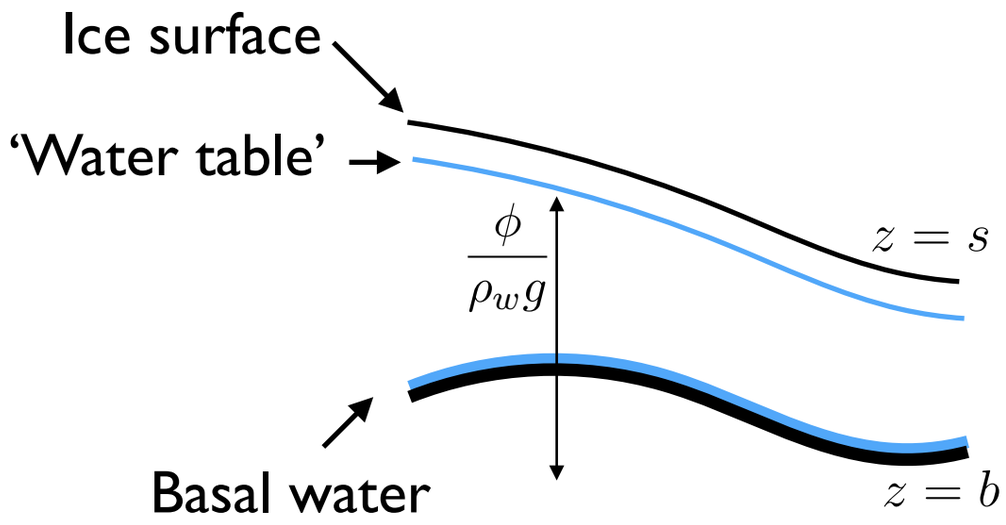
Mass conservation $\frac{\partial h}{\partial t} + \nabla \cdot \mathbf{q} = m + r$

Flux law $\mathbf{q} = -\mathcal{K}(h)\nabla\phi$

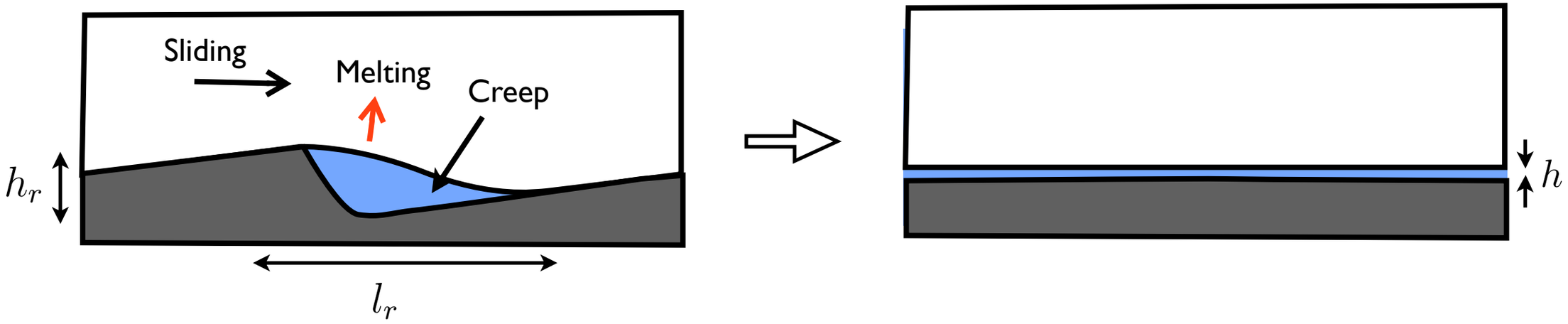
Hydraulic potential $\phi = \rho_w g b + p_w$

$= \rho_i g s + (\rho_w - \rho_i) g b - N$

Evolution law $\frac{\partial h}{\partial t} = \dots$



A distributed drainage model



Evolution law

$$\frac{\partial h}{\partial t} = \frac{\rho_w}{\rho_i} m + \frac{U_b}{\ell_r}(h_r - h) - \tilde{c}\frac{hN}{\eta}$$

Sliding

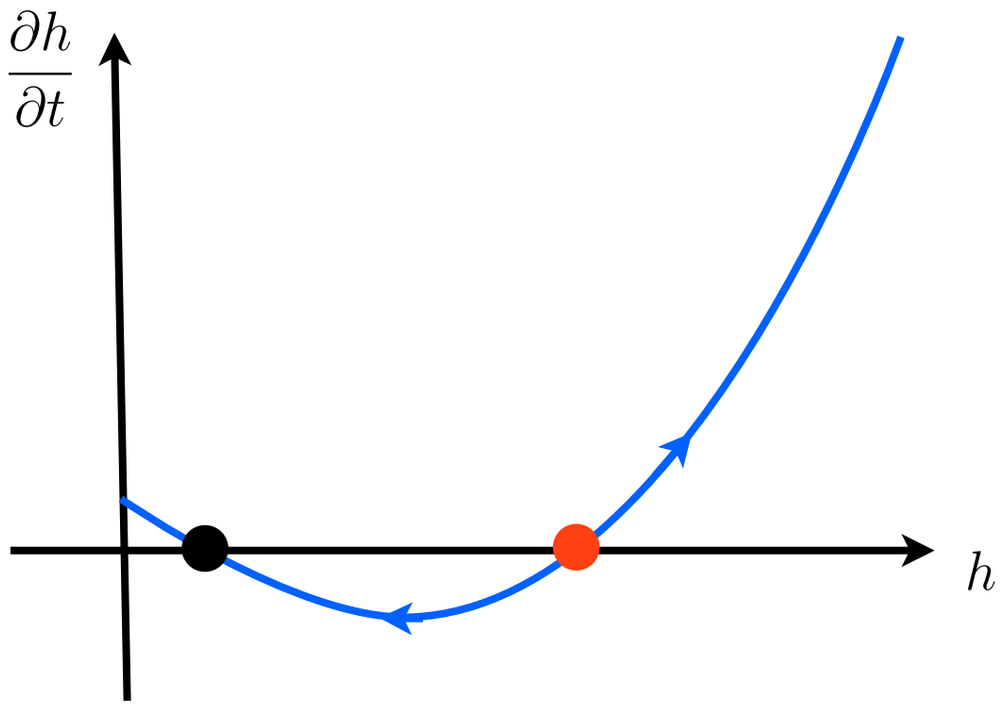
Creep closure

Melting

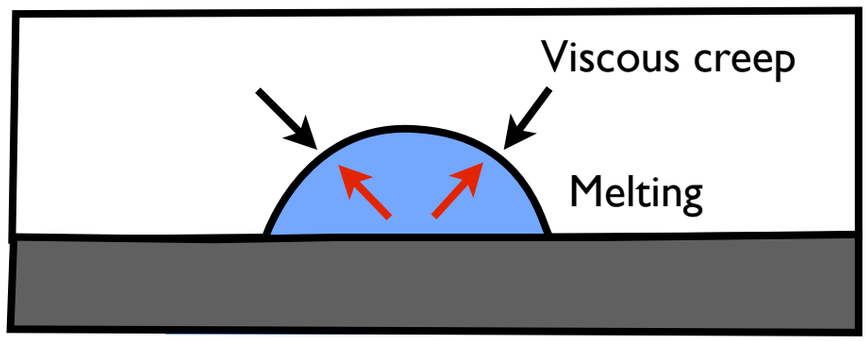
$$m = \frac{G + kT_z + \boldsymbol{\tau}_b \cdot \mathbf{u}_b + |\mathbf{q} \cdot \nabla \phi|}{\rho_w L}$$

Walder 1986, Fowler 1986, Kamb 1987, Schoof et al 2012

A distributed drainage model



A conduit drainage model



Mass conservation $$\frac{\partial S}{\partial t} + \frac{\partial Q}{\partial s} = M + q_{in}$$

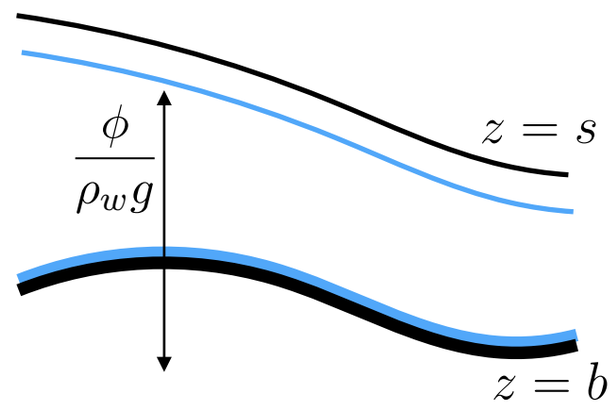
Flux law $$Q = -K_c S^{\alpha} \left| \frac{\partial \phi}{\partial s} \right|^{-1/2} \frac{\partial \phi}{\partial s}$$

Hydraulic potential $$\phi = \rho_w g b + p_w$$
$$= \rho_i g s + (\rho_w - \rho_i) g b - N$$

Cross-sectional area evolution

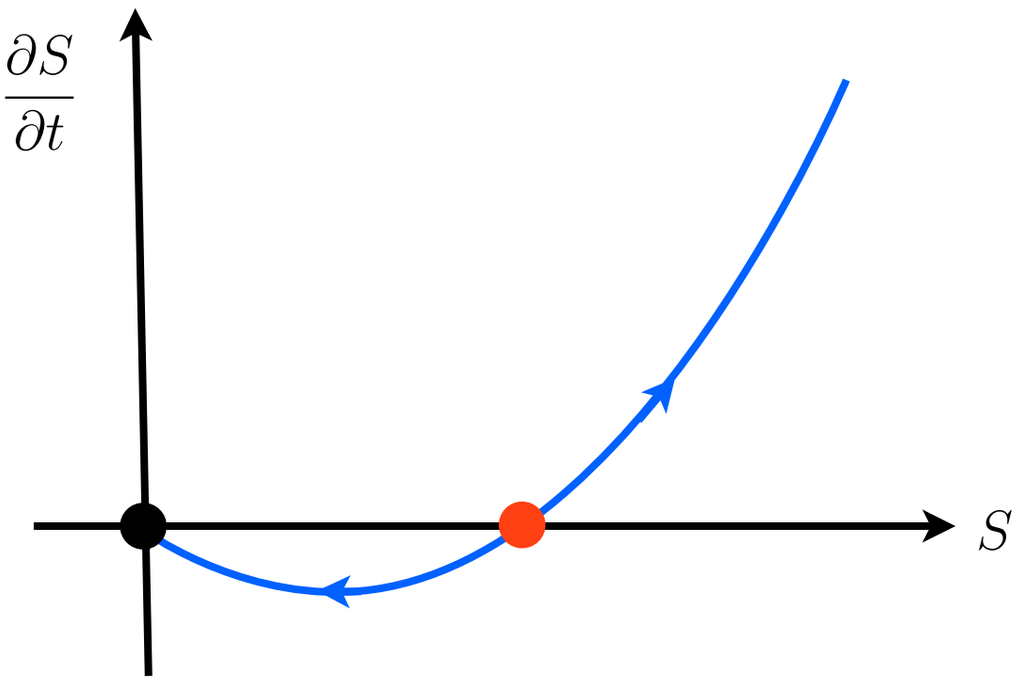
$$\boxed{\frac{\partial S}{\partial t} = \frac{\rho_w}{\rho_i} M - \tilde{C} \frac{S N}{\eta}}$$

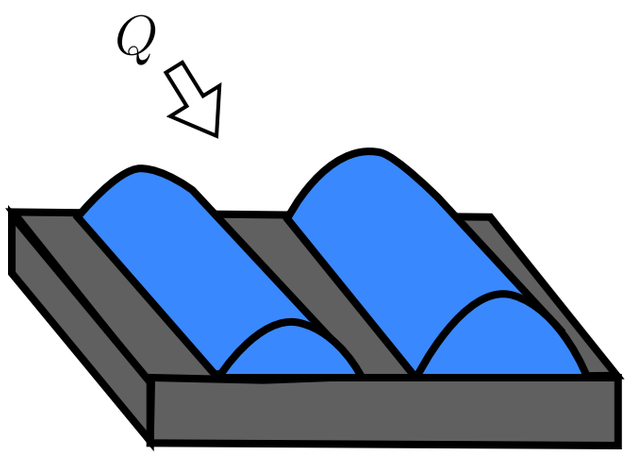
Melting $$M = \frac{1}{\rho_w L} \left| Q \frac{\partial \phi}{\partial s} \right|$$



Röthlisberger 1972, Nye 1976

Subglacial conduits

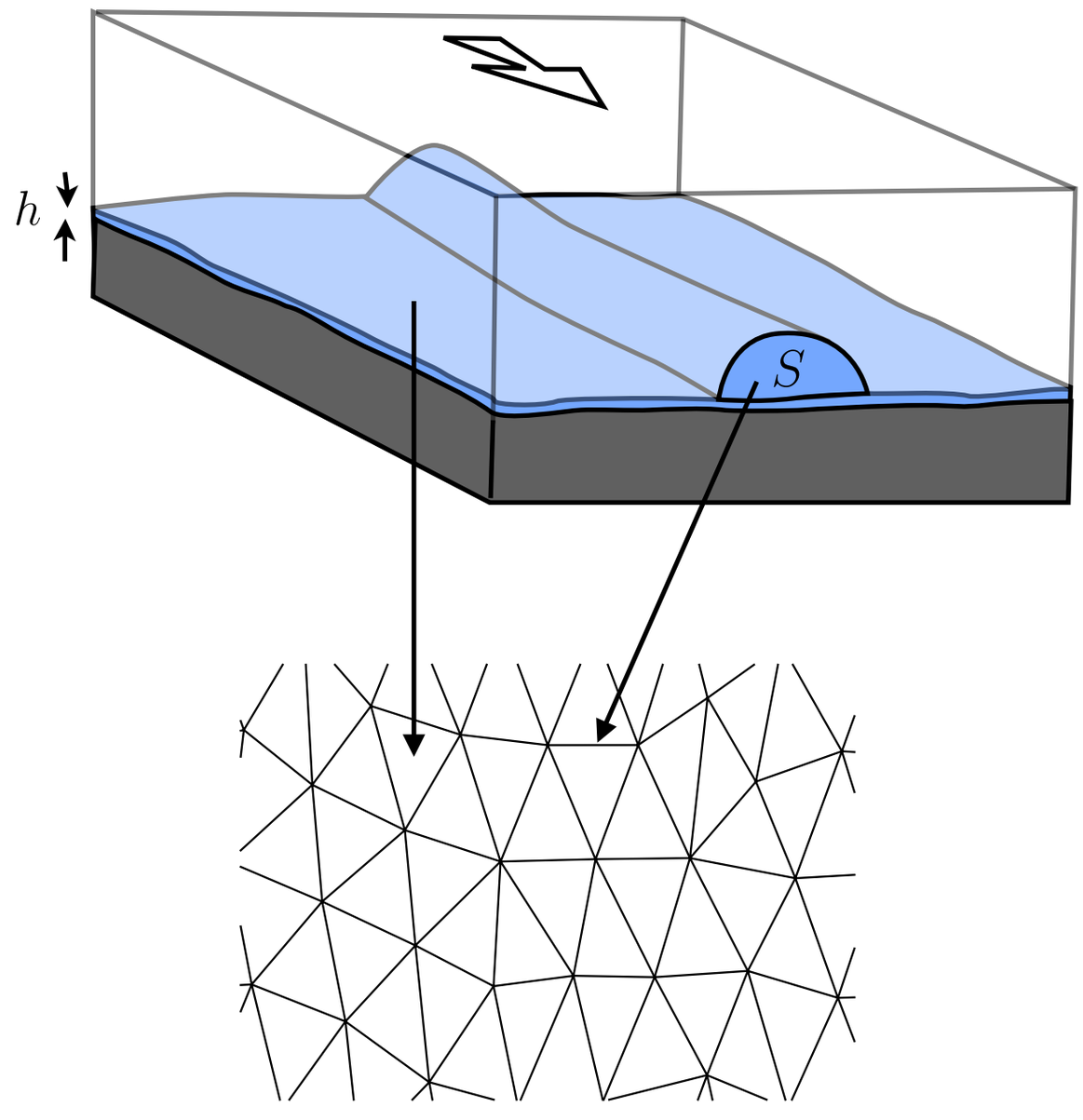




Mass conservation prevents unbounded growth

... but neighbouring channels compete with one other

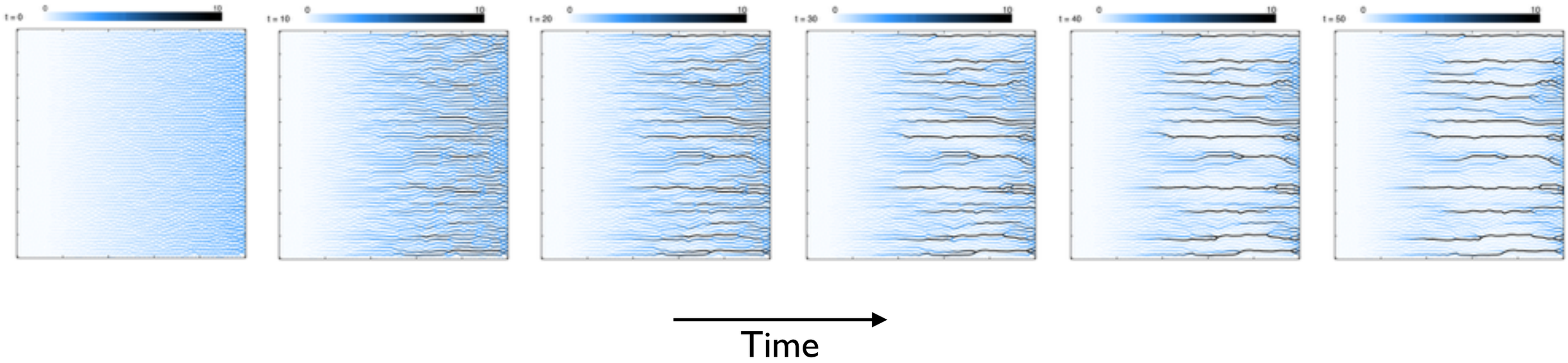
A hybrid sheet-conduit drainage model



Numerical method combines 1d and 2d finite elements

Werder et al 2013

A hybrid sheet-conduit drainage model



Werder et al 2013

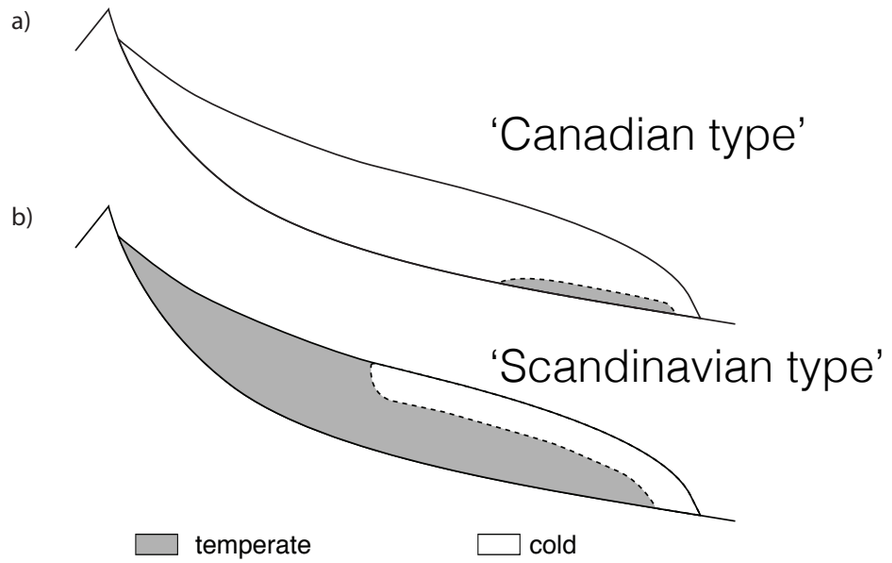
1. Subglacial drainage systems as a compacting/reacting porous medium

2. Two-phase mechanics of temperate ice

Temperature of glaciers and ice sheets

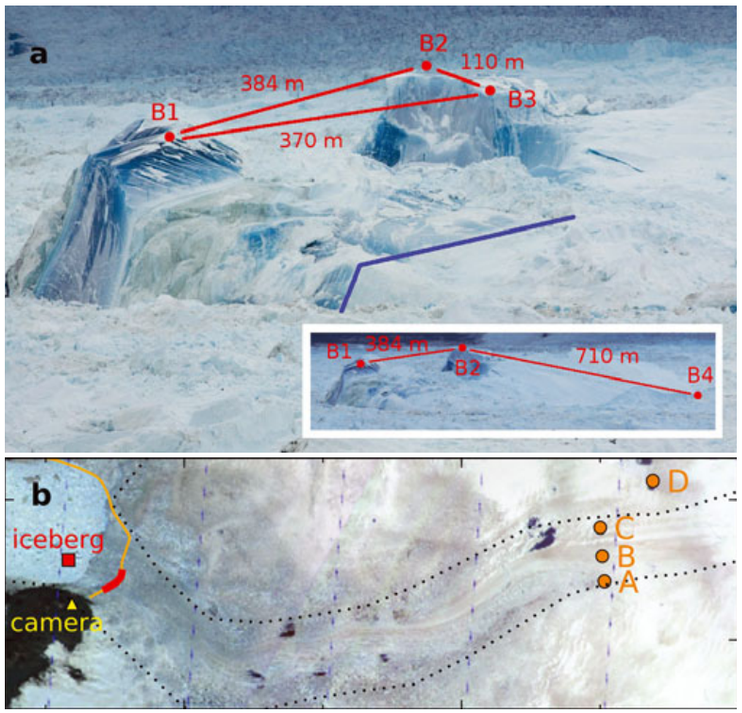
Many mid-latitude alpine glaciers are entirely ‘temperate’.

In high latitudes many glaciers are ‘polythermal’.



Aschwanden et al (2012)

Most of the large ice-sheets (Antarctica & Greenland) are ‘cold’, but may be locally temperate near base



Lüthi et al (2009)

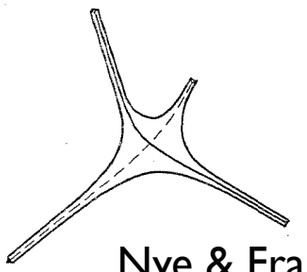
Temperate = in thermodynamic equilibrium at $T = T_m(p)$

History

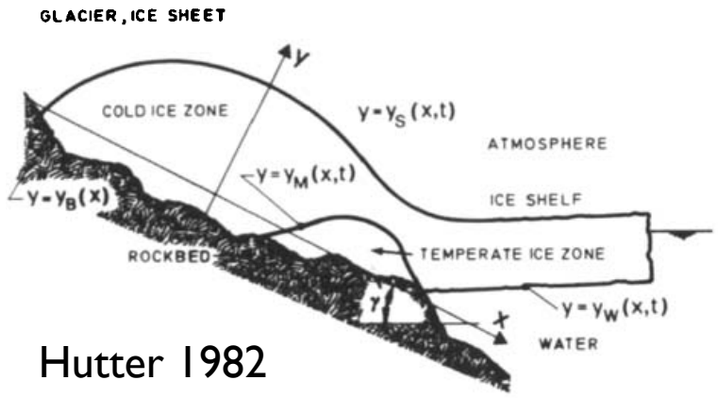
Lliboutry (1971,1976), Nye & Frank (1973) - permeability

Duval (1977) - melt-dependent viscosity

Fowler & Larson (1978) - continuum formulation, no moisture movement



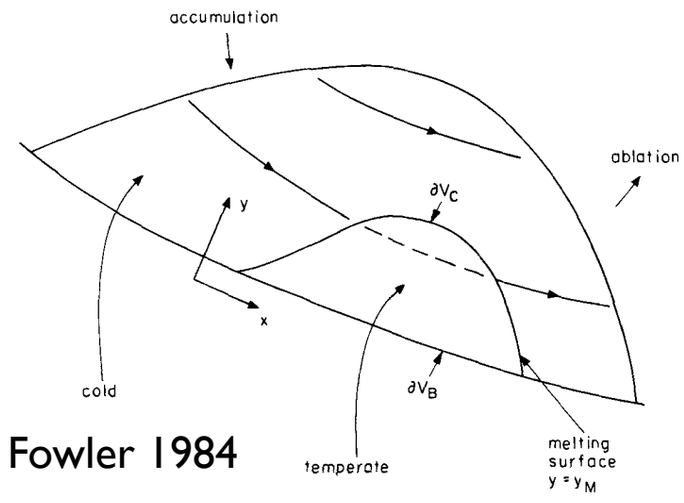
Nye & Frank 1973



Hutter 1982

Hutter (1982) - mixture theory, diffusive moisture transport

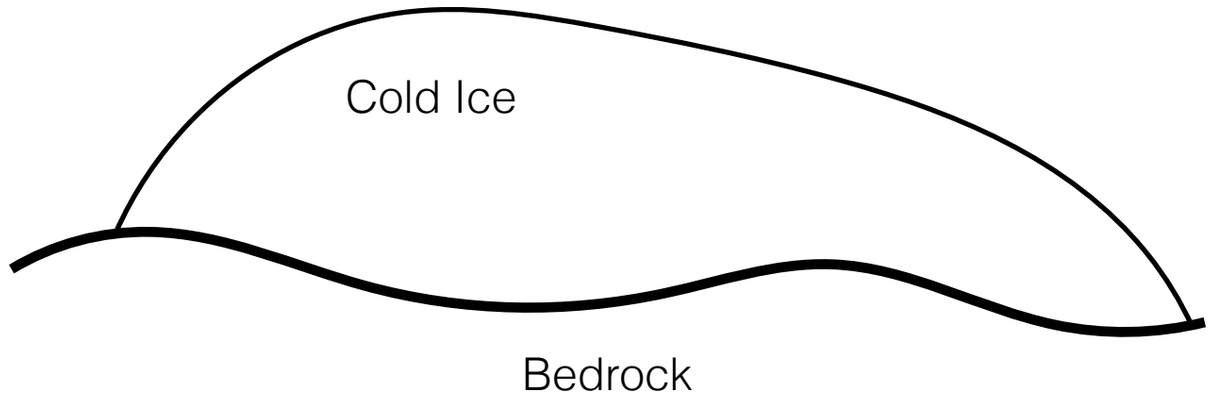
Fowler (1984) - two-phase theory, Darcy's law for moisture transport



Fowler 1984

Greve (1997), Aschwanden et al (2012) - computation, enthalpy methods

Cold ice



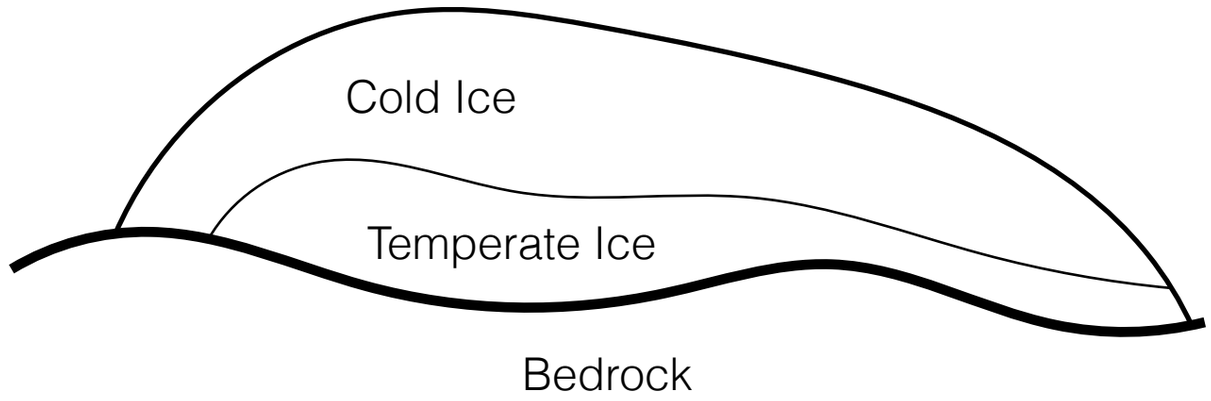
Stokes flow $\nabla \cdot \mathbf{u} = 0$

$$\frac{\partial \tau_{ij}}{\partial x_j} - \frac{\partial p}{\partial x_i} = -\rho g_i$$

Temperature dependent viscosity $\eta = \eta(T)$

Energy $$\rho c \left(\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) = \nabla \cdot (k \nabla T) + \tau_{ij} \dot{\varepsilon}_{ij}$$

Polythermal ice - no moisture movement



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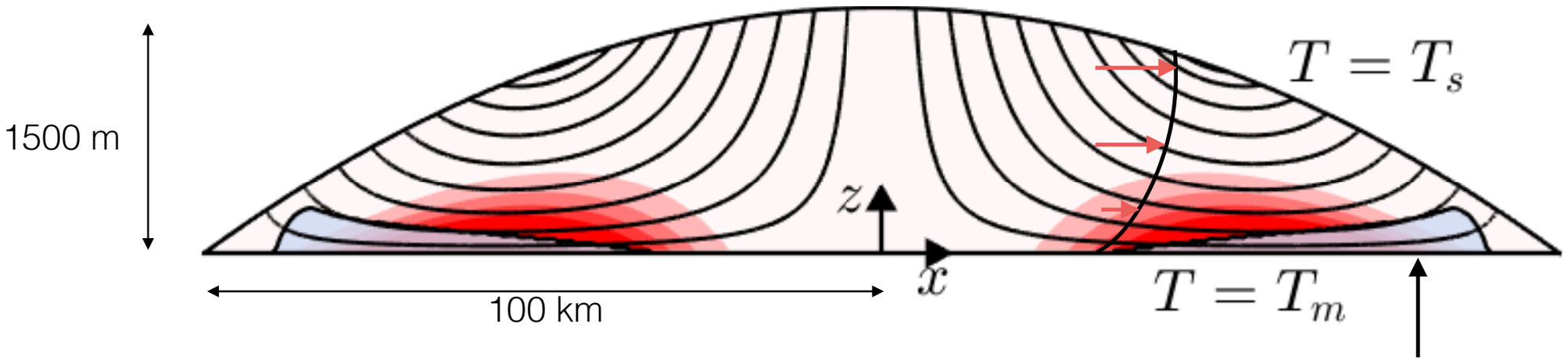
$$\frac{\partial \tau_{ij}}{\partial x_j} - \frac{\partial p}{\partial x_i} = -\rho g_i$$

Temperature and porosity dependent viscosity $\eta = \eta(T, \phi)$

Energy

$$\begin{cases} \rho c \left(\dfrac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) = \nabla \cdot (k \nabla T) + \tau_{ij} \dot{\varepsilon}_{ij}, & \phi = 0, \quad T \leq T_m \\ \rho_w L \left(\dfrac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi \right) = \tau_{ij} \dot{\varepsilon}_{ij}, & T = T_m, \quad \phi > 0 \end{cases}$$

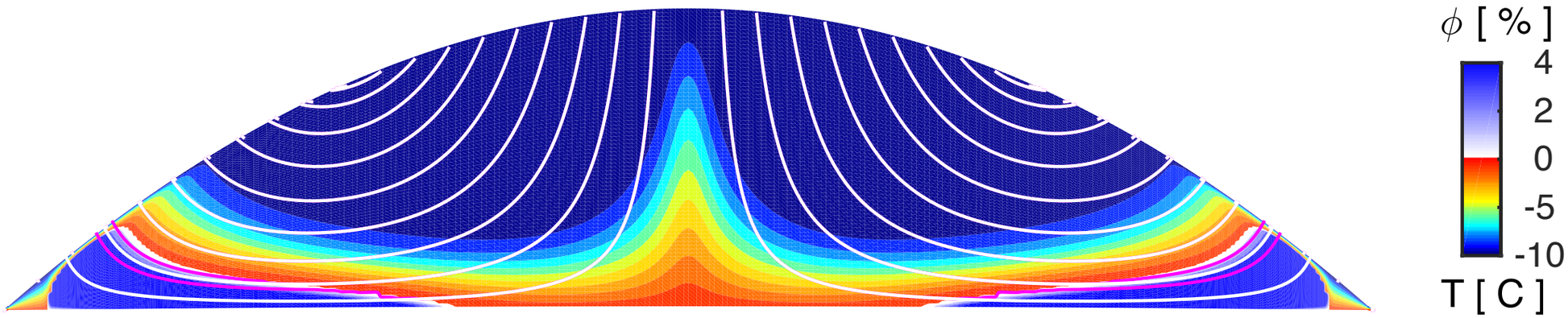
Ice divide example



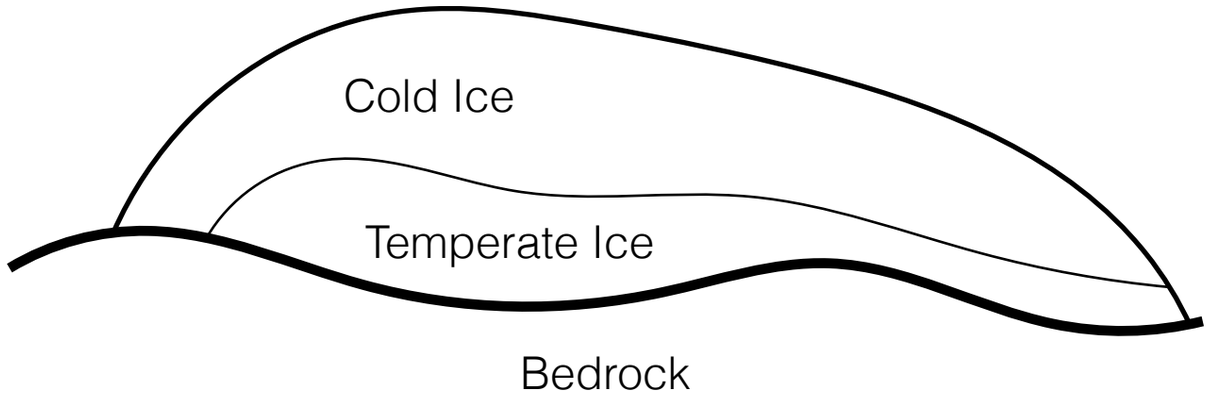
Pore pressure = subglacial drainage pressure $p_e = N_0$

- Velocity from shallow ice approximation (thermodynamically decoupled).

Ice divide example



Polythermal ice - with moisture movement



Stokes flow $\nabla \cdot \mathbf{u} = -\nabla \cdot \mathbf{j}$

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Temperature and porosity dependent viscosity $\eta = \eta(T, \phi)$

Energy

$$\begin{cases} \rho c \left(\dfrac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) = \nabla \cdot (k \nabla T) + \tau_{ij} \dot{\varepsilon}_{ij}, & \phi = 0, \quad T \leq T_m \\ \rho_w L \left(\dfrac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi \right) + \rho_w L \nabla \cdot \mathbf{j} = \tau_{ij} \dot{\varepsilon}_{ij}, & T = T_m, \quad \phi > 0 \end{cases}$$

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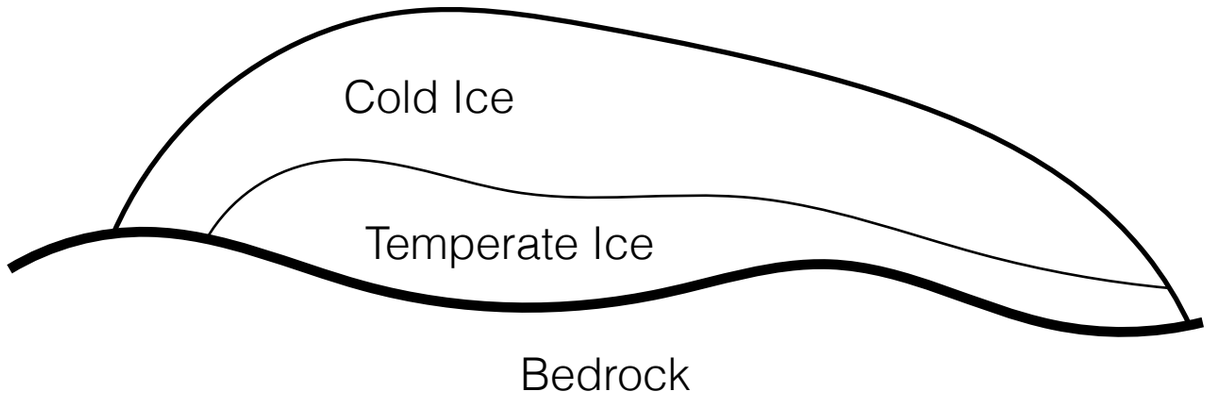


In terms of 'enthalpy',

$h = \rho c T + \rho_w L \phi$

$$\left(\frac{\partial h}{\partial t} + \mathbf{u} \cdot \nabla h\right) + \nabla \cdot \mathbf{Q} = \tau_{ij} \dot{\varepsilon}_{ij}, \qquad \mathbf{Q} = \begin{cases} -k \nabla T & h < \rho c T_m, \\ \rho_w L \, \mathbf{j} & h \geq \rho c T_m. \end{cases}$$

Relative moisture flux (Darcy's law)

$$\mathbf{j} = \frac{k_0 \phi^2}{\eta_w} (\rho_w \mathbf{g} - \nabla p_w)$$

Effective pressure

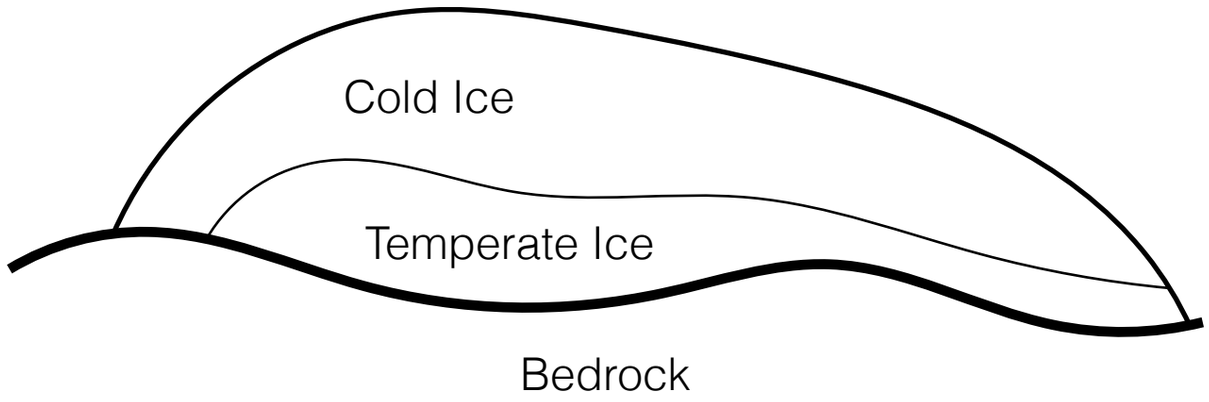
$$p_e = p - p_w - p_s(\phi) \longrightarrow \mathbf{j} = \frac{k_0 \phi^2}{\eta_w} ((\rho_w - \rho)\mathbf{g} + \nabla p_r + \nabla p_e + \nabla p_s)$$

Viscous compaction

$$p_e = \frac{\eta}{\phi} \nabla \cdot \mathbf{j}$$

Schoof & Hewitt 2015 *in review*

Polythermal ice - with moisture movement



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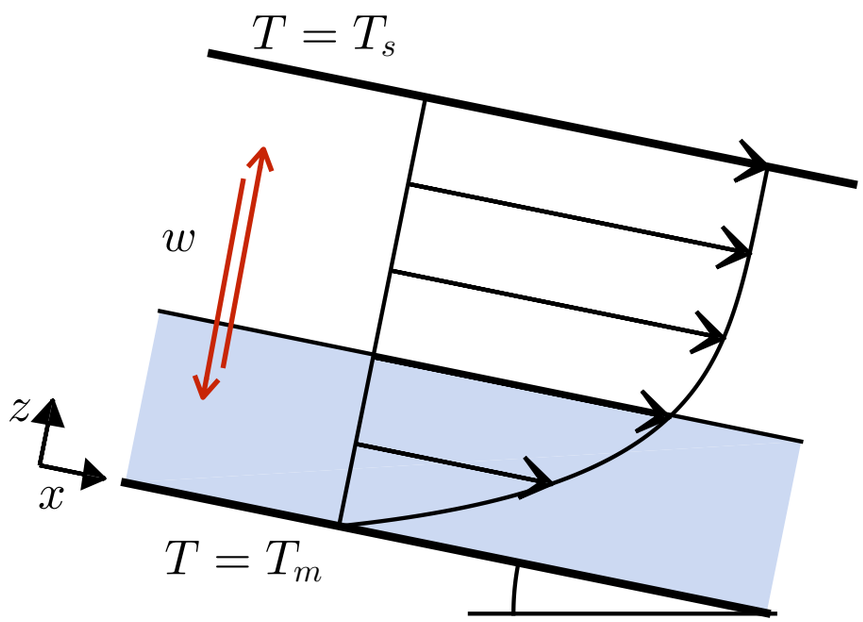
1) Compaction model

$$\mathbf{j} = \frac{k_0 \phi^\alpha}{\eta_w} \left((\rho_w - \rho)\mathbf{g} + \nabla p_e \right) \qquad \nabla \cdot \mathbf{j} = \frac{\phi p_e}{\eta}$$

2) Modified enthalpy model

$$\mathbf{j} = \frac{k_0 \phi^2}{\eta_w} \left((\rho_w - \rho)\mathbf{g} - \nu \nabla \phi \right) \qquad \nu = -\frac{\mathrm{d}p_s}{\mathrm{d}\phi}$$

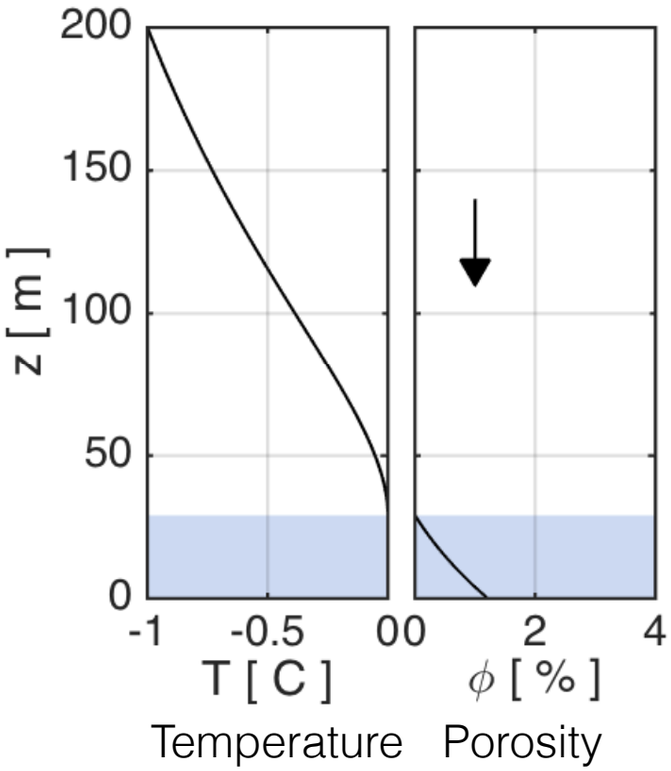
Slab glacier test case

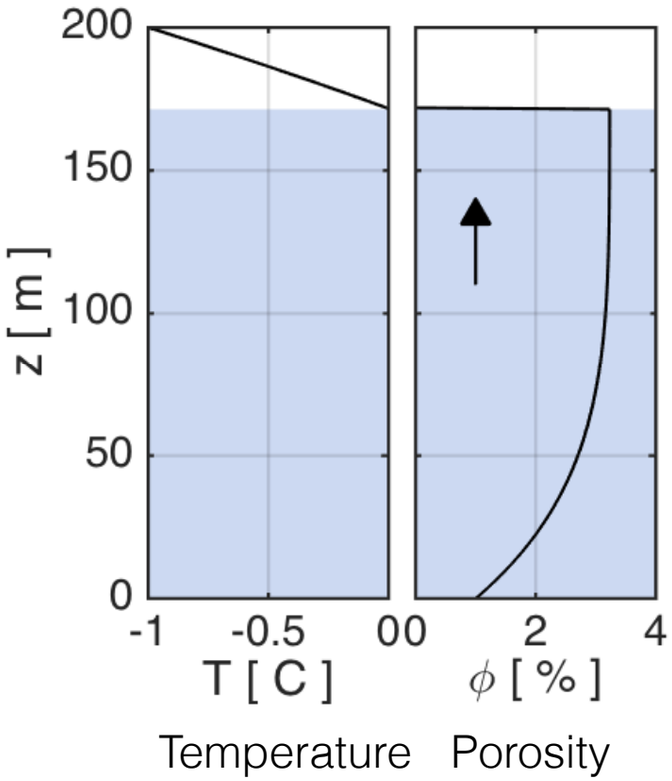


- Greve & Blatter (2009), Kleiner et al (2015), Blatter & Greve (2015)

Slab glacier test case

- Solutions for no relative water transport



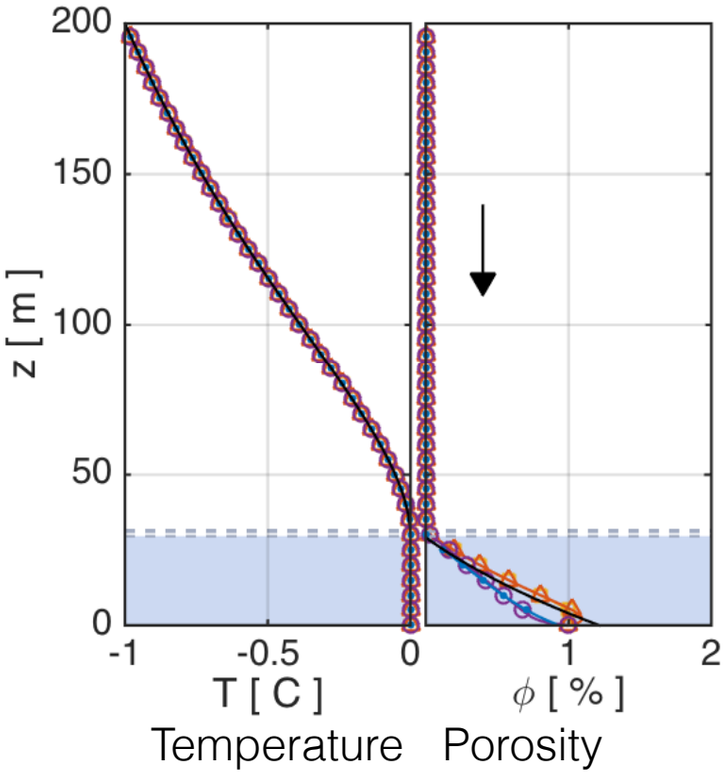


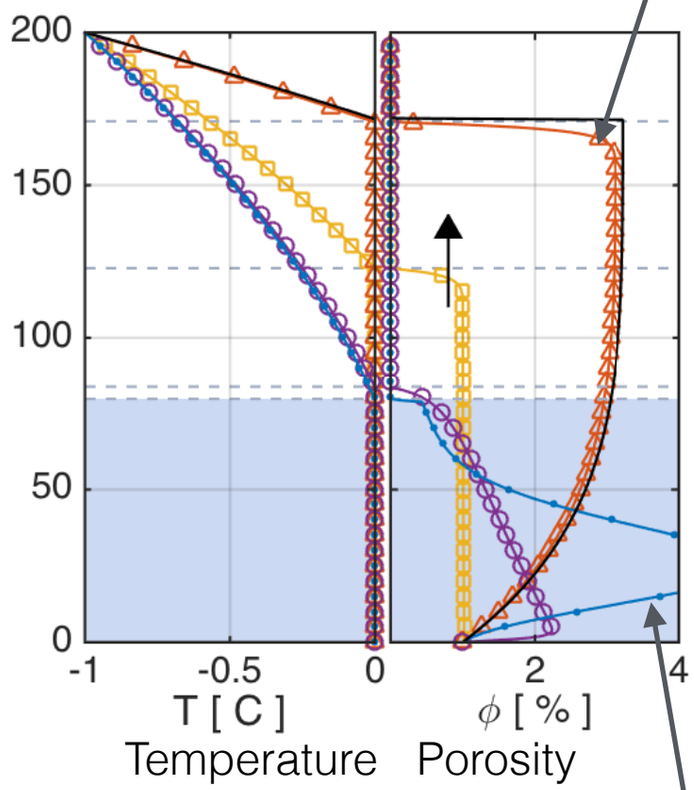
- Jump conditions $\rho_w L[\phi(\mathbf{u}-\mathbf{v})]^+ \cdot \mathbf{n} = -k\nabla T^- \cdot \mathbf{n} - \rho_w L\,\mathbf{j}\cdot\mathbf{n}$ Stefan condition

$\mathbf{j}\cdot\mathbf{n} = 0$

Slab glacier test case

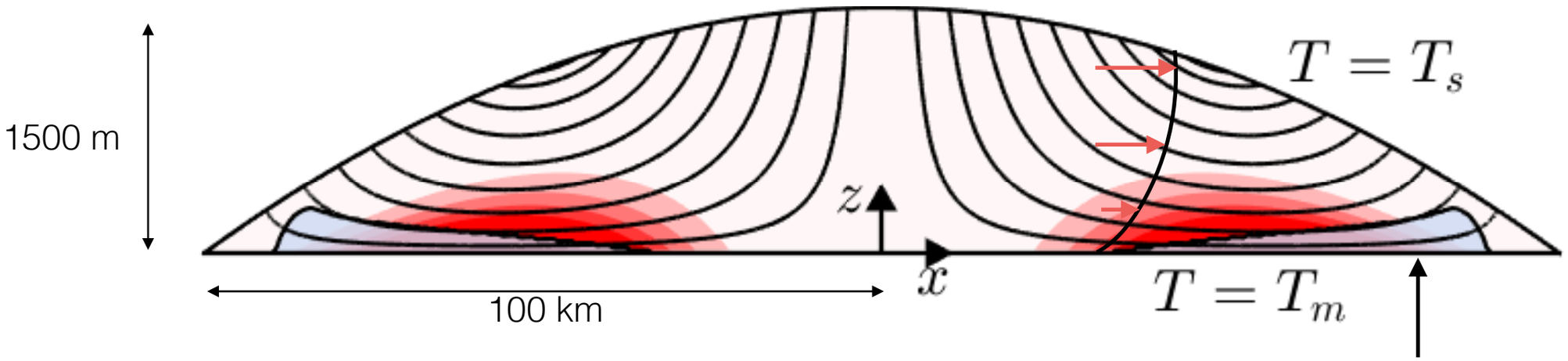
- Comparison of different models





- If gravity-driven drainage opposes advection, leads to formation of high porosity bands.
- Bands can descend as porosity waves.

Ice divide example

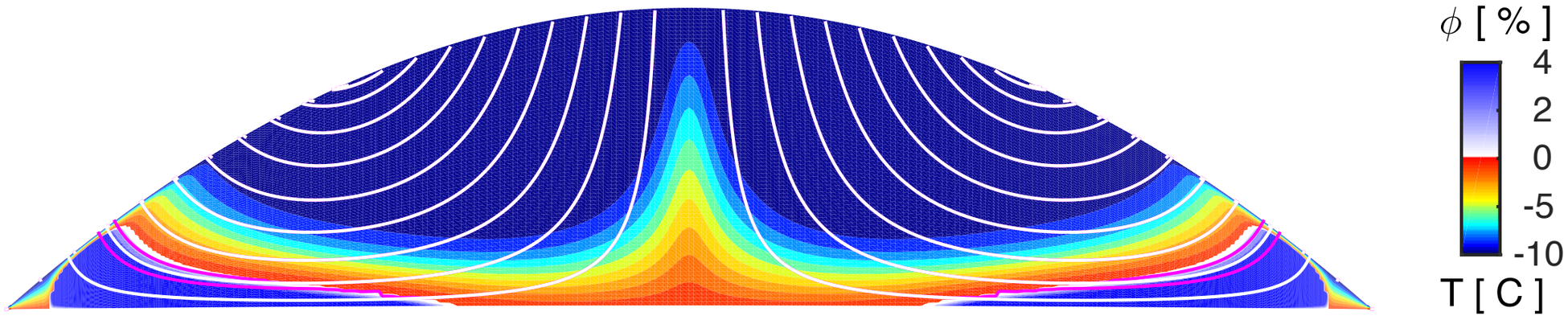


Pore pressure = subglacial drainage pressure $p_e = N_0$

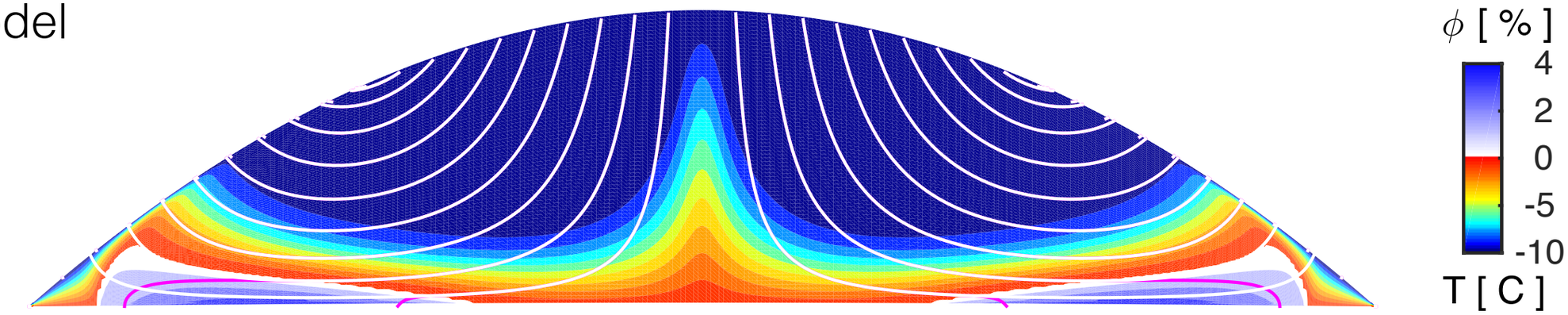
- Velocity from shallow ice approximation (thermodynamically decoupled).

Ice divide example

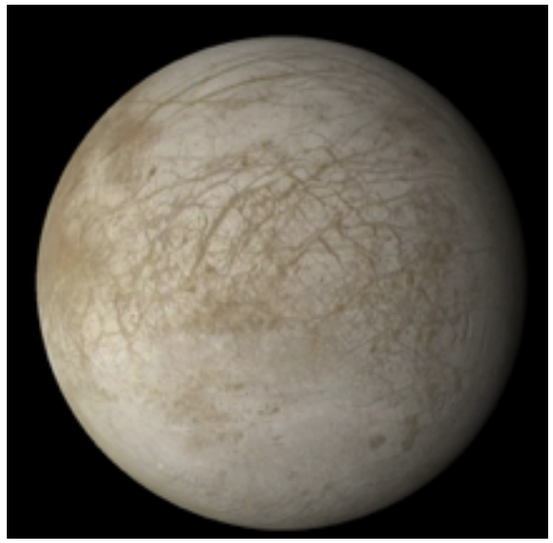
Standard enthalpy gradient model



Compaction pressure model

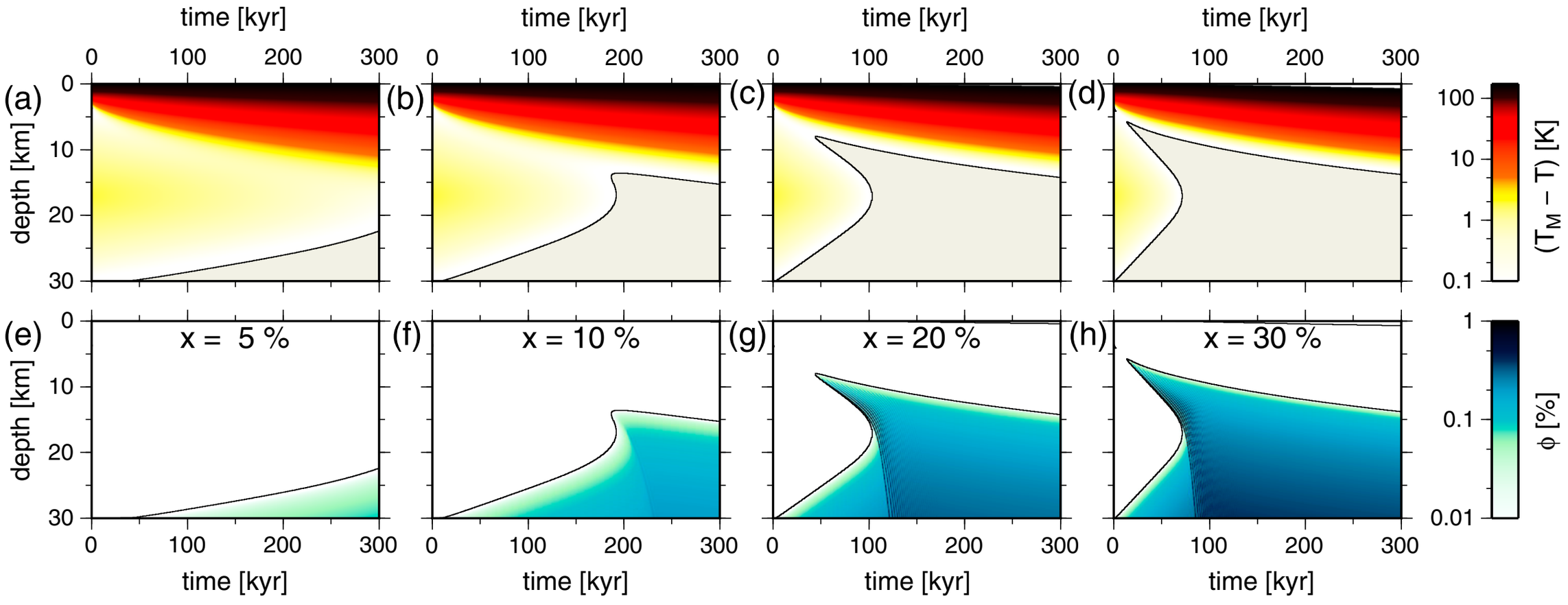


Something different - partial melting on Europa



Tidal heating results in partial melting of ice shell

Melt drains gravitationally into underlying ocean



Kalousova et al 2014

Summary

Subglacial drainage systems exhibit similar behaviour to compacting mantle - compaction, porosity waves, reactive feedbacks

Partially molten ice has porosity-weakening viscosity - may be dynamically important for ice sheets

Two phase compaction equations useful to understand moisture distribution in temperate ice sheets