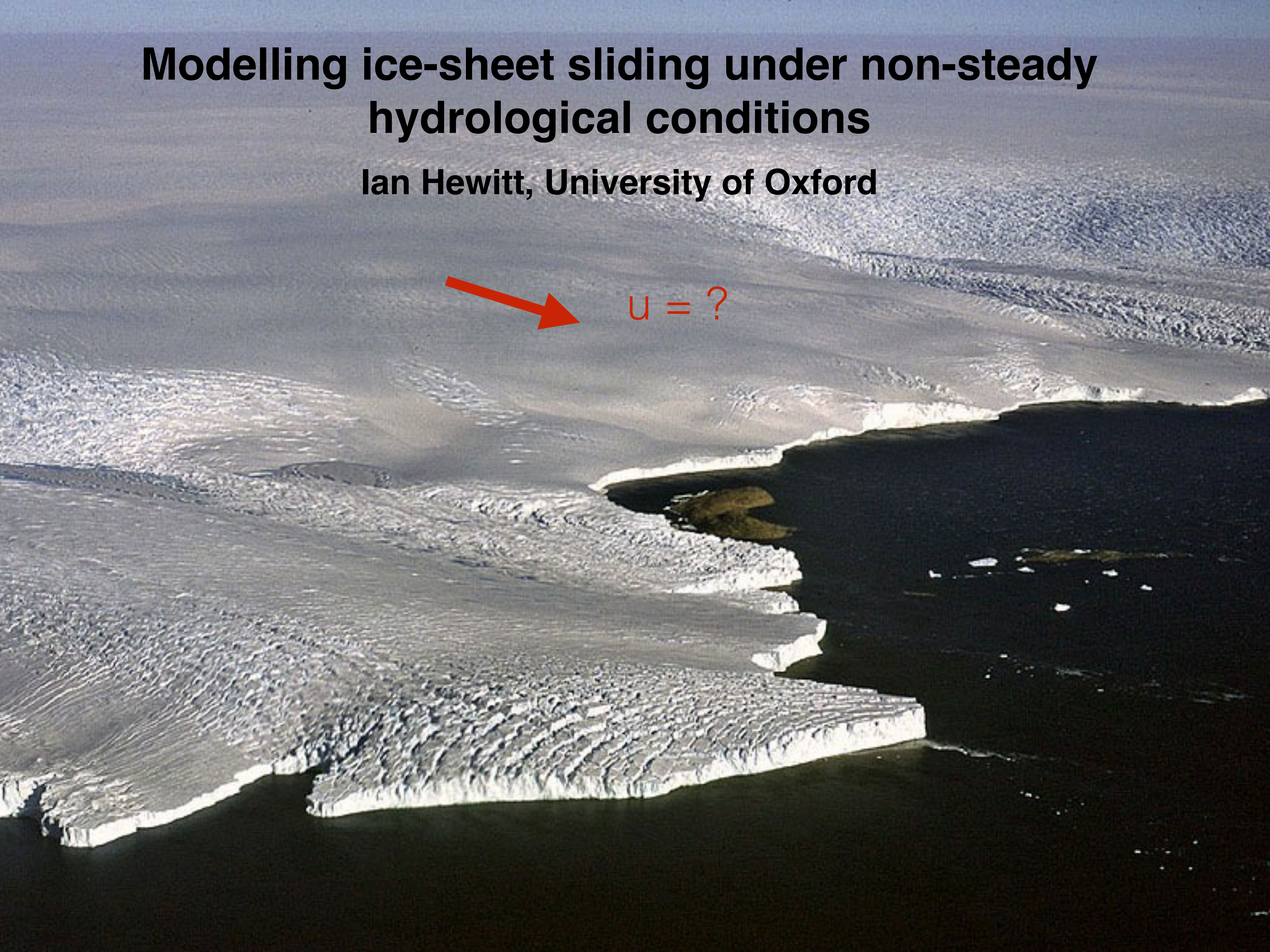
Modelling ice-sheet sliding under non-steady hydrological conditions

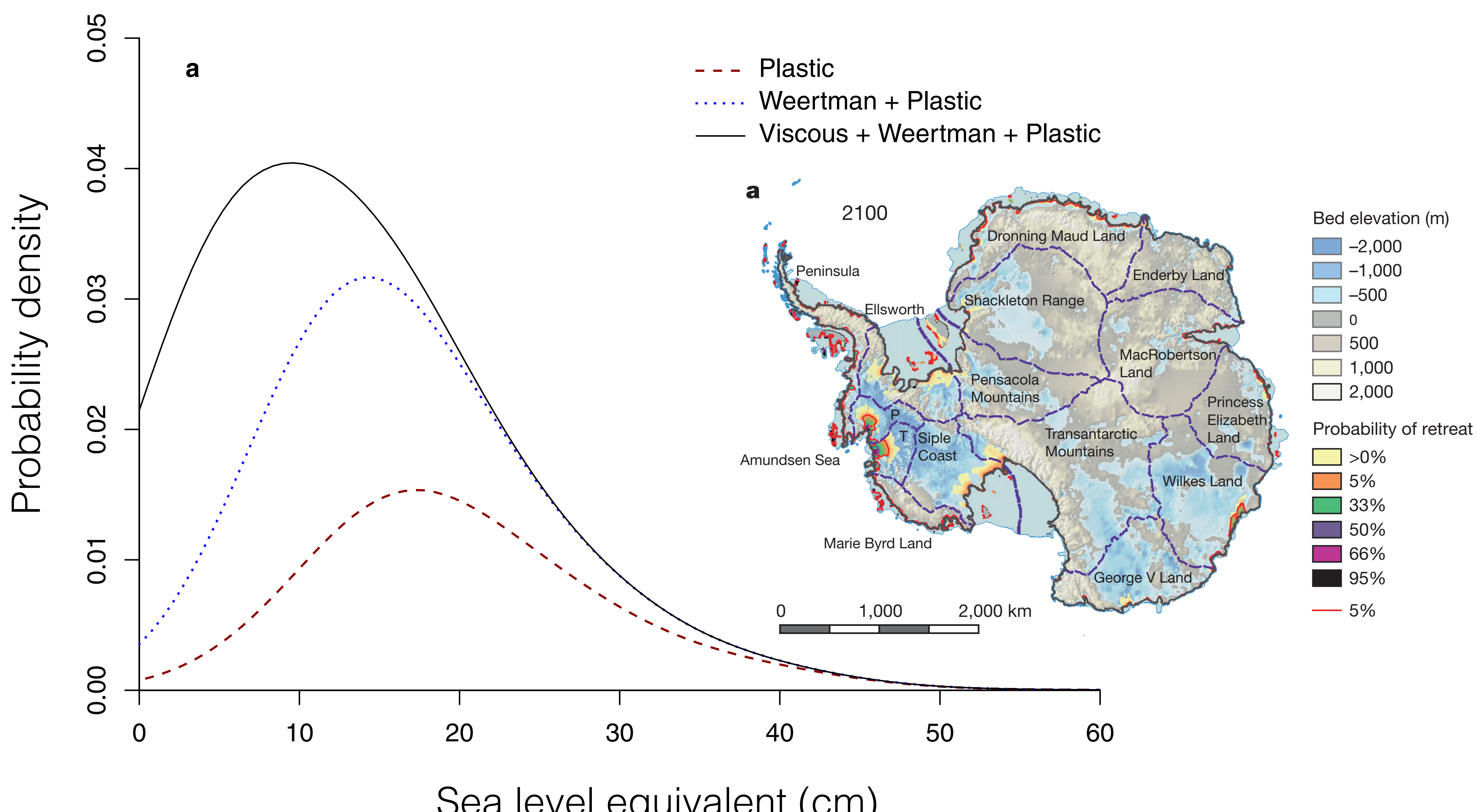
Ian Hewitt, University of Oxford



Motivation: large-scale ice-sheet flow

The basal **sliding law / friction law** represents a (perhaps **the**) major uncertainty in current forecasts of sea-level rise.

Using different sliding laws results in different modelled behaviour.

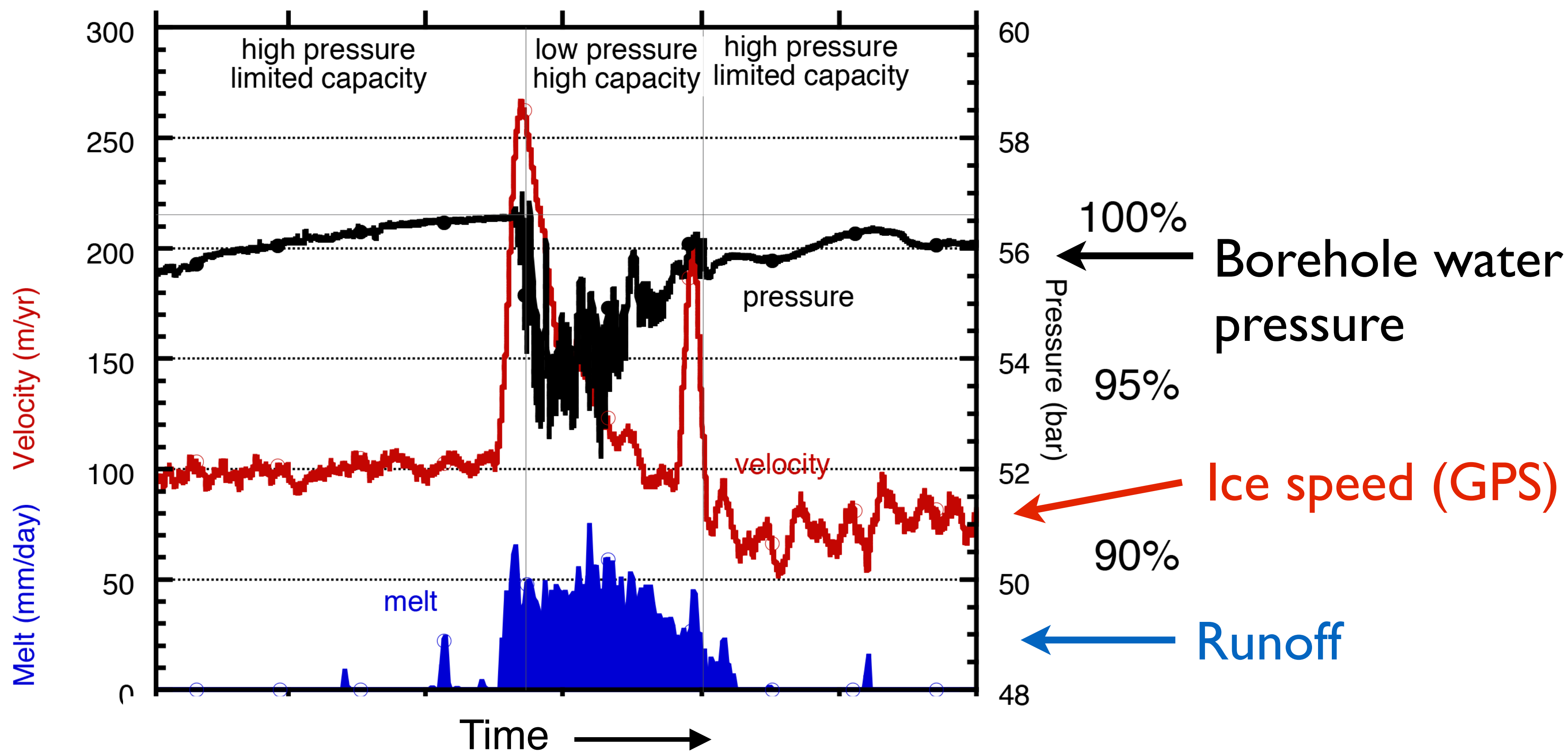


Ritz et al 2015

We'd like to understand more about what controls ice-sheet sliding

Motivation: subglacial water

Sliding is strongly affected by the presence of **water** at the ice-bed interface.



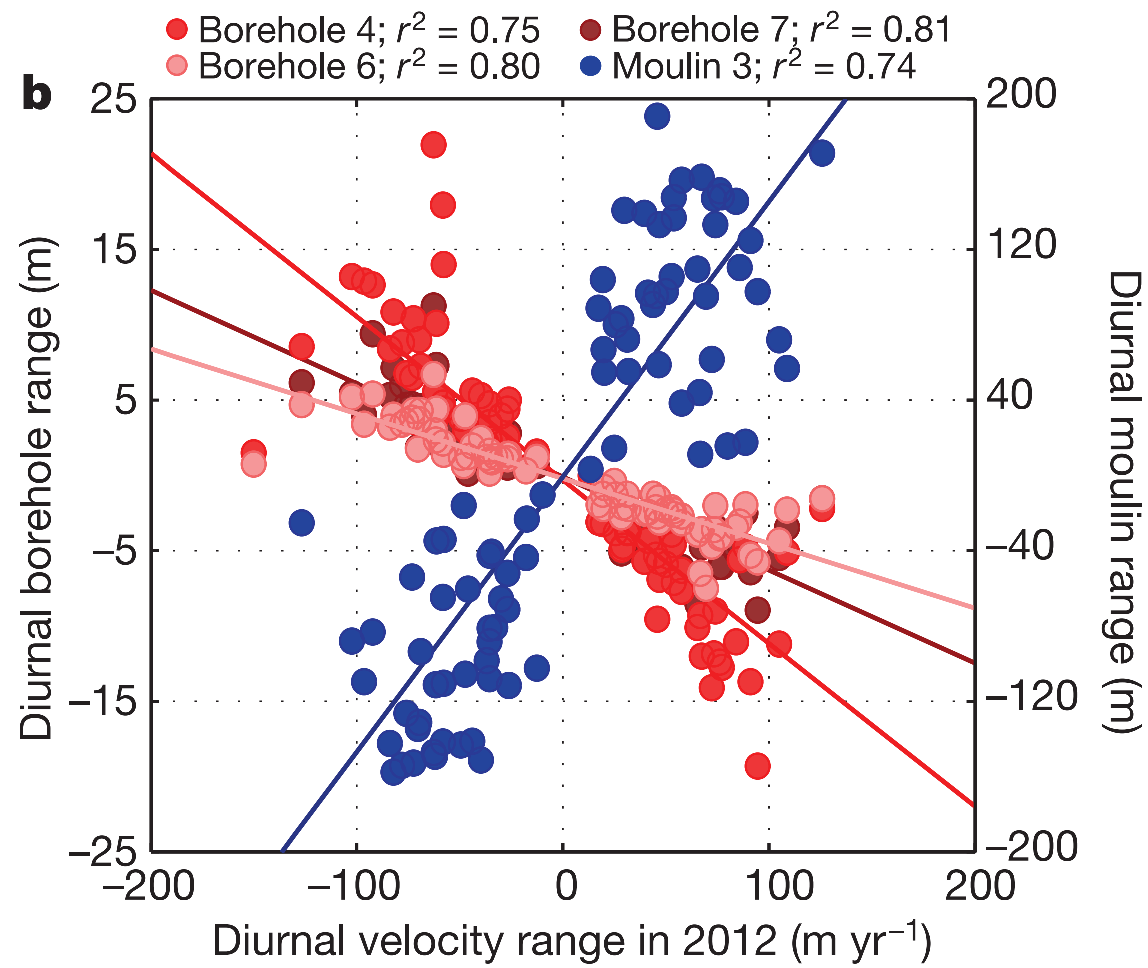
van de Wal et al 2015

Existing sliding models relate basal **shear stress** to **sliding speed** and **effective pressure**

- the relationship does not agree all that well with observations

Motivation: field measurements (Greenland ice sheet)

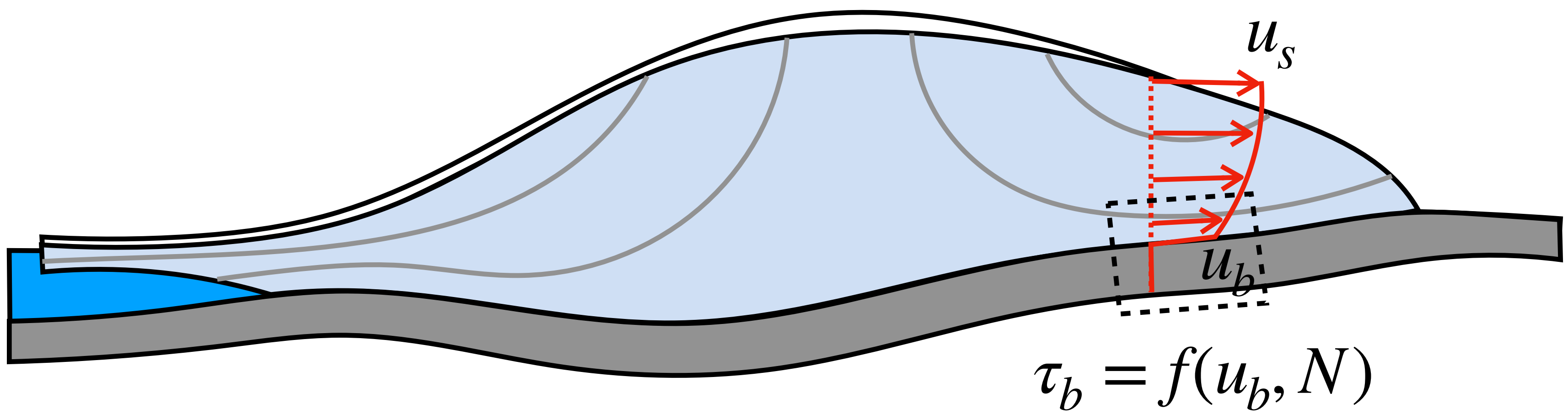
Velocity changes are roughly **in phase** with changes in moulin water pressure, but **out of phase** with changes in borehole water pressure

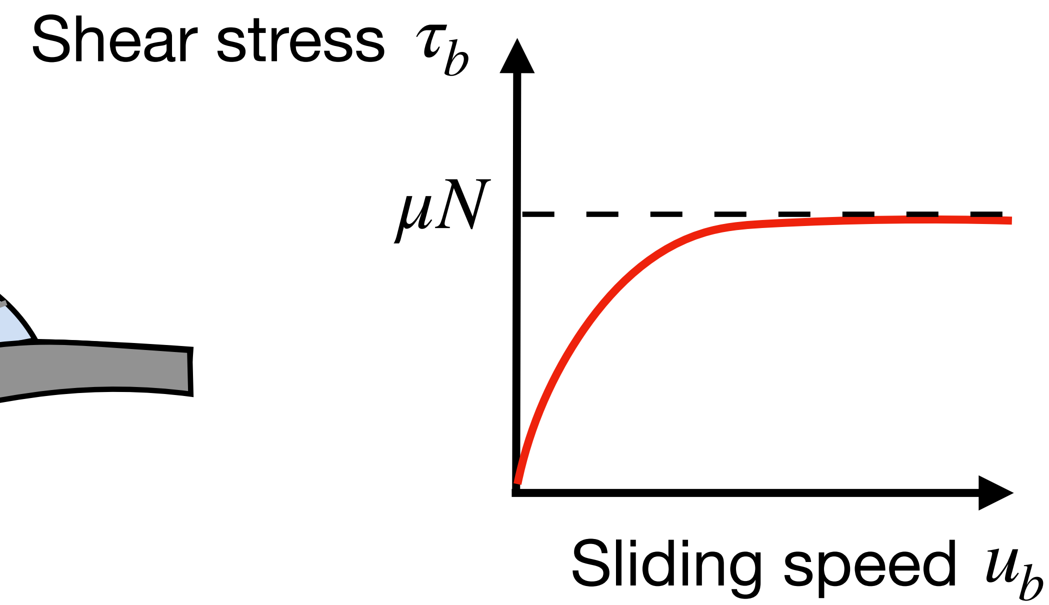


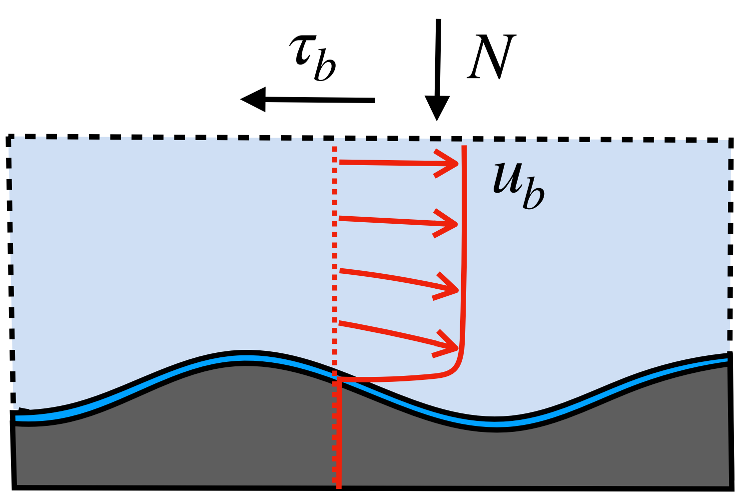
Andrews et al 2014

➔ **Investigate hydrologically-controlled sliding in more detail**

The ice-sheet sliding law

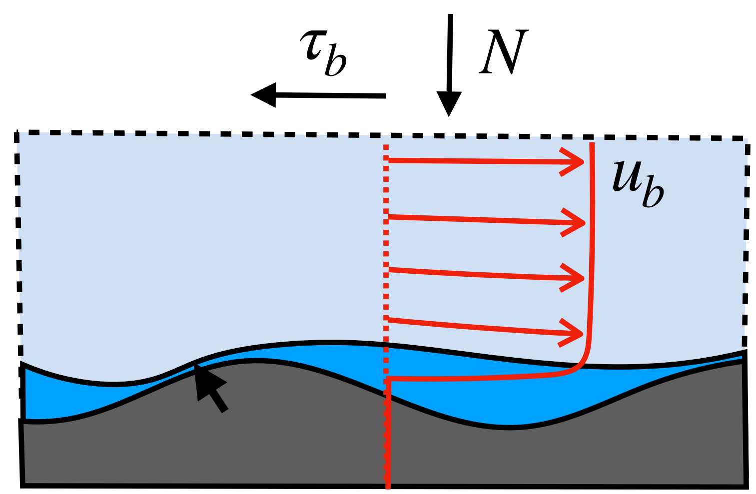






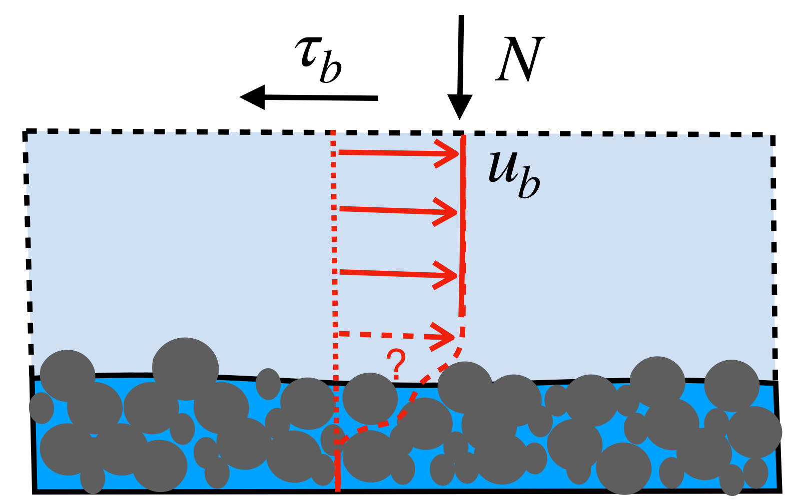
Hard-bed sliding (‘Weertman’)

$$\tau_b = C u_b^m$$



Hard-bed sliding (‘Schoof’)

$$\tau_b = \mu N \left(\frac{u_b}{u_b + \lambda N^n} \right)^{1/n}$$

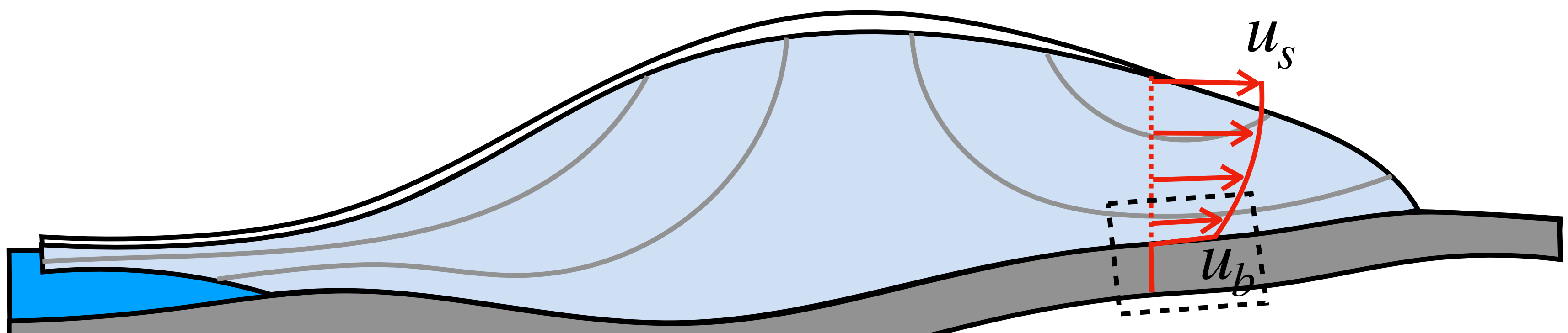


Soft-bed sliding

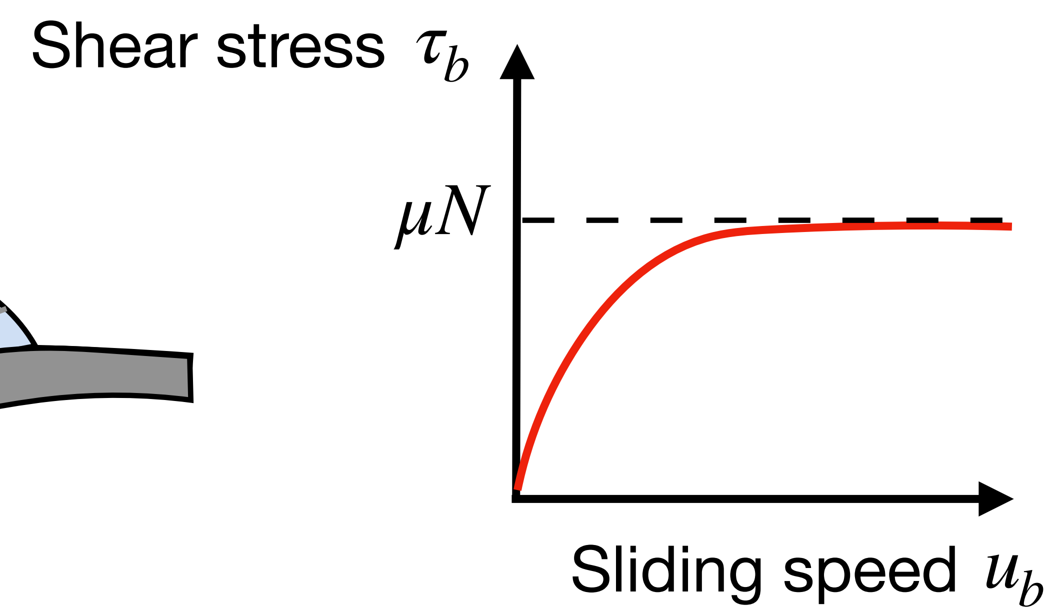
$$\tau_b = \mu N$$

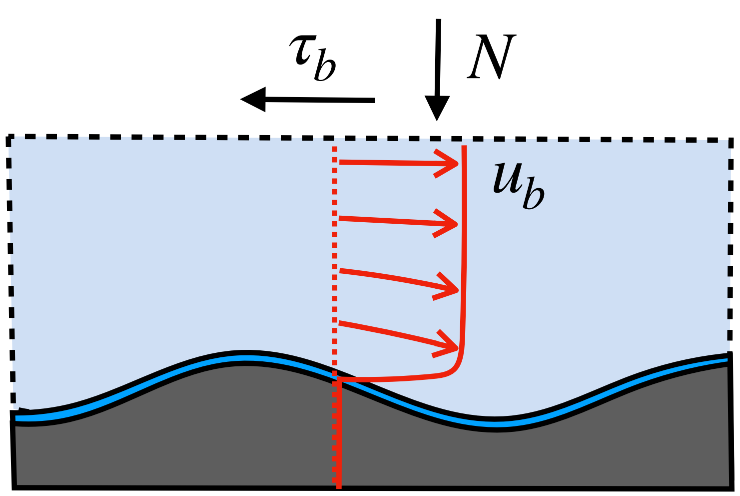
Weertman 1957, Nye 1969, Kamb 1970, Lliboutry 1968, Fowler 1986, Schoof 2005, Gagliardini et al 2007, Iverson 2010

The ice-sheet sliding law



$$\tau_b = f(u_b, N)$$





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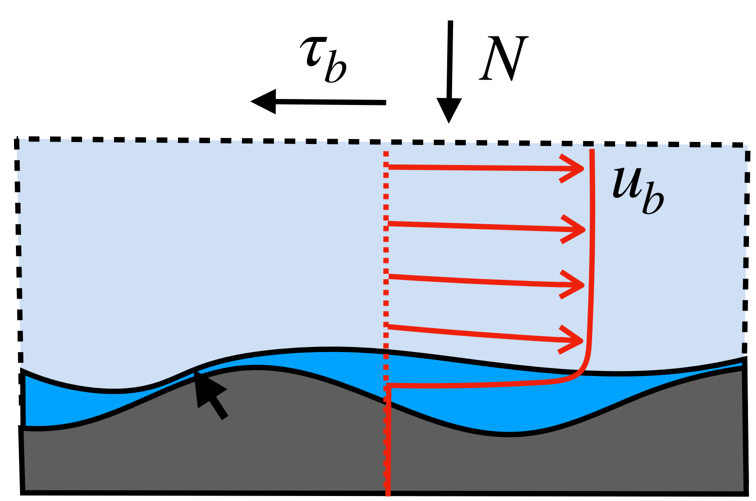

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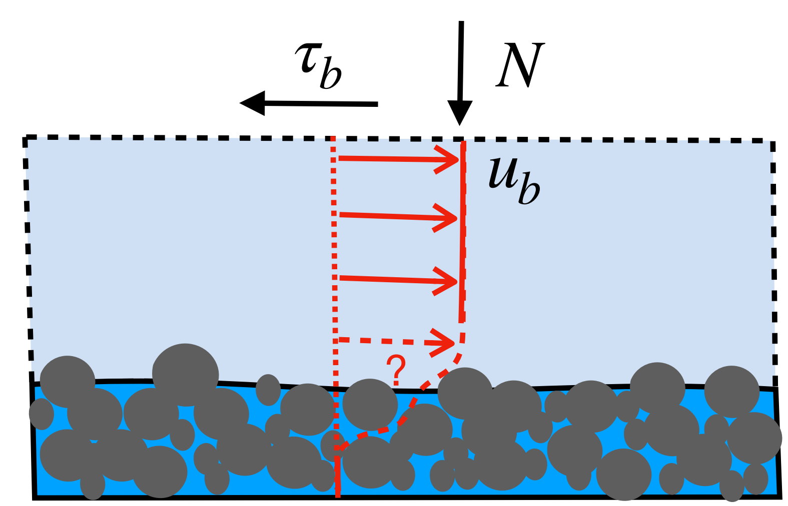

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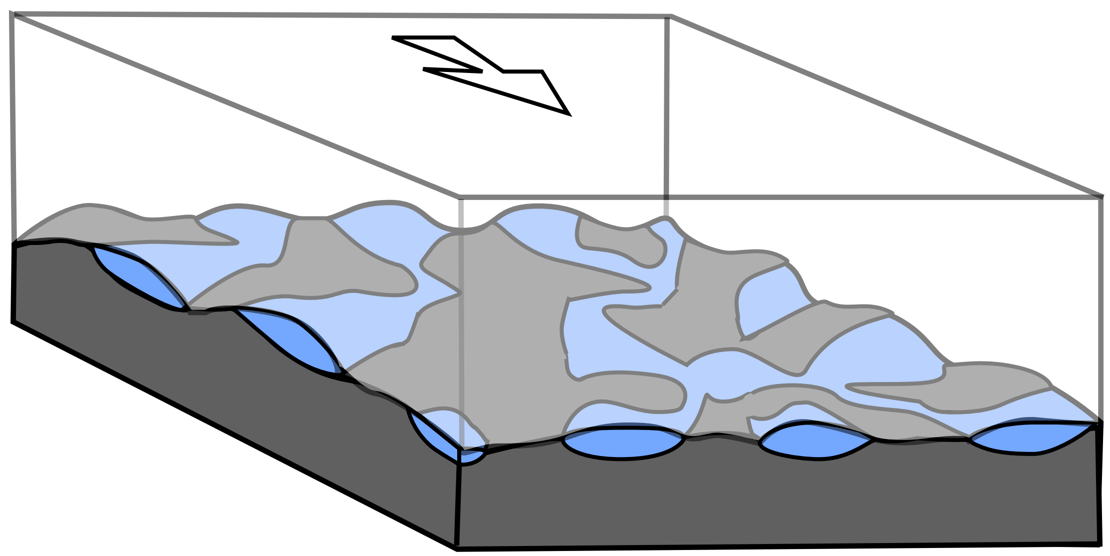
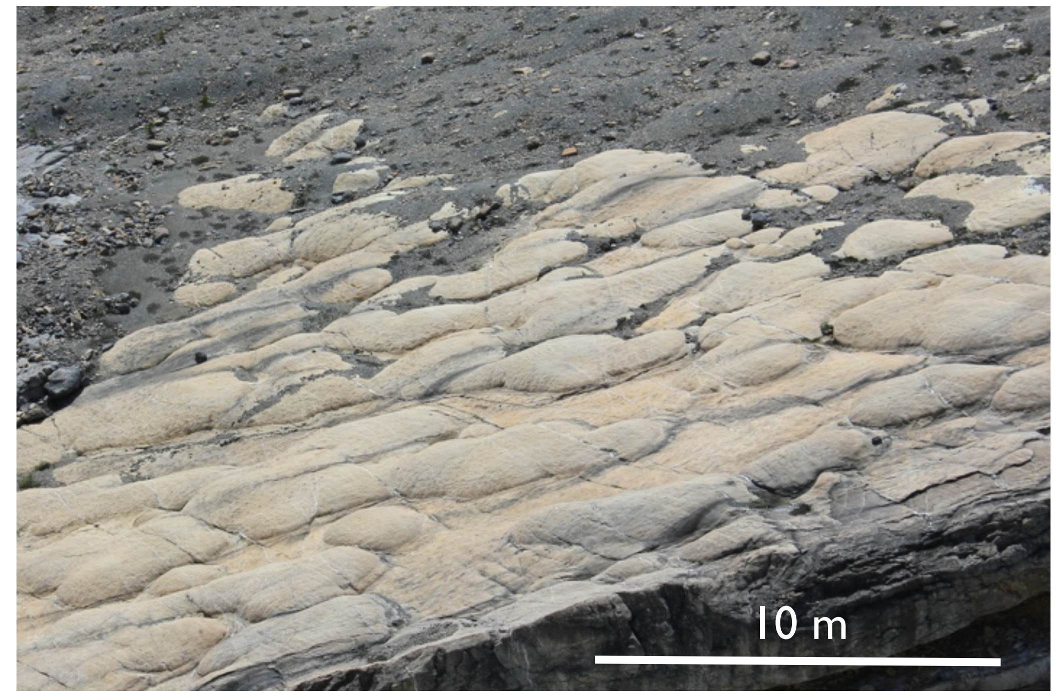
Subglacial cavitation

Water-filled cavities form downstream of bedrock bumps, where local normal stress is low





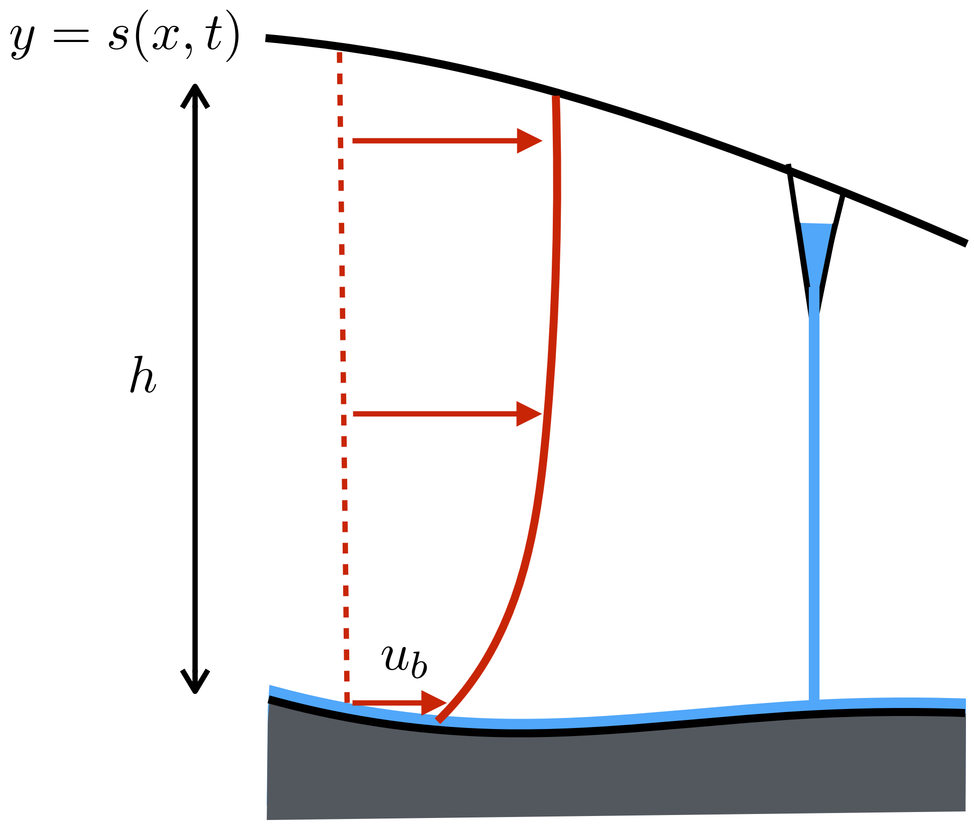
Previous **modelling** work: Lliboutry 1968, Iken 1981, Kamb 1987, Fowler 1986, 1987, Schoof 2005, Gagliardini et al 2007, Helanow et al 2019

Laboratory-based **experimental** work: Zoet & Iverson 2015

→ **Existing models assume a steady state**

This is inconsistent with an evolving subglacial hydrological system

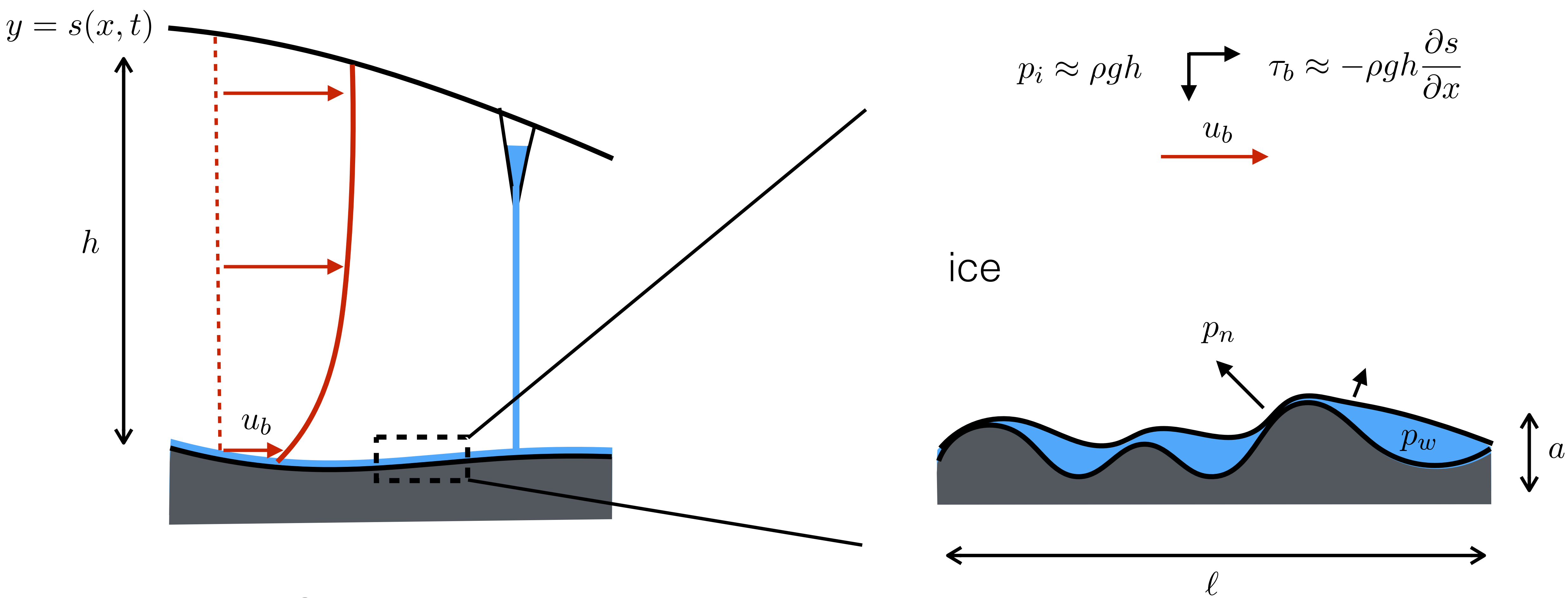
Mathematical formulation



Sliding law

$$\tau_b = f(u_b, \ldots)$$

Mathematical formulation



Sliding law

$$\tau_b = f(u_b, \ldots)$$

Assume zero shear stress microscopically

Macroscopic shear stress arises from local variations of normal stress:

$$\tau_b = \frac{1}{\ell} \int_0^\ell p_n \frac{\partial b}{\partial x} \, \mathrm{d}x$$

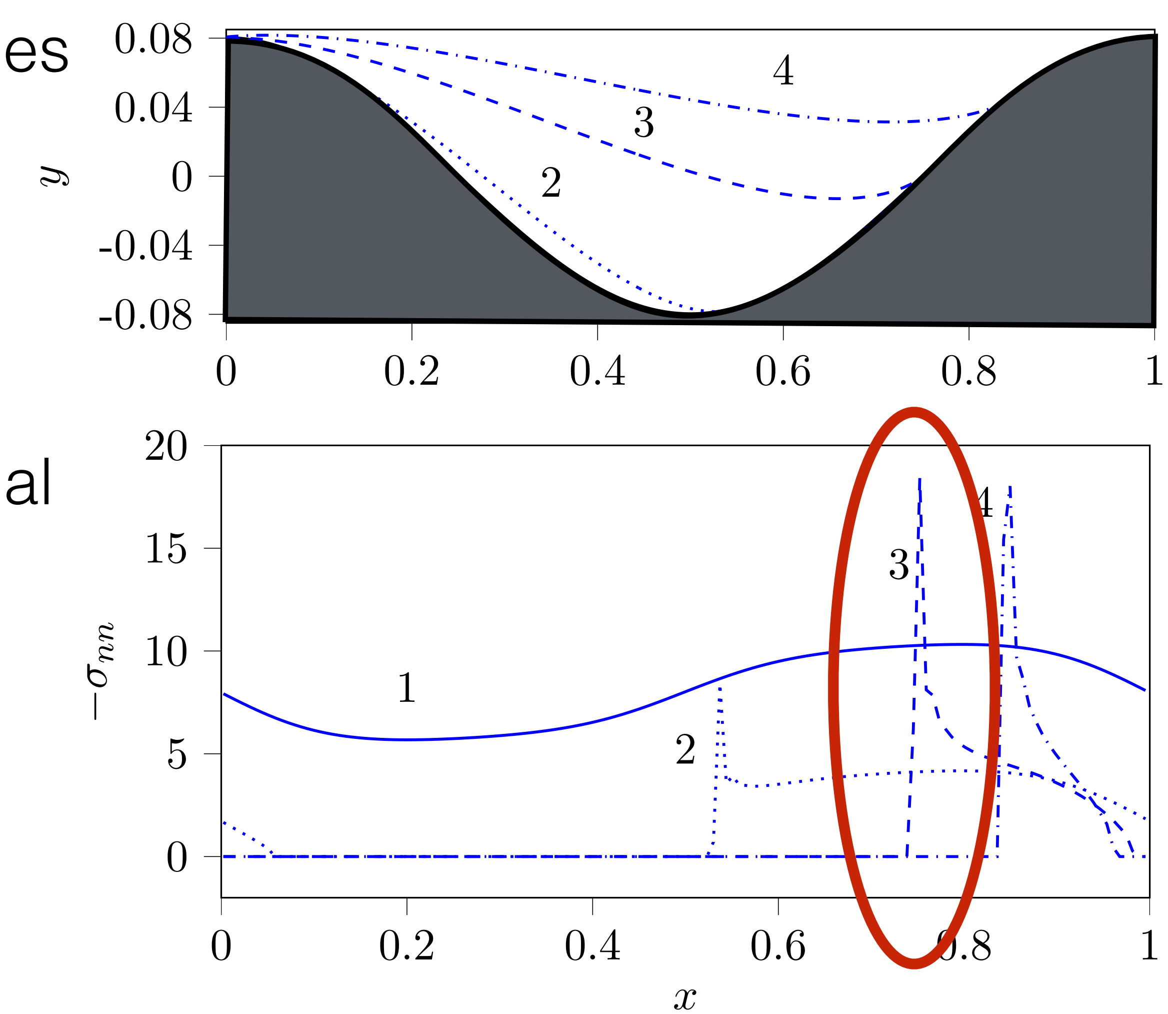
Finite-element calculations

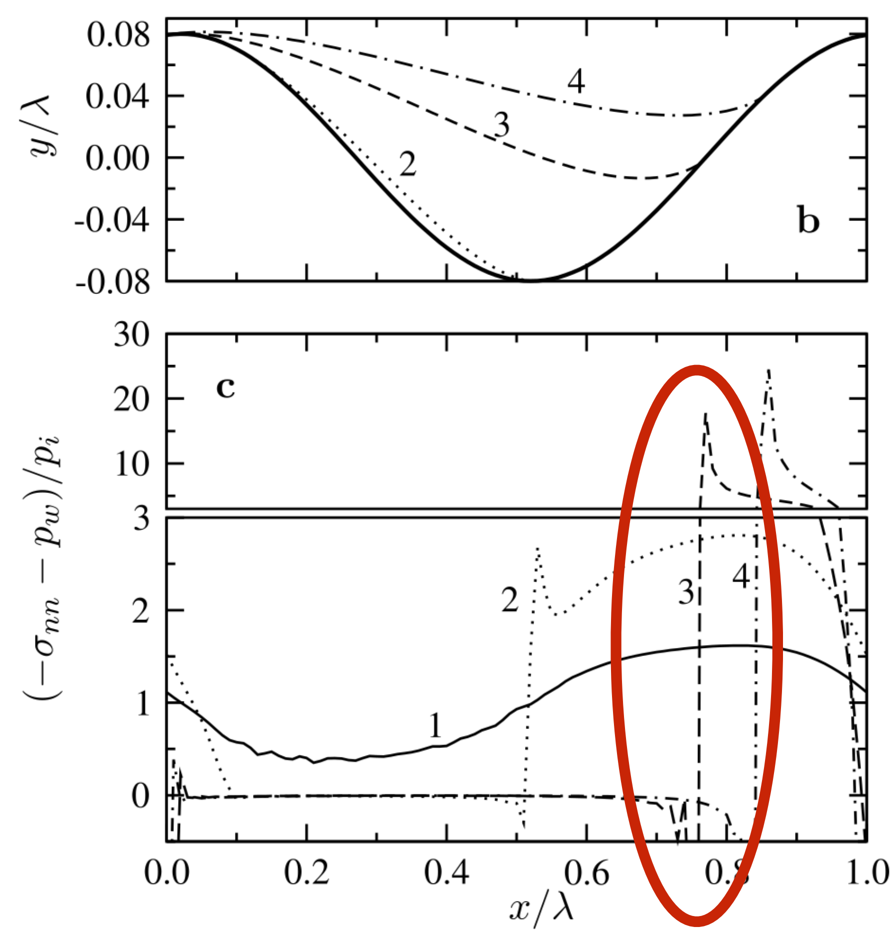
with Gonzalo Gonzalez de Diego & Patrick Farrell

Formulate **viscous contact problem** as a variational inequality.
Contact conditions enforced using a Lagrange multiplier.

Cavity shapes

Local normal stress





cf. **Gagliardini et al 2007**

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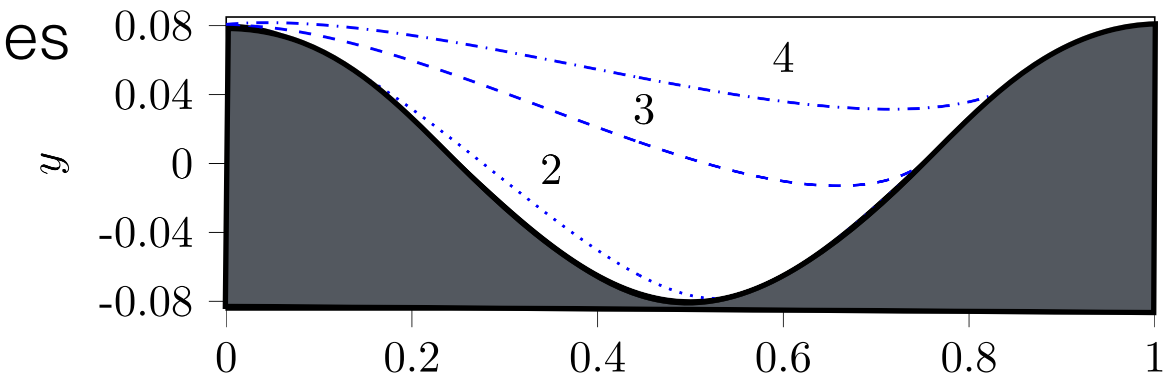
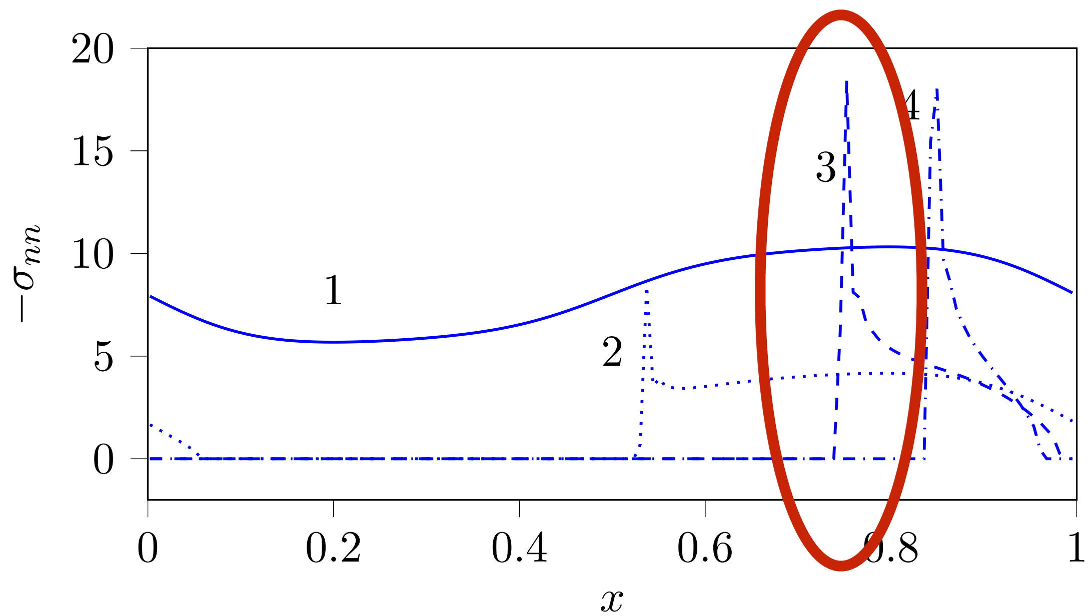
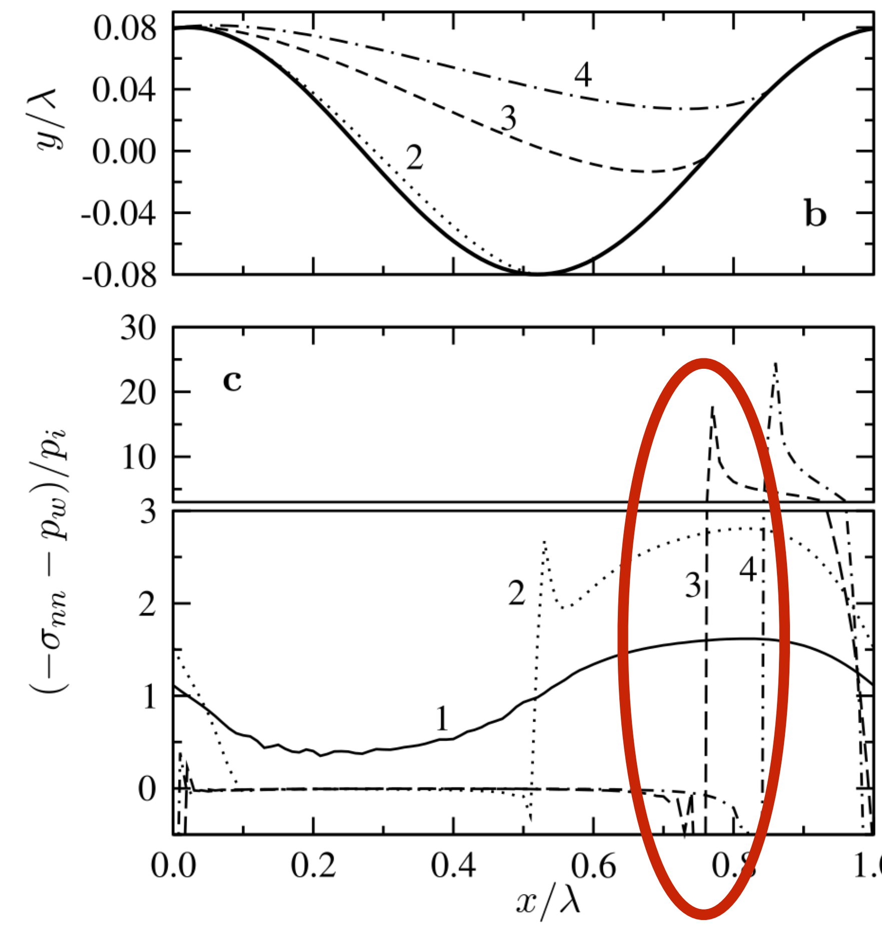


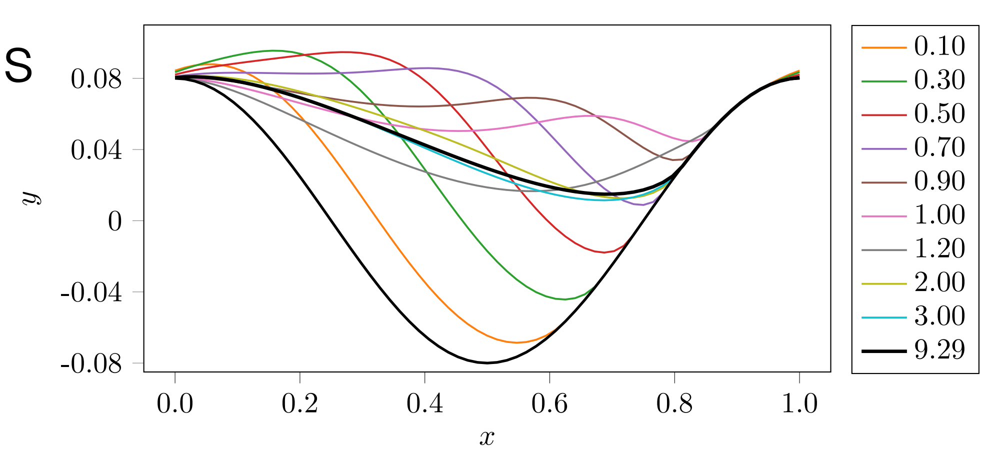
Local normal stress





cf. **Gagliardini et al 2007**

Evolving cavities (in progress)



Linearised problem

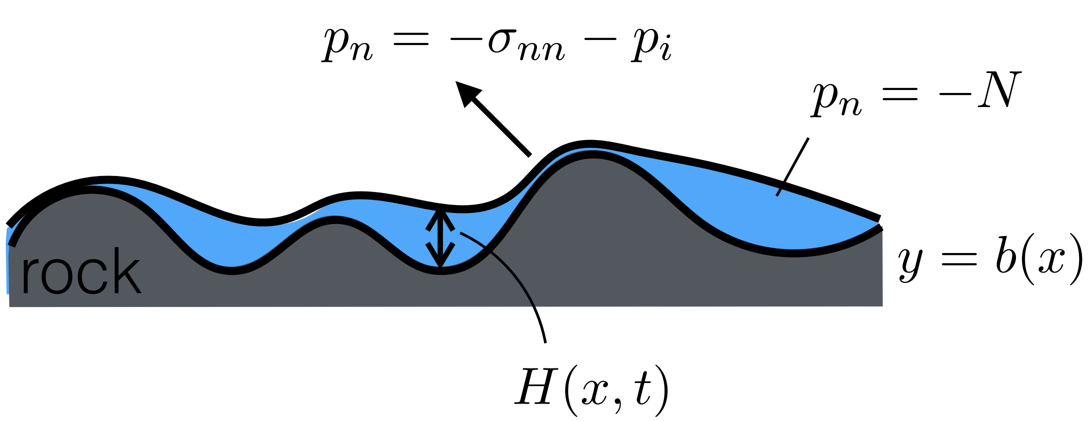
$p_i \approx \rho g h$ $\tau_b \approx -\rho g h \dfrac{\partial s}{\partial x}$

u_b

$$\nabla \cdot \mathbf{u} = 0$$

$$0 = -\nabla p + \eta \nabla^2 \mathbf{u}$$

ice



Effective pressure $N = p_i - p_w$

Fowler 1987, Schoof 2005, Hewitt 2018 unpub.

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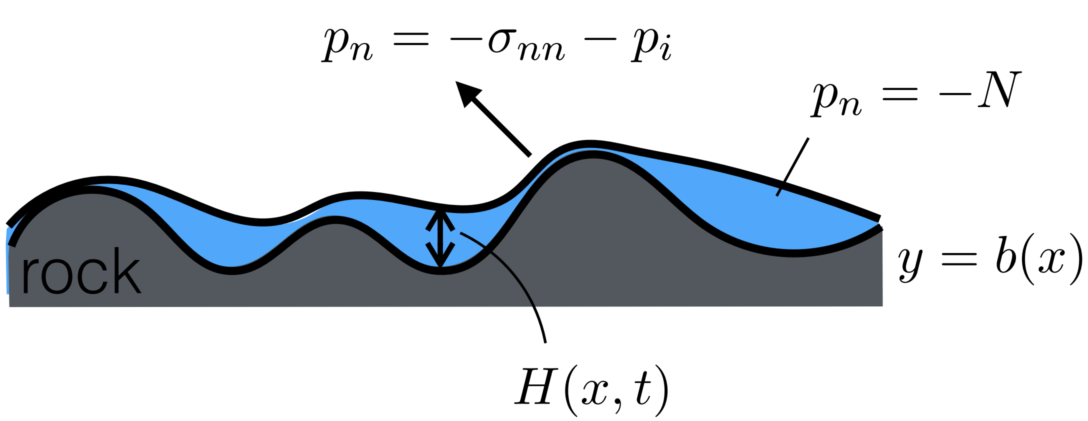


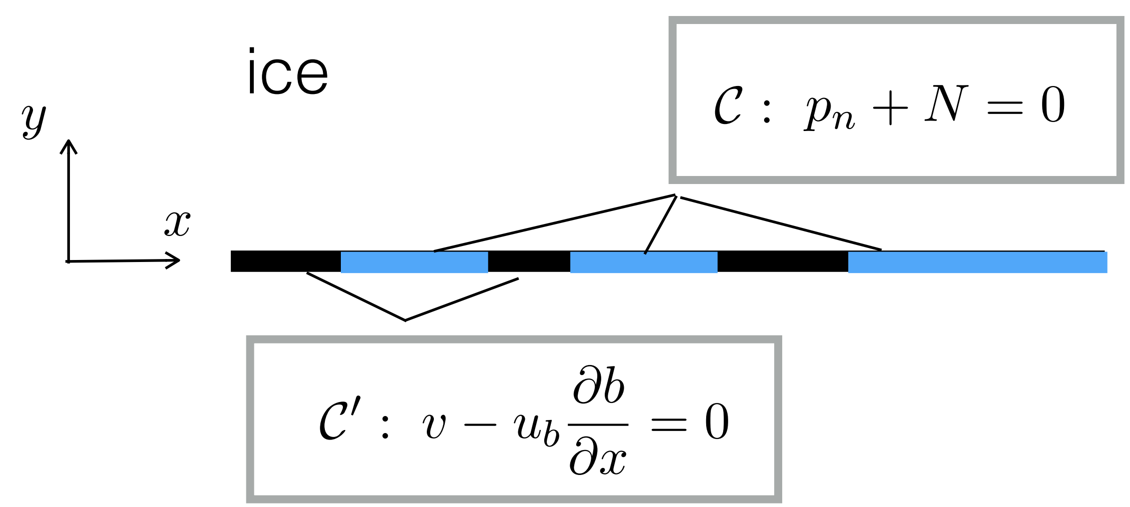
Effective pressure $N = p_i - p_w$

Linearise

$$\mathbf{u} = (u_b, 0) + \mathcal{R}\,(u, v)$$

$$\mathcal{R} = \frac{a}{\ell} \ll 1$$

Viscous flow in a half plane -
solve using complex variable methods



Evolution of cavity depth $\quad \frac{\partial H}{\partial t} + u_b \frac{\partial H}{\partial x} = v - u_b \frac{\partial b}{\partial x}$

Fowler 1987, Schoof 2005, Hewitt 2018 unpub.

Linearised problem

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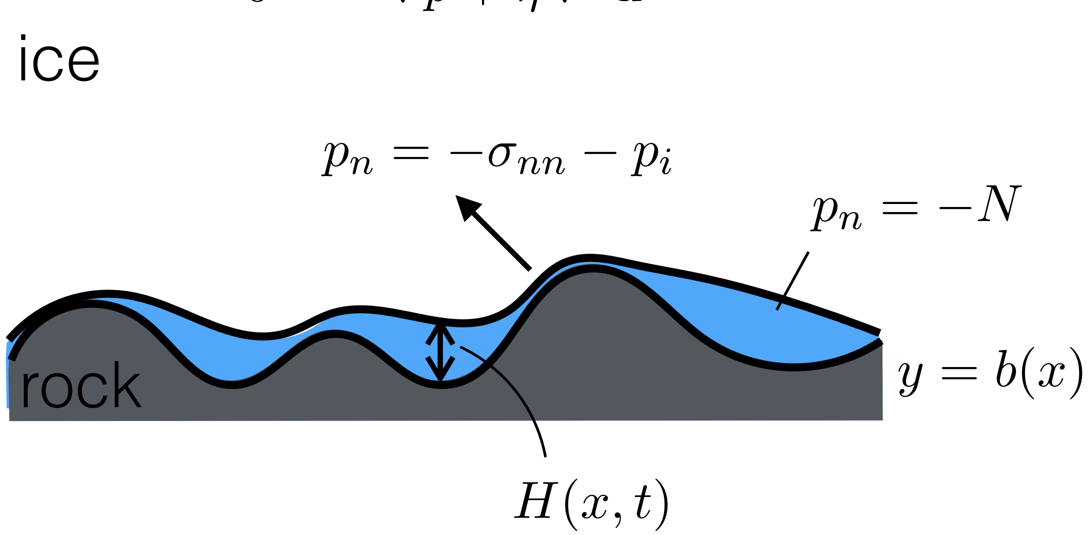

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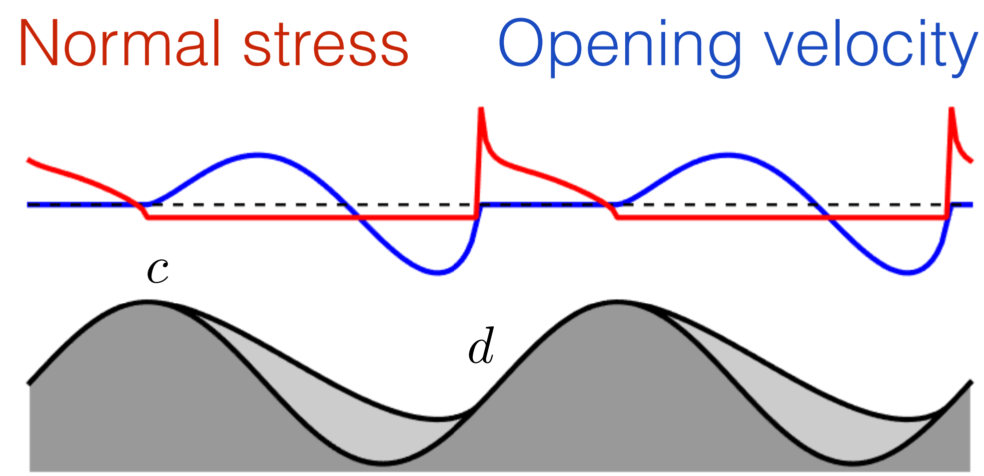

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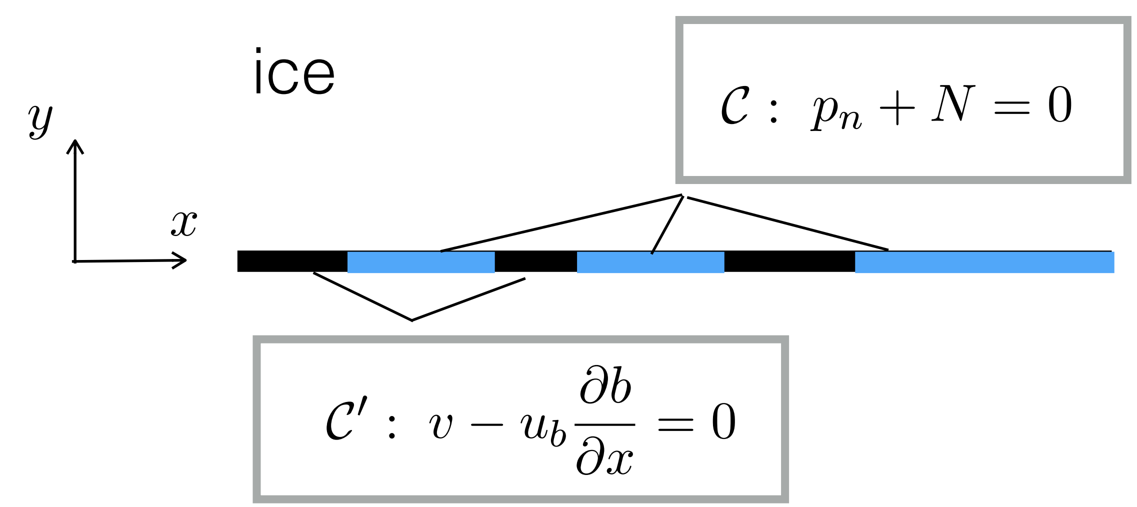

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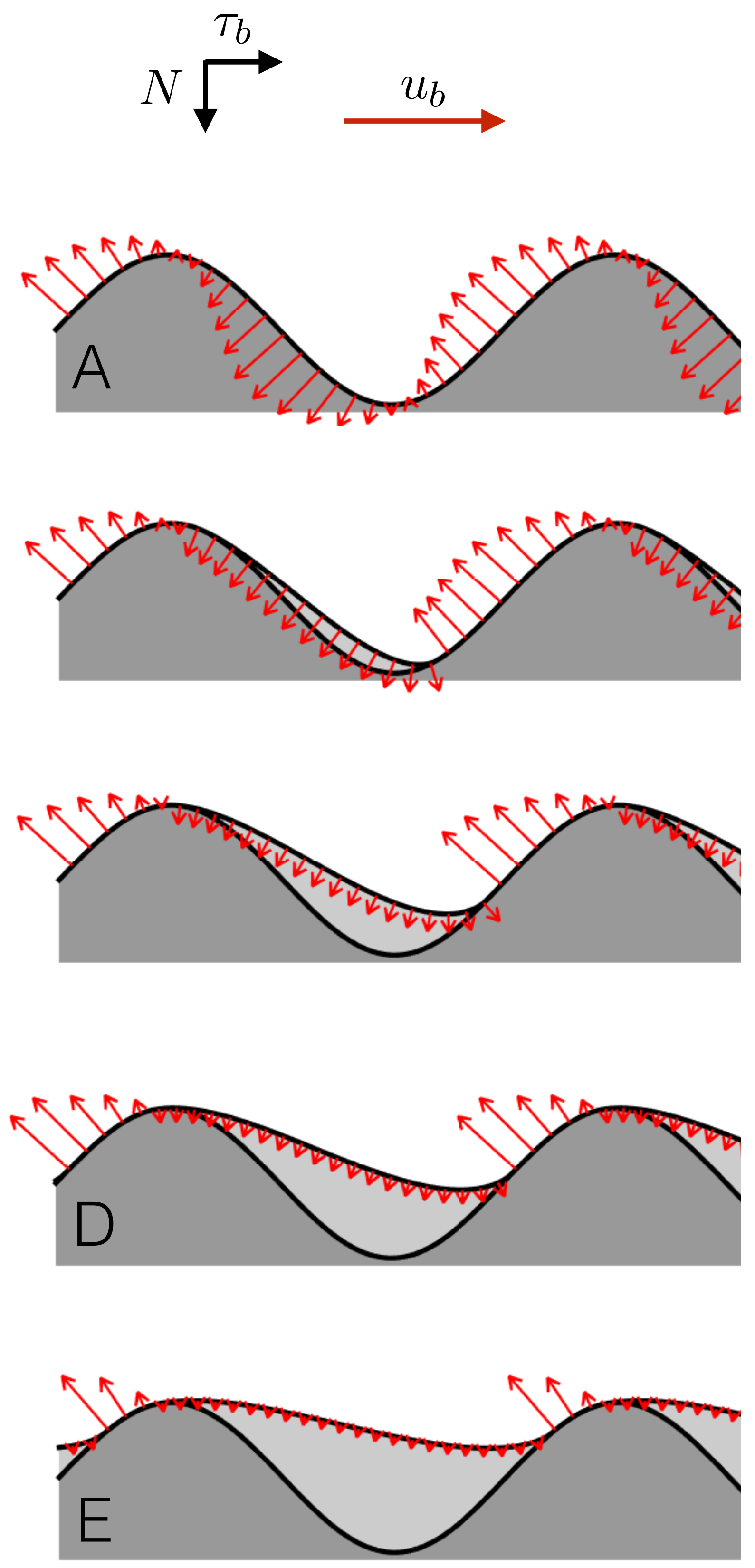
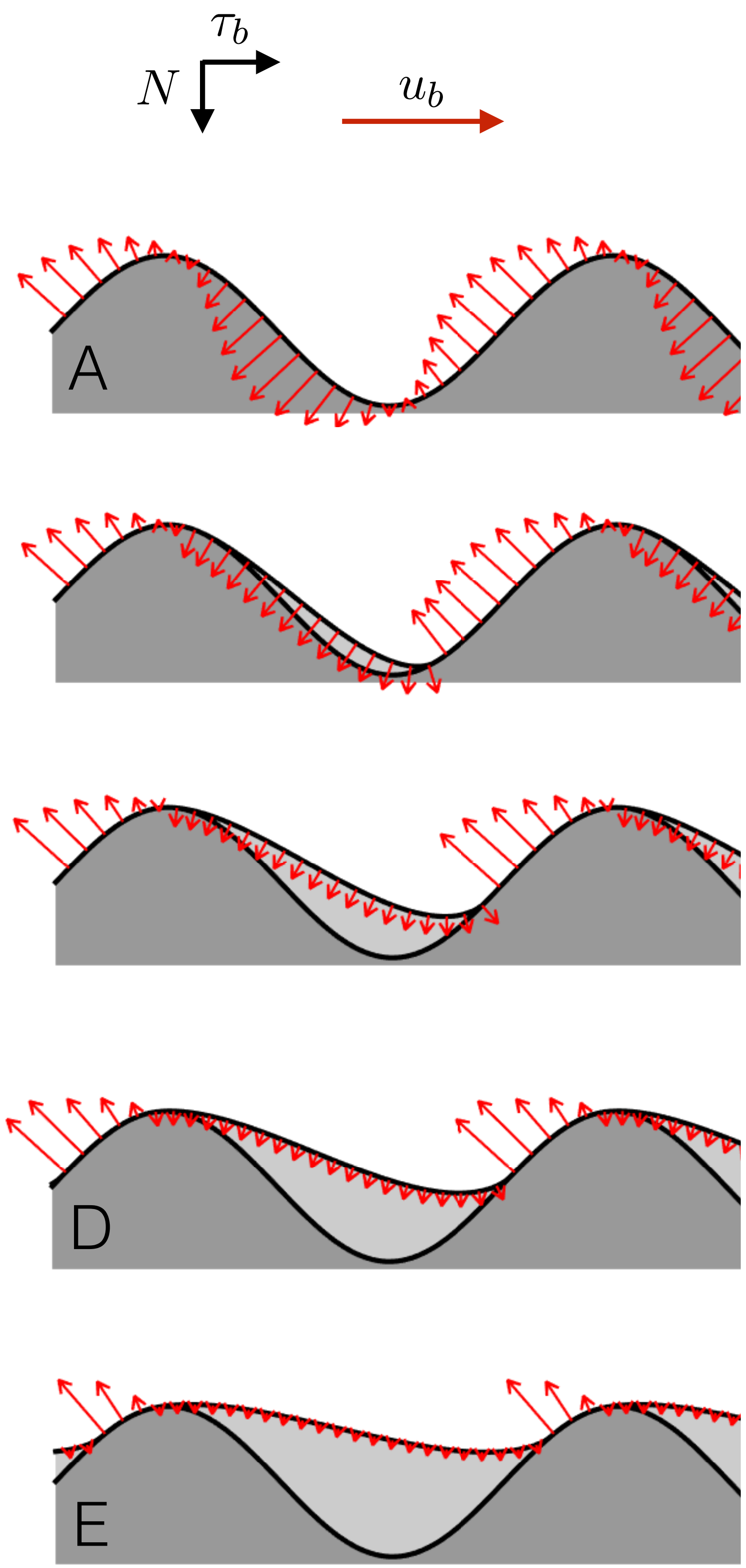
Steady-state cavities

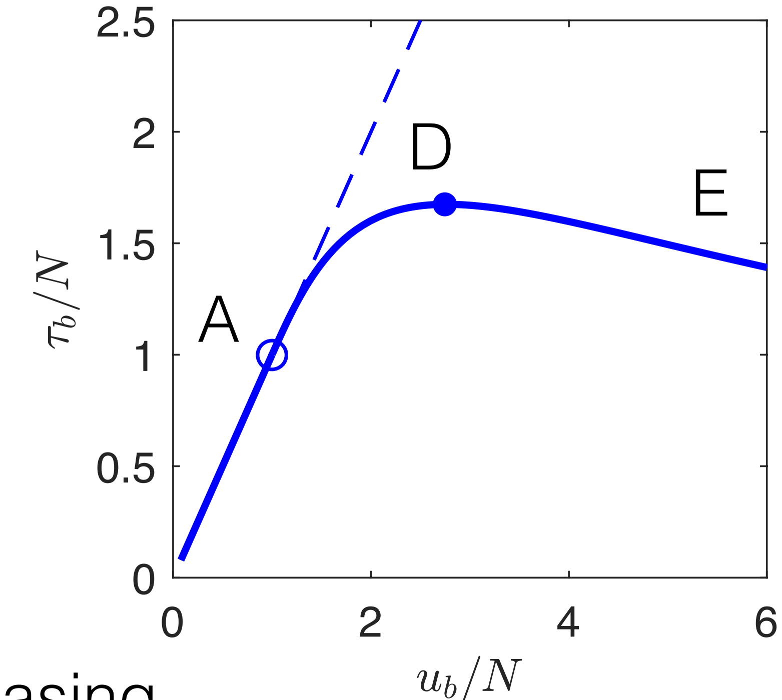


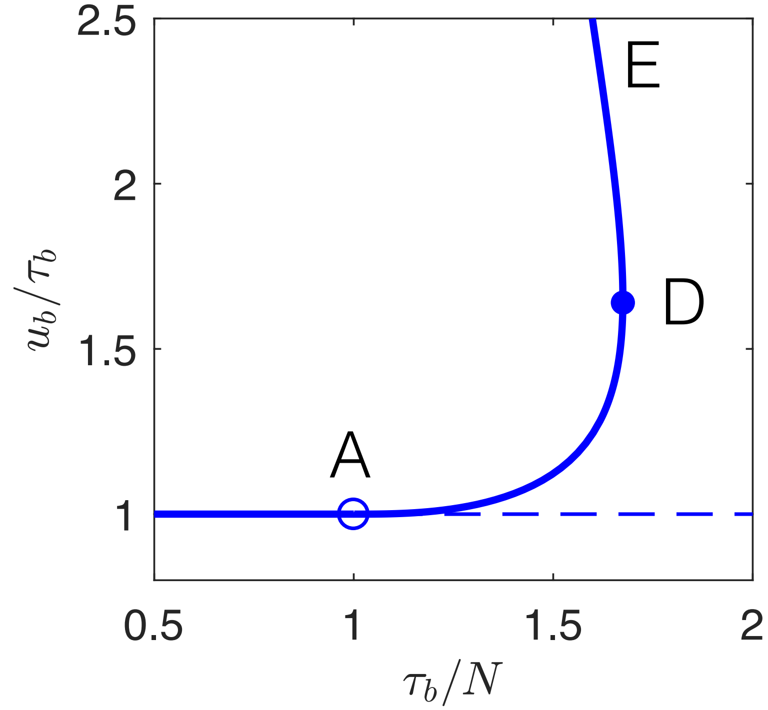
Increasing water pressure

Steady-state cavities



Relationship between τ_b , u_b , and N





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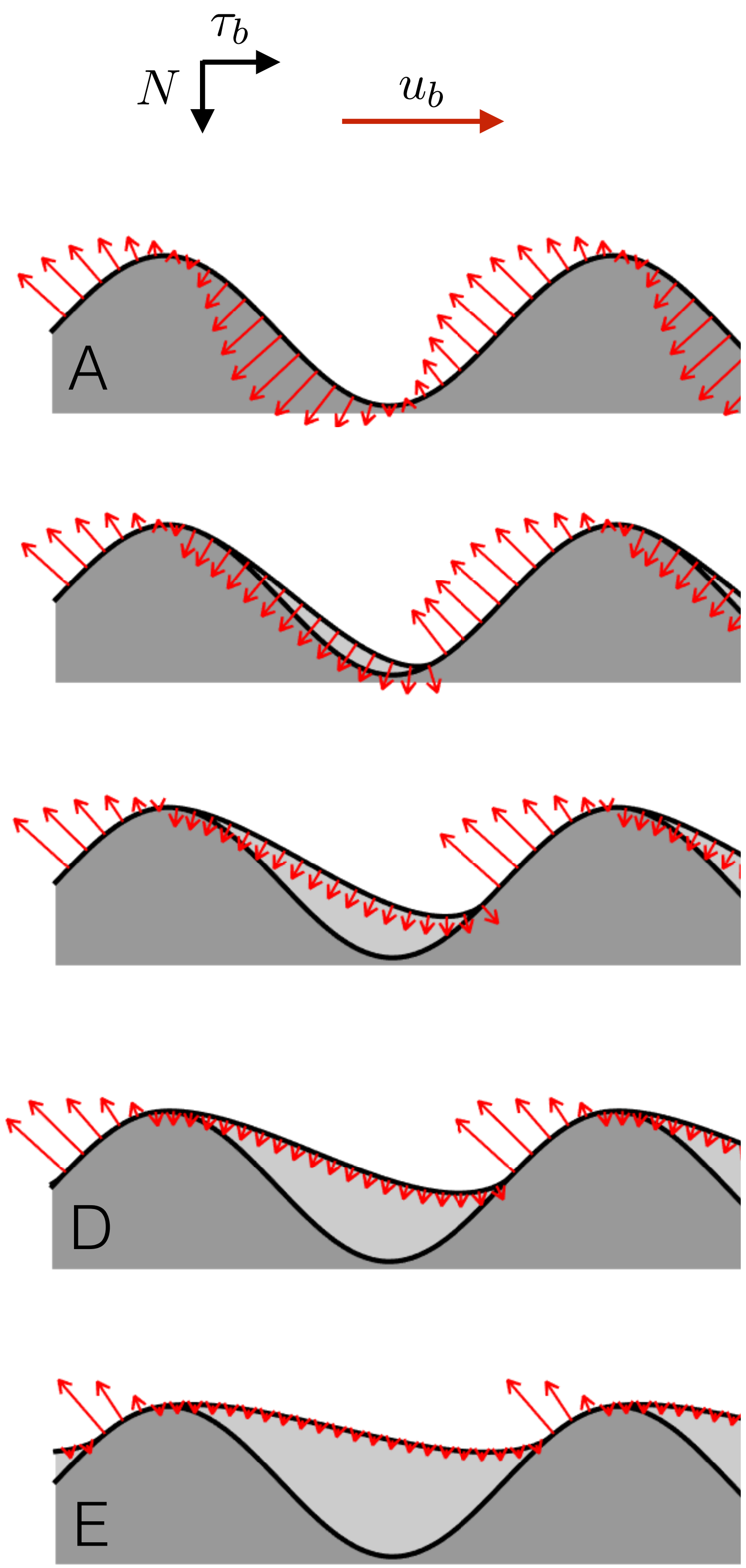

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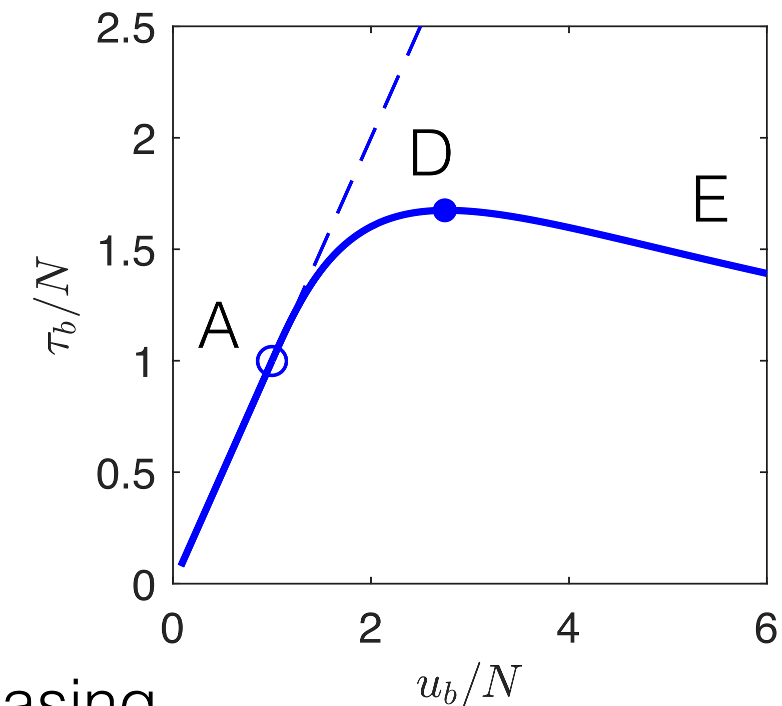
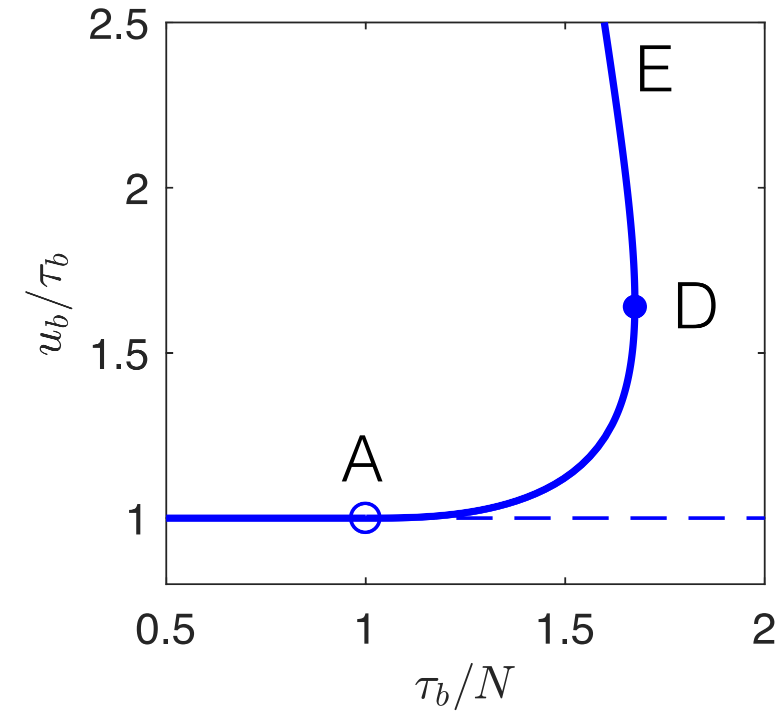
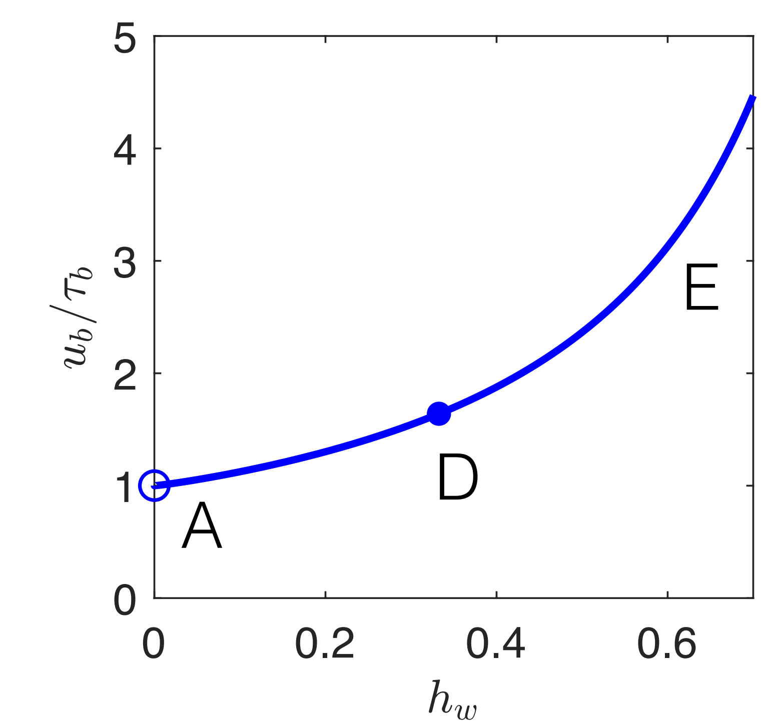


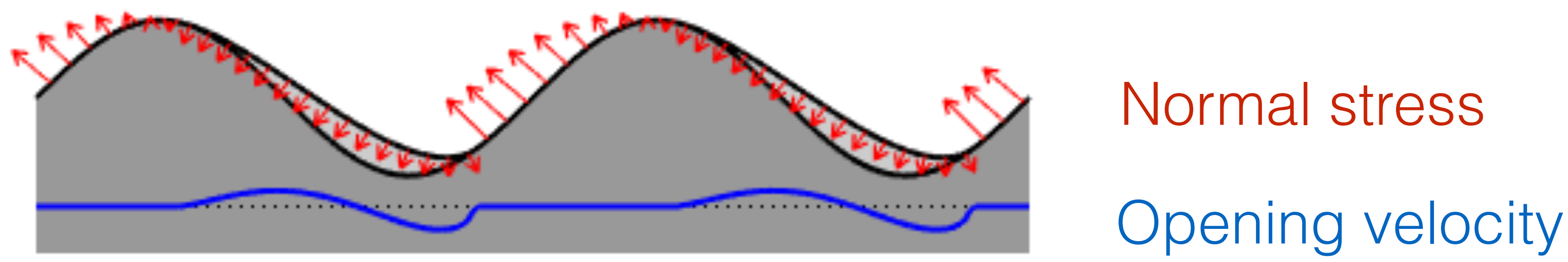


Alternatively, in terms of cavity volume (average water sheet thickness h_w)

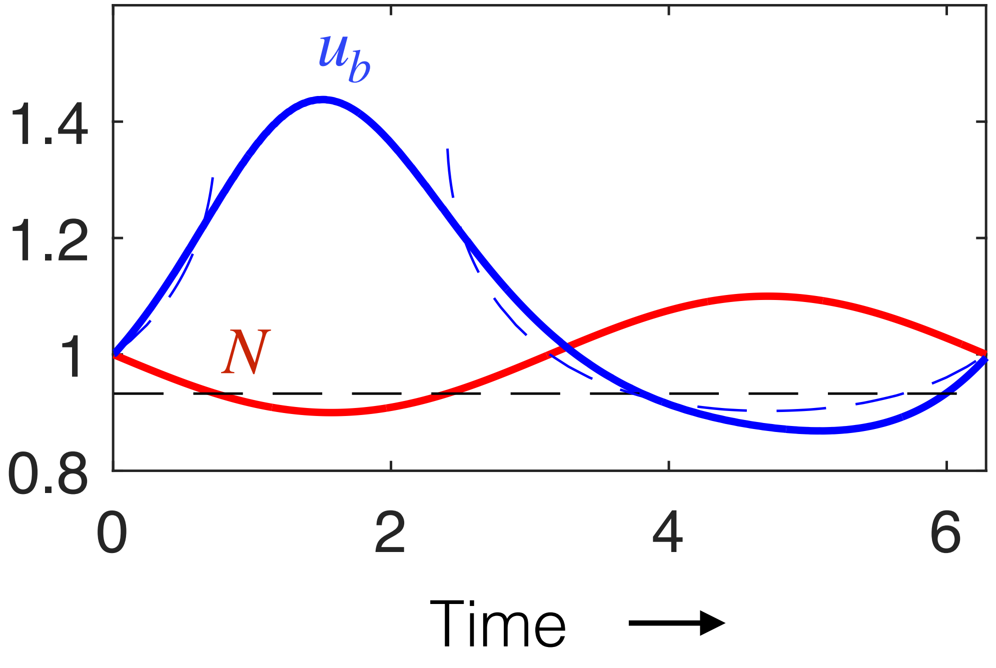


Time-dependent fluctuations

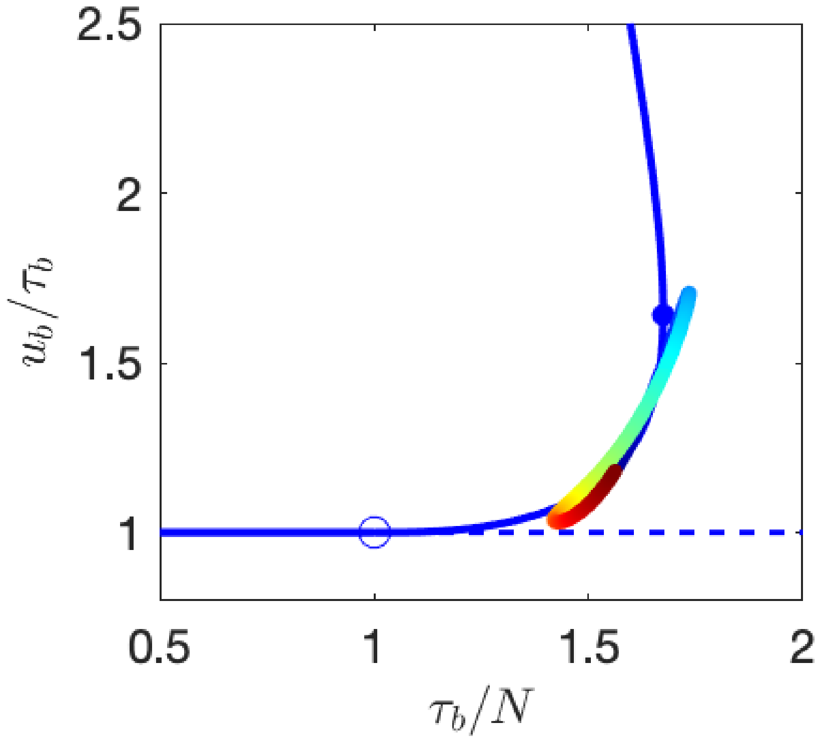
Effective pressure forced to oscillate, basal shear stress held constant.



Causes oscillations in sliding speed:



Compare to steady-state sliding law:



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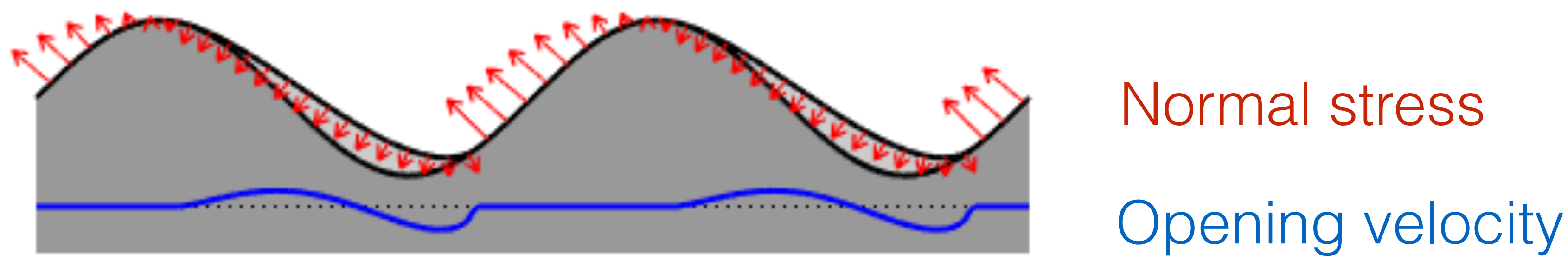

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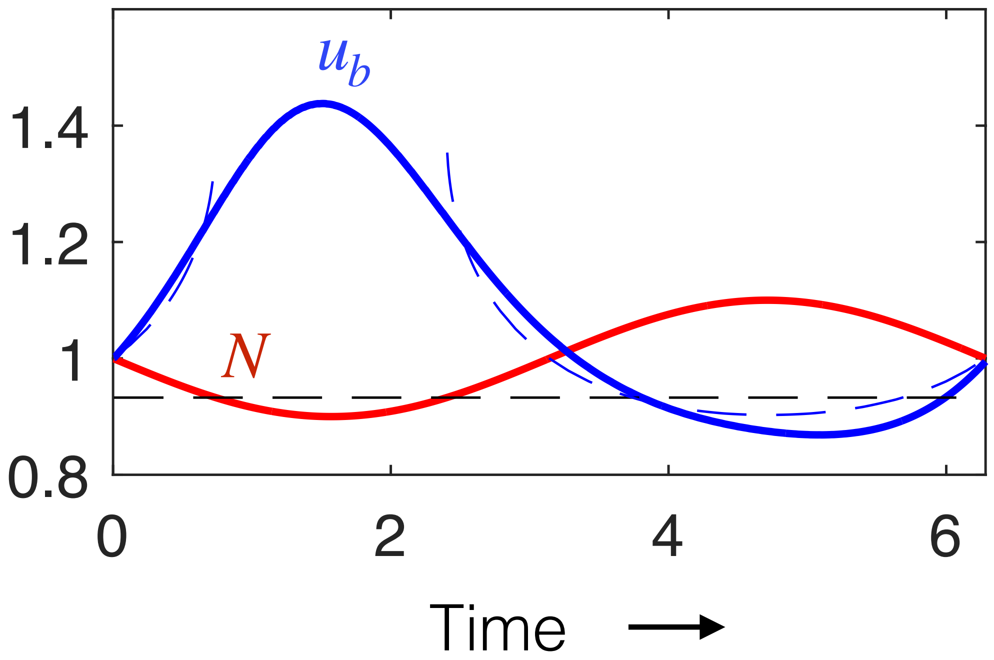

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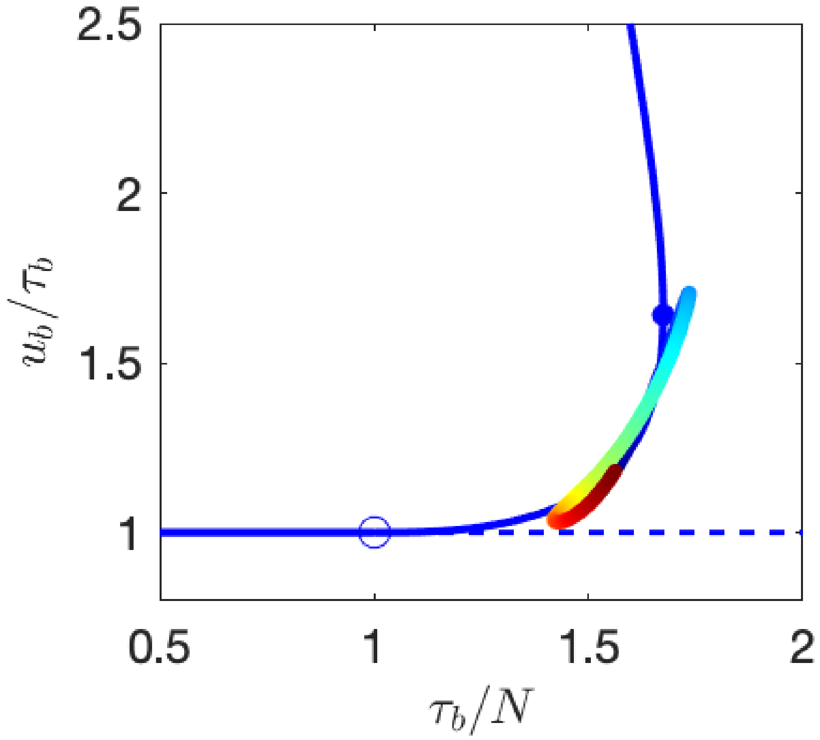
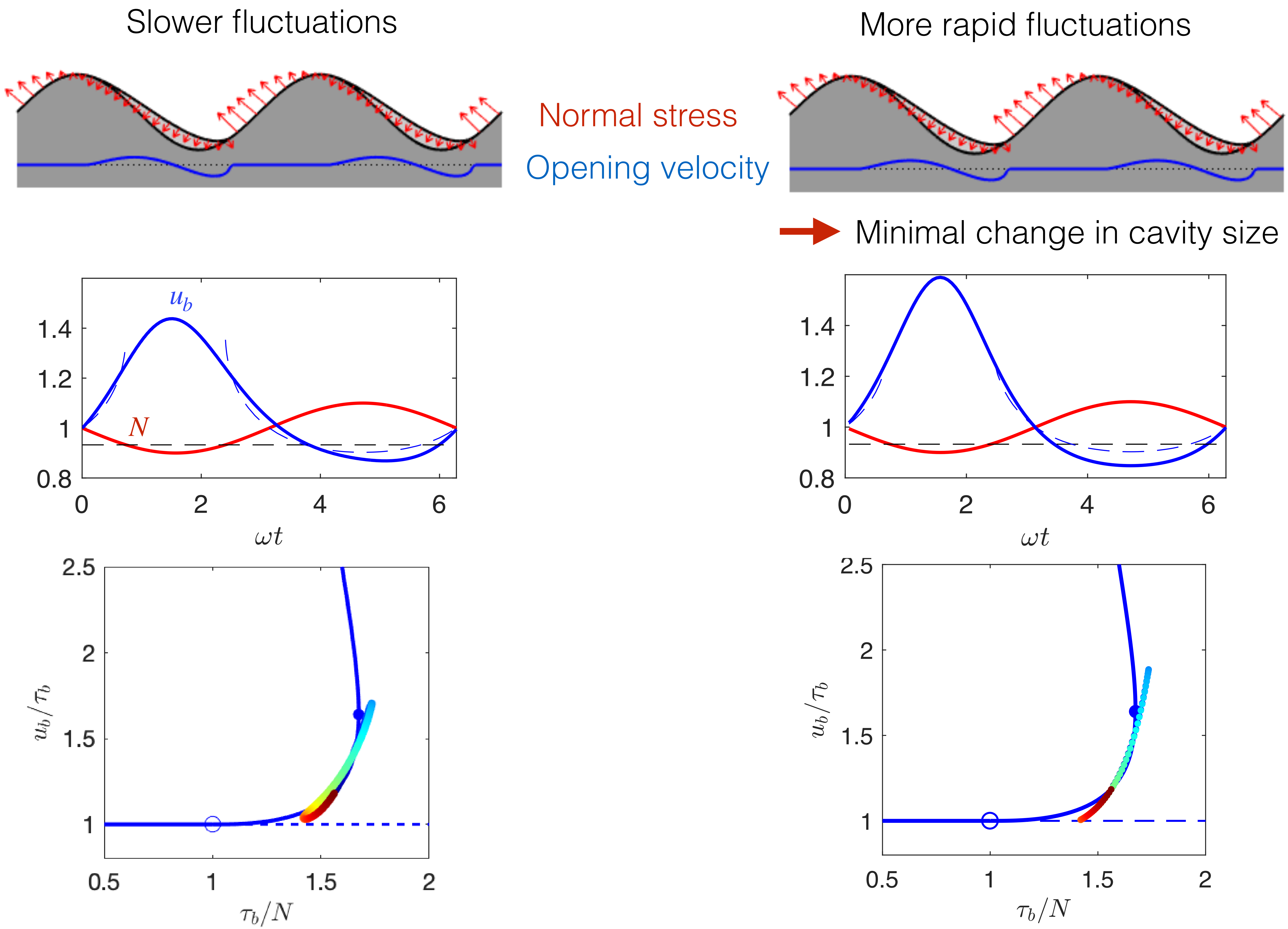


Frequency of fluctuations



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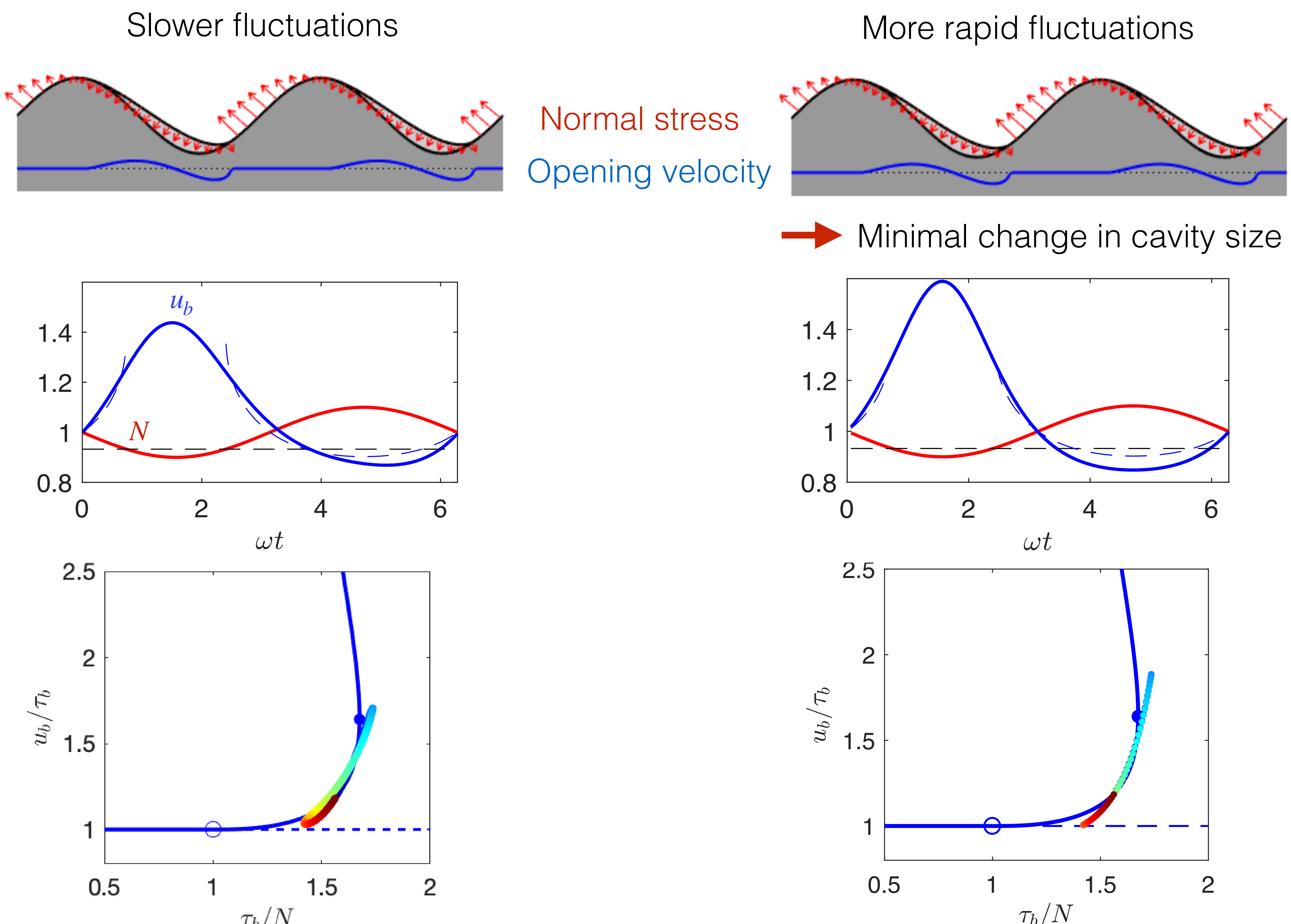
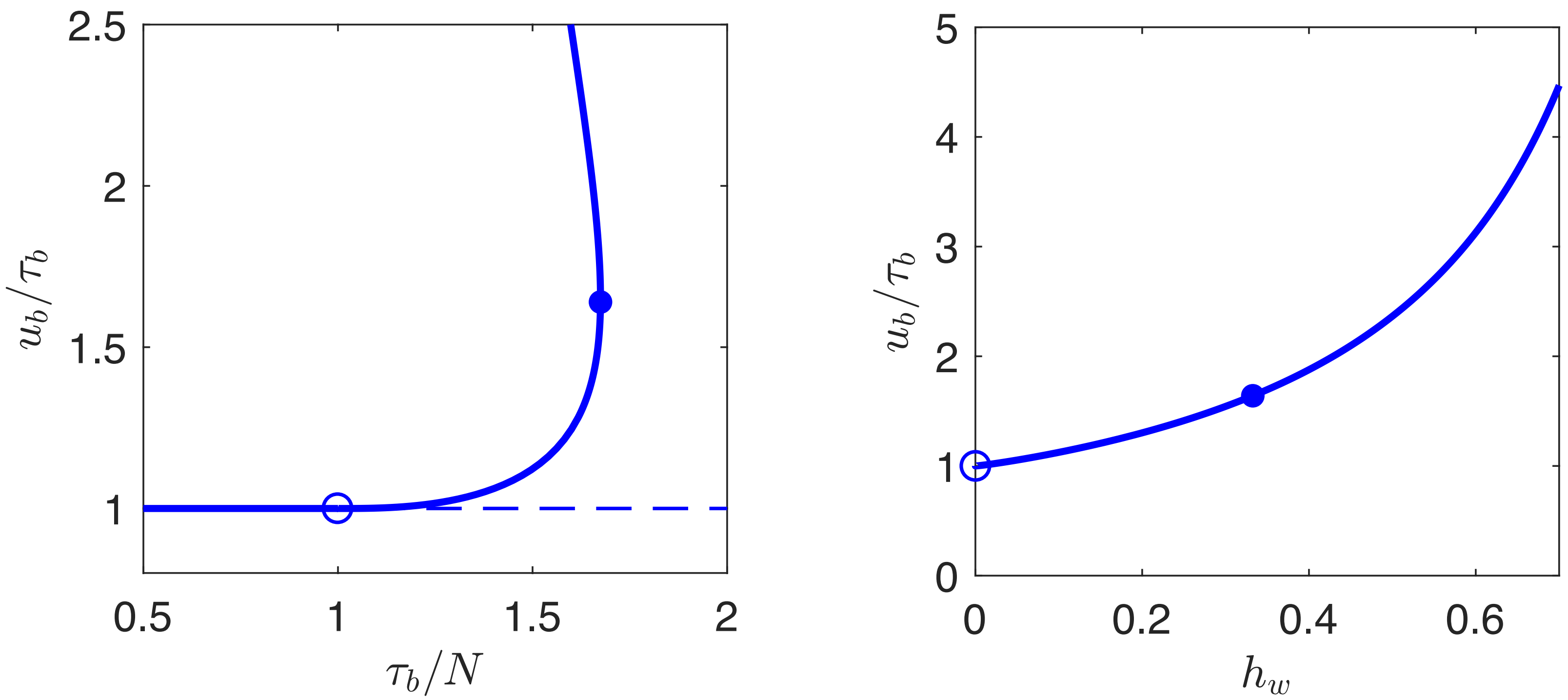


Summary

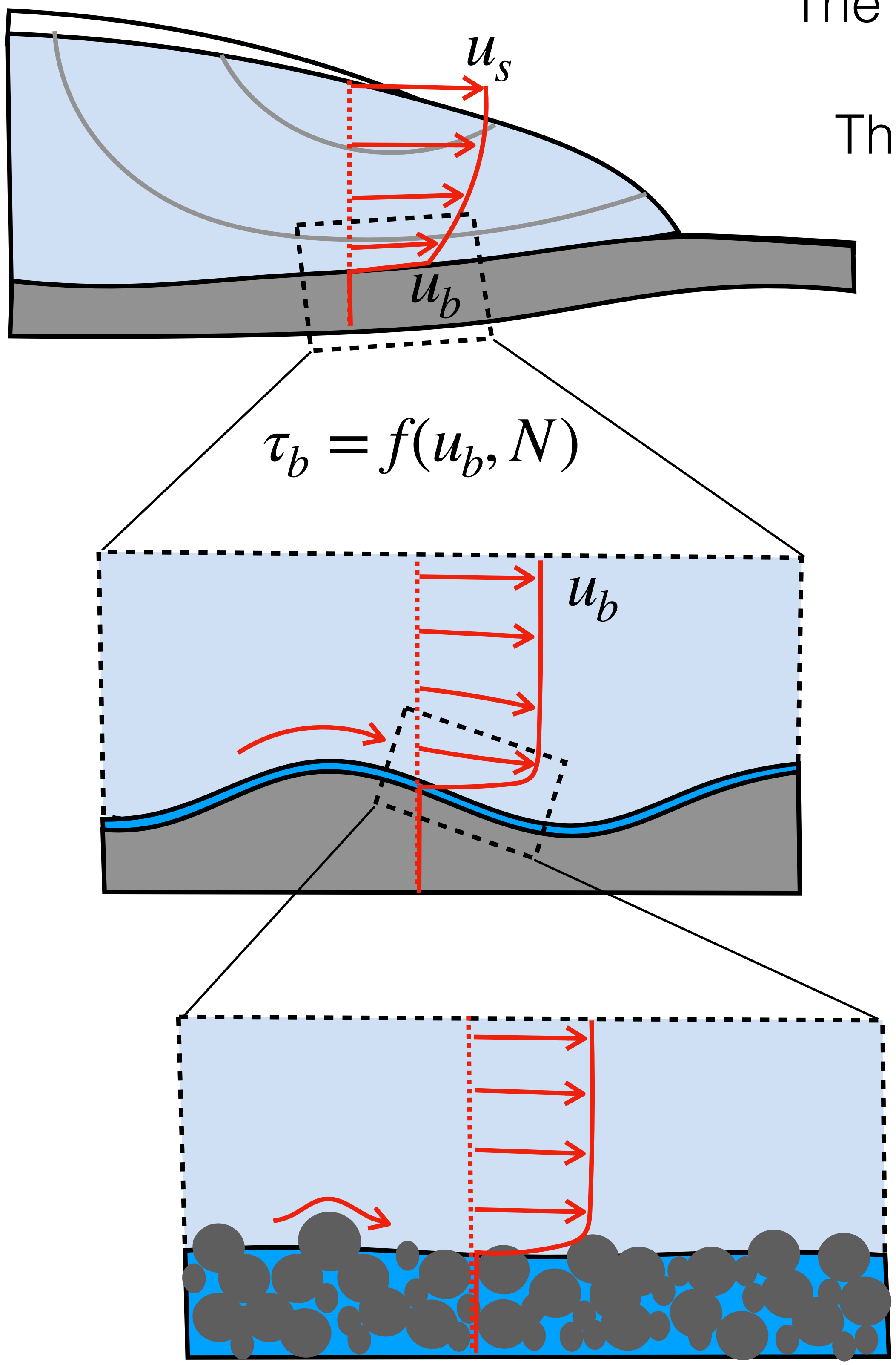
The usual 'steady-state' sliding law relates **shear stress** and **sliding speed** for given **effective pressure** or given **cavity volumes.**



Forced **fluctuations** in water pressure do not seem to result in too large perturbations from the steady-state law.

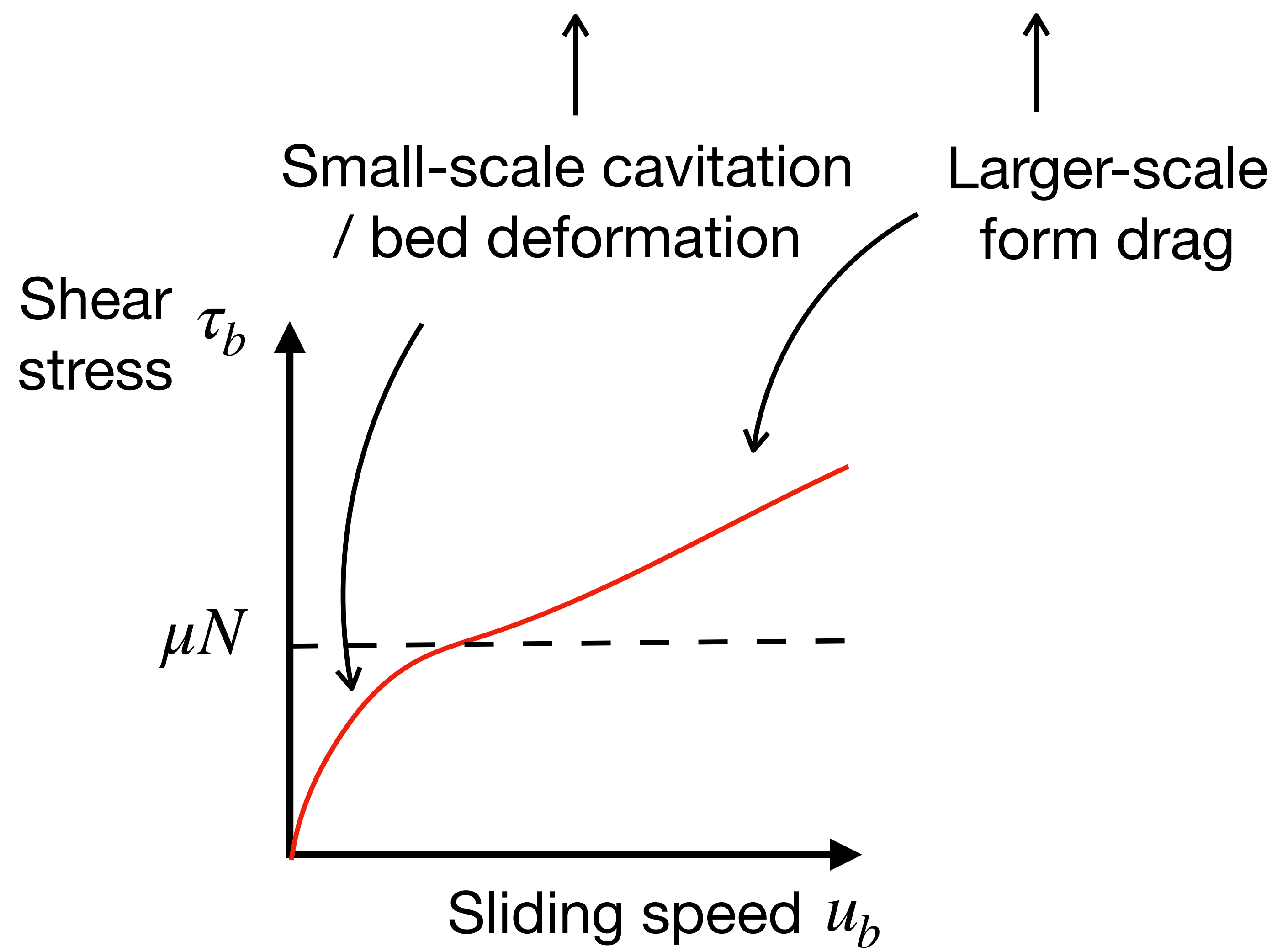
However, many cavities may be hydraulically isolated, and are **constrained by cavity volume**, rather than effective pressure. This acts as a 'brake' on surface-meltwater-driven velocity changes.

The importance of 'form drag'



The sliding law in numerical models needs to account for **all sub-grid scale 'roughness'**.
That often includes larger scales than those for which cavitation / bed deformation are relevant.

$$\tau_b = \mu N \left(\frac{u_b}{u_b + \lambda N^n} \right)^{1/n} + C u_b^{1/n}$$



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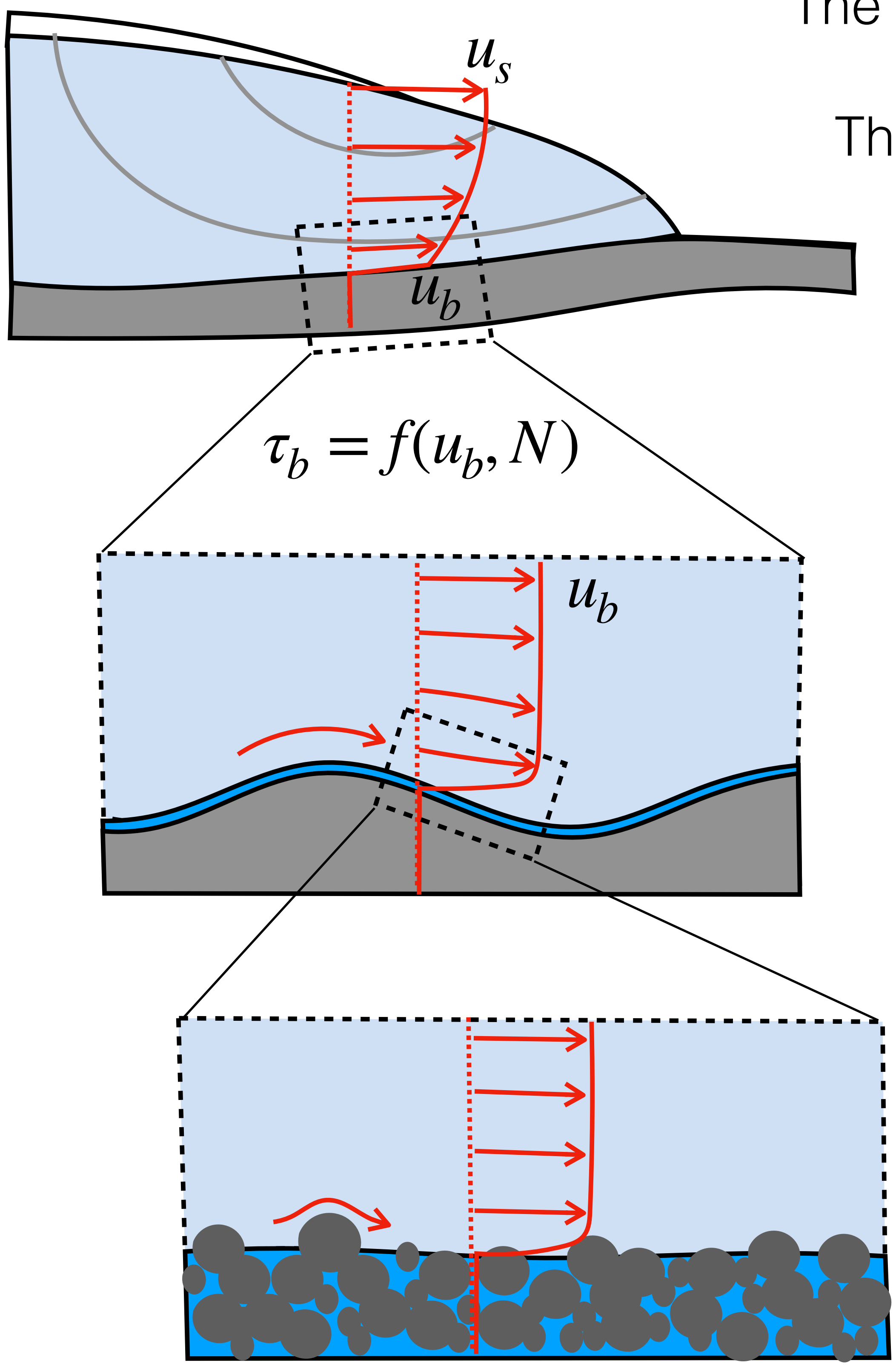

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