

## 5. The symbol $\aleph$

*The Greek letters  $\zeta$  and  $\xi$   
Caused a young Oxford student to sigh,  
“All those  $p$ 's and those  $\rho$ 's  
Only add to my woes.  
And don't mention  $X$ ,  $\aleph$ , and  $\chi$ .”*

### 5.1. How does one write it?

Over the years I have seen many varied attempts to write the symbol  $\aleph$ , and this has convinced me that people experience some difficulty in knowing where to start. I think there are two main relevant points:

(1) If you are writing Hebrew, it doesn't matter that much exactly how you write  $\aleph$ , because no other letter of the Hebrew alphabet has even approximately the same shape.

(2) In mathematics, the vitally important thing is to make sure that whatever you write is easily distinguishable from other similar symbols, such as  $X$ ,  $K$ ,  $N$ ,  $x$ ,  $\kappa$ ,  $\chi$  and so on. Provided that is the case, one can go easy on the number of twiddles etc.

If one is reading Hebrew, the letter  $\aleph$  is easily recognisable by being roughly cross-shaped. Other features of the letter are to some extent analogous to serifs on Roman letters; they are unnecessary but nonetheless make it even easier to recognise the character.\* The font mostly used in this document is, I suspect, designed to imitate characters written with an italic pen whose nib is held horizontally in the right hand and is drawn, on the whole, from left to right (notwithstanding the fact that Hebrew script runs from right to left).

Aleph in some other Hebrew fonts:  $\aleph\aleph\aleph$

### 5.2. Why this letter in particular?

This is not really a mathematical question, and I don't know the answer for sure. It has been suggested† that the notation was chosen because  $\aleph$  is the initial letter of the Hebrew phrase  $\text{אין סוף}$  (*ein soph*) meaning “without end” ( $\aleph$  representing the glottal stop at the beginning of the first word). Whether that's true or not, the choice of symbol is very convenient, because it's instantly recognisable in a piece of mathematical text. It's a distinctive symbol for a unique and important concept. That can't be bad.‡

### 5.3. It sounds like $\alpha$ but doesn't look like it

This isn't mathematical either, but I'll address it anyway.

The answer is very simple: the Greek and Hebrew alphabets have a common origin.

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\* So it is not necessary to do what one student did whose work I remember marking, and end each of the four arms of the letter with three or four oscillations, like a resistor in a circuit diagram. That work must have taken hours to write out.

† See A. Aczel, *The Mystery of the Aleph*.

‡ And it's possible that this is Cantor's major reason for choosing the notation; see page 179 of Dauben's excellent *Georg Cantor*.

It goes roughly like this.\* It seems (from discoveries made in the last few years†) that the alphabet was invented in Egypt, in imitation of Egyptian hieroglyphics, about 2000BC, to write a Semitic language related to Hebrew. The symbols were pictures of animals or everyday objects, and each one represented the first letter of the corresponding word, so that a picture of a house stood for *b*, because that was the first sound of the word for “house”. Also, a picture of a bull’s head stood for a glottal stop, being the first sound of the word for “bull”‡§. This evolved into the symbol ⚡ in the Phoenician alphabet, which had the same value, and a name something like *aleph*. (The bull is facing to the left.) The same alphabet was used to write Hebrew, and is found particularly in inscriptions. A variety of it, better for writing fast when keeping records, evolved to write Aramaic, and this later replaced the original Hebrew alphabet, and became the modern Hebrew alphabet. (The old one is still used by the Samaritans.) In the modern Hebrew letter, the nose of the bull has been compressed to a line, and one horn has disappeared.

The Phoenician alphabet was adopted by the Greeks, who took over the names and the basic idea. The letter name became *alpha*, the symbol was changed to *A* (so that the bull now has no ears and is balanced on its horns), and the Greeks, not having a glottal stop in their language, used the letter for the vowel *a* instead, because it was the first sound in the name *alpha*. We get our own alphabet from a version of the Greek, though somewhere along the way the original letter names were lost.

#### 5.4. Other Hebrew letters in mathematics

Other letters of the Hebrew alphabet do not seem to be much used in mathematics. Perhaps perhaps part of the reason is that many mathematicians are not familiar with the

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\* For further details, consult one of any number of excellent books. Jensen’s *Sign, Symbol and Script* (Allen and Unwin, 1970) is a classic. John F. Healey’s *The Early Alphabet* (British Museum Publications) is more up to date and is a fantastic read.

† Various inscriptions, more or less enigmatic, have been found at Wadi el-Hol in Egypt; see [www.usc.edu/dept/LAS/wsrp/information/wadi\\_el\\_hol/](http://www.usc.edu/dept/LAS/wsrp/information/wadi_el_hol/).

‡ The system wasn’t strictly speaking an alphabet of the sort we are used to, since, like Egyptian writing, it didn’t represent vowels. Most early writing systems were based on words and syllables; a picture of an object could come to stand for the sound of that object, as if we used a picture of a rose to write *rows* and *rhos*, or a picture of an eye to write *I*. But in Egyptian, it seems that different forms of a word might have different vowels (as in English *man/men* or *sing/sang/sung*) so the Egyptians decided to ignore their vowels; or, equivalently, represented syllables or strings of syllables with the same consonants by using the same signs. (It is as if a picture of a man were used to represent both *man* and *men*, and later was also used to write *main*, *moan* and *mine*.) And the inventors of the alphabet did the same. For more about Egyptian, try Mark Collier’s *How to read Egyptian hieroglyphs* (British Museum Press, 1998), or A. Loprieno’s *Ancient Egyptian: a linguistic introduction* (Cambridge University Press, 1995). For more on the Semitic languages, see Hetzron, *The Semitic Languages*, Routledge 1997.

§ For those with a lot of time on their hands, Beatrix Potter’s *Peter Rabbit* has been translated into Ancient Egyptian. Blackwell’s had it in stock at one point. Or you could try Amazon.

Hebrew alphabet. But then, many of them are; and besides which, mathematicians are used to learning or even creating new symbols, such as  $\phi$ ,  $\nabla$ ,  $\exists$ ,  $\sqsubseteq$ , or  $\downarrow$ .

I suspect that the very importance of the concept symbolised by  $\aleph$  has been a deterrent. Maybe there is a feeling that any other Hebrew letter has to be used for a concept that is just as important; and such concepts are difficult to find. That having been said, the second and third letters,  $\beth$  (beth) and  $\gimel$  (gimel) are used in Set Theory; Cantor himself used the last letter  $\daleth$  (taw), and current implementations of  $\text{\TeX}$ , the typesetting system now all but universally used in mathematics, also provide the fourth letter  $\daleth$  (daleth). I have used this myself (with trepidation), but have not yet seen it used by anyone else.\*

The course website, [www.maths.ox.ac.uk/~knight/lectures/b1st.html](http://www.maths.ox.ac.uk/~knight/lectures/b1st.html), contains other information, relevant links, and announcements. If you haven't already, you might consider bookmarking it.

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